Craig Hane, Ph.D., Founder

Workforce Development:

Module 8

1.1 Lessons Abbreviation Key Table

C = Calculator Lesson
P = Pre-algebra Lesson
A = Algebra Lesson
G = Geometry Lesson
T = Trigonometry Lesson
S = Special Topics

The number following the letter is the Lesson Number.

E = Exercises with Answers: Answers are in brackets **[ ]**.
EA = Exercises Answers: (only used when answers are not on the same page as the exercises.)
ES = Exercises Supplemental: Complete if you feel you need additional problems to work.

1.2 Exercises Introduction

Why do the Exercises?

Mathematics is like a "game." The more you practice and play the game the better you will understand and play it.

The Foundation's Exercises, which accompany each lesson, are designed to reinforce the ideas presented to you in that lesson's video.

It is unlikely you will learn math very well by simply reading about it or listening to Dr. Del, or anyone else, or watching someone else doing it.

You WILL learn math by "doing math."

It is like learning to play a musical instrument, or write a book, or play a sport, or play chess, or cooking.

You will learn by practice.

Repetition is the key to mastery.

You will make mistakes. You will sometimes struggle to master a concept or technique. You may feel frustration sometimes “**WE ALL DO**.”

But, as you learn and do math, you will begin to find pleasure and enjoyment in it as you would in any worthwhile endeavor. Treat it like a sport or game.

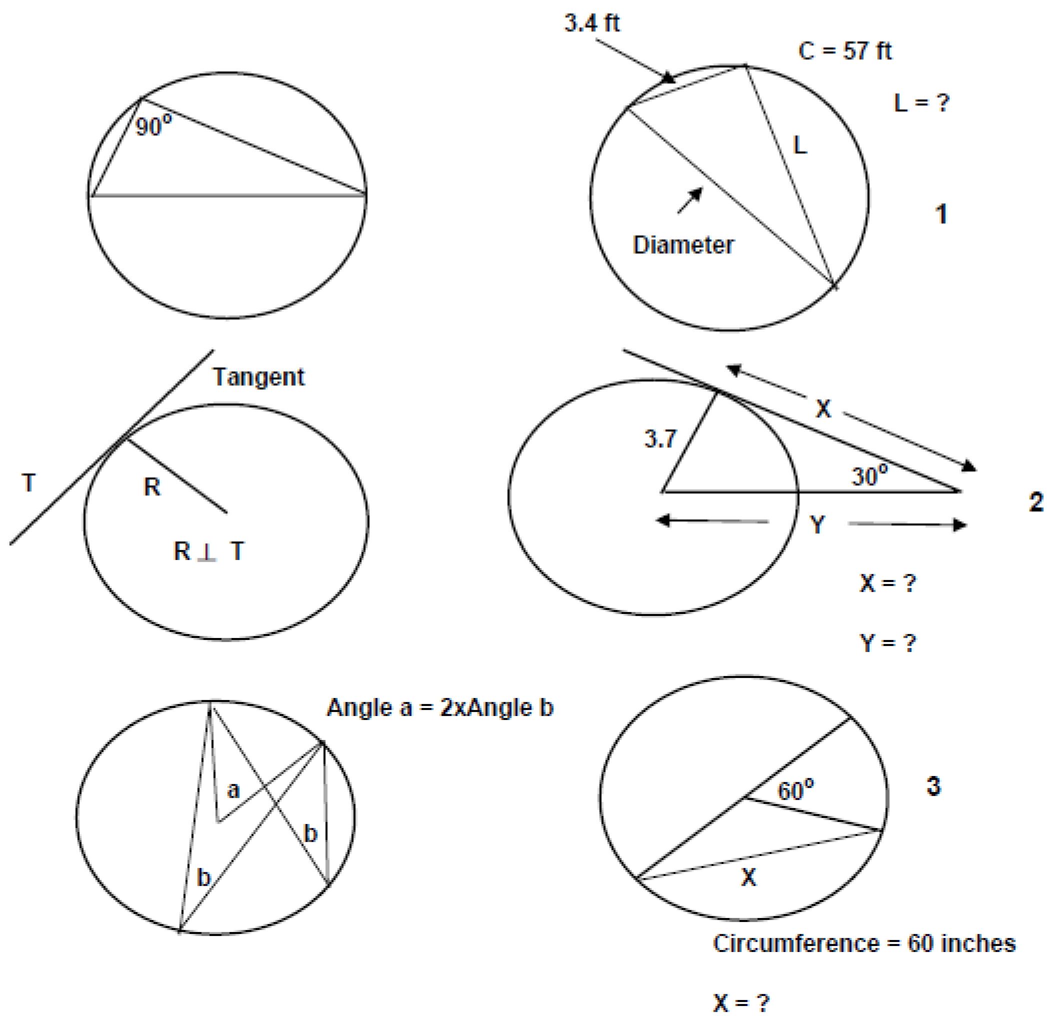
These exercises are the KEY to your SUCCESS!

ENJOY!

G12 LESSON: CIRCLES SPECIAL PROPERTIES

There are three facts about **circles** that I find useful sometimes in a practical problem.

I will present them to you with examples below:



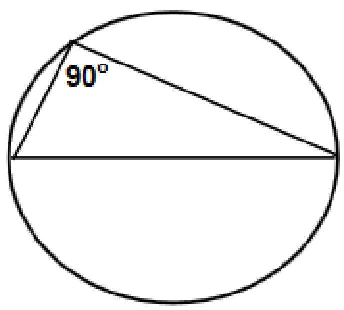
Answers: 1. 17.8 2. Y = 7.4, X = 6.4 3. X = 16.5

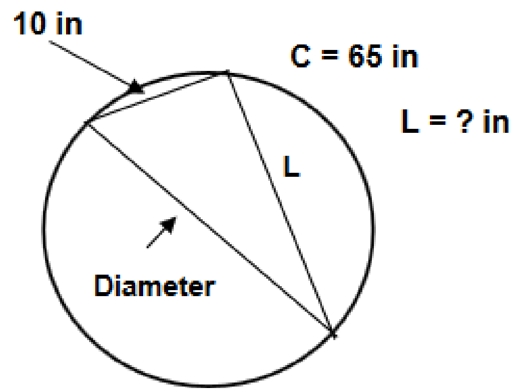
G12E

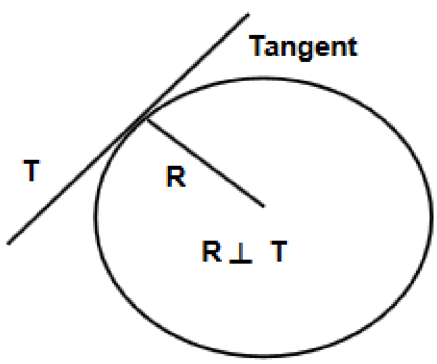
CIRCLES SPECIAL PROPERTIES

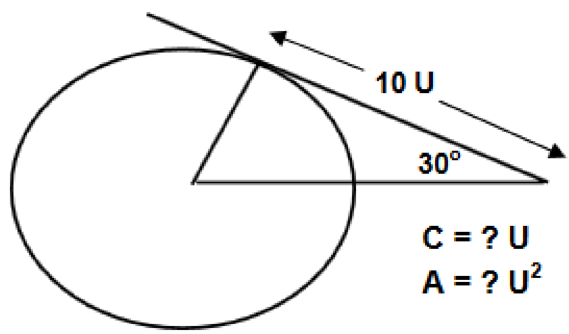
There are three facts about **circles** that I find useful sometimes in a practical problem.

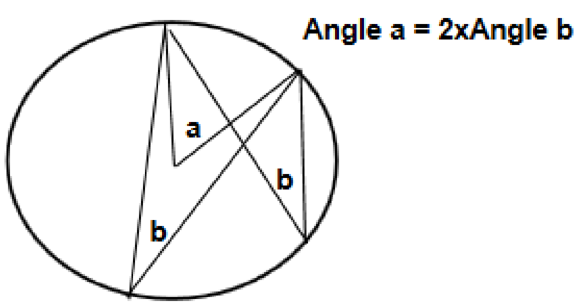
Find the Unknowns.

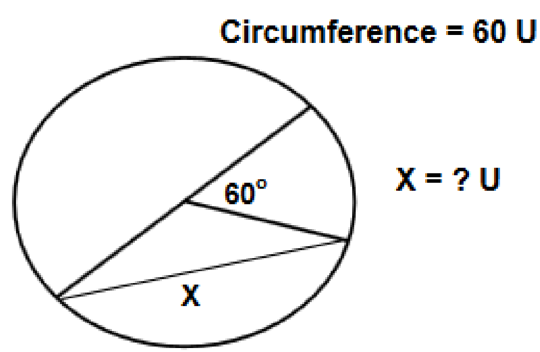










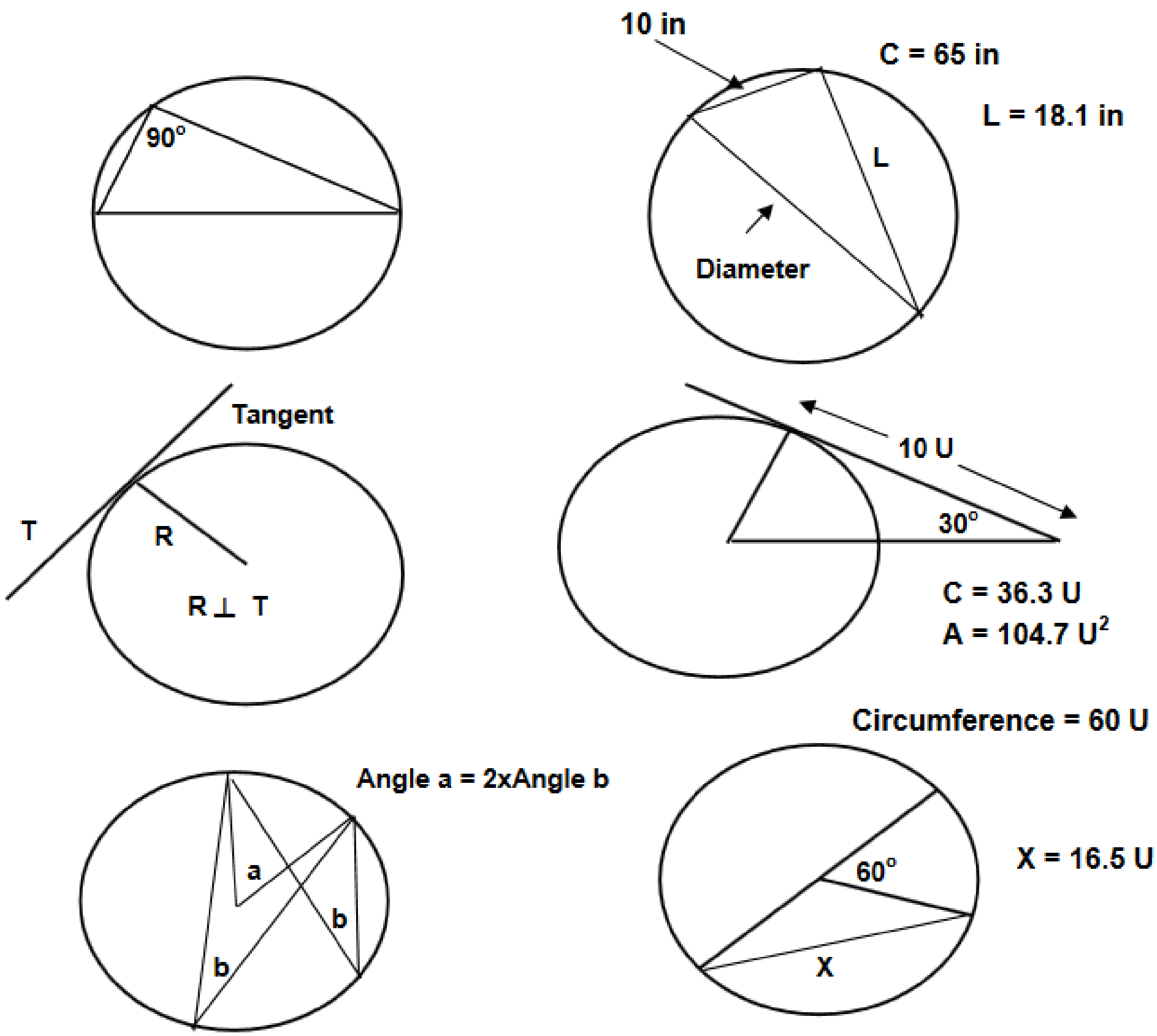


G12EA

CIRCLES SPECIAL PROPERTIES

There are three facts about **circles** that I find useful sometimes in a practical problem.

Find the Unknowns.

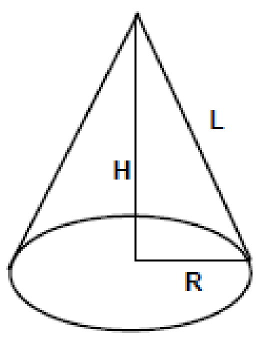


G14 LESSON: SURFACE AREAS CONES

If a **Cone** has **Radius**, **R**, for its **Base** and has **Height**, **H**, and **Length**, **L**, then its **Surface Area** consist of the area of the **Base** plus its **Lateral Area**.

Base Area = πR^2 and **Lateral Area** = $\pi RL = \pi R\sqrt{R^2 + H^2}$

Total Area = $\pi R(R + L)$ measured in U^2

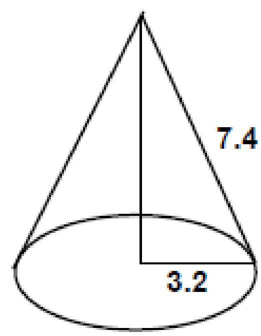


Base Area = πR^2

Lateral Area = πRL

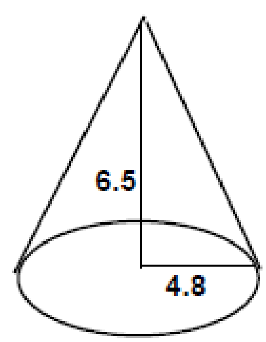
Total Area = $\pi R(R + L)$

1



Find Base, Lateral, Total Areas

2



Find Base, Lateral, Total Areas

Answers: 1. 32.2, 74.4, 106.6 U2 **2. 72.4, 121.8, 194.2 U^2**

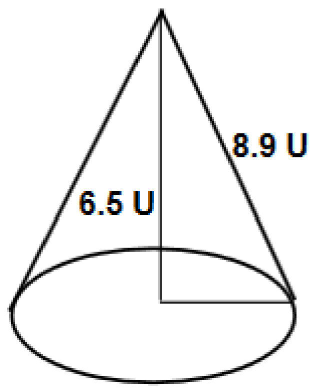
G14E

SURFACE AREAS   CONES

Find the **Base Area, BA**

Find the **Lateral Area, LA**

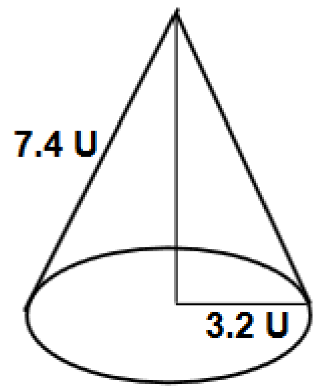
Find the **Total Area, TA**



BA = ? U^2

LA = ? U^2

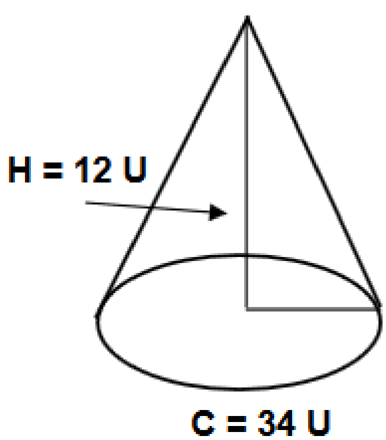
TA = ? U^2



BA = ? U^2

LA = ? U^2

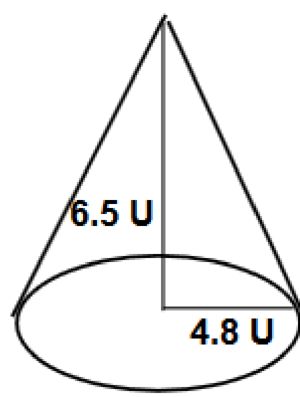
TA = ? U^2



BA = ? U^2

LA = ? U^2

TA = ? U^2



BA = ? U^2

LA = ? U^2

TA = ? U^2

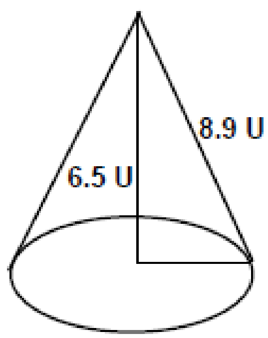
G14EA

SURFACE AREAS CONES

Find the **Base Area, BA**

Find the **Lateral Area, LA**

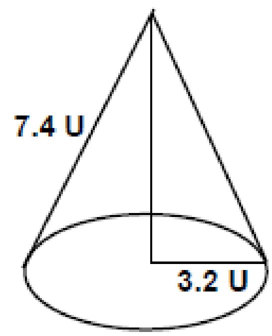
Find the **Total Area, TA**



BA = 116 U^2

LA = 170 U^2

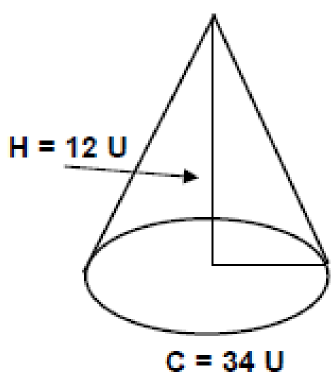
TA = 286 U^2



BA = 32.2 U^2

LA = 74.4 U^2

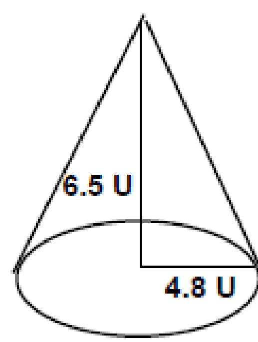
TA = 107 U^2



BA = 92 U^2

LA = 224 U^2

TA = 316 U^2



BA = 72.4 U^2

LA = 121.8 U^2

TA = 194 U^2

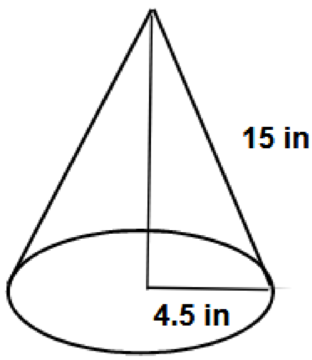
G14ES

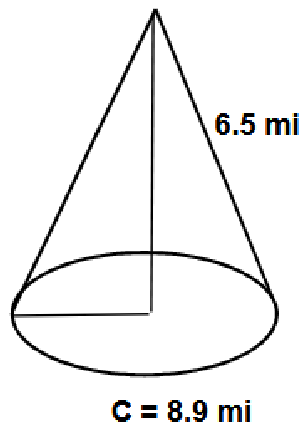
SURFACE AREAS CONES

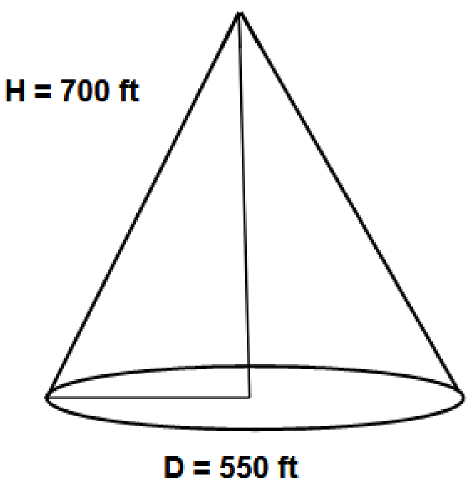
Find the **Base Area, BA**

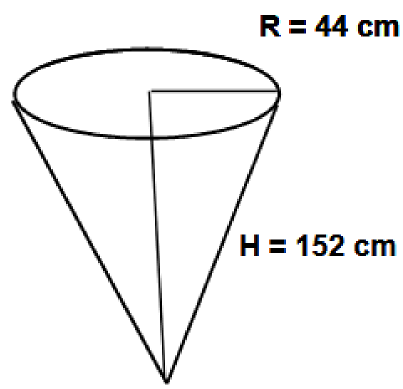
Find the **Lateral Area, LA**

Find the **Total Area, TA**









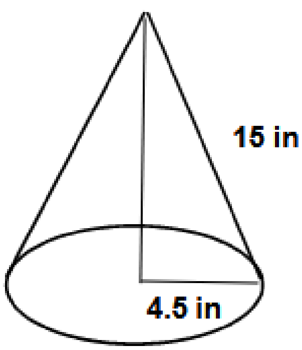
G14ESA

SURFACE AREAS CONES

Find the **Base Area, BA**

Find the **Lateral Area, LA**

Find the **Total Area, TA**



BA = 63.6 in^2

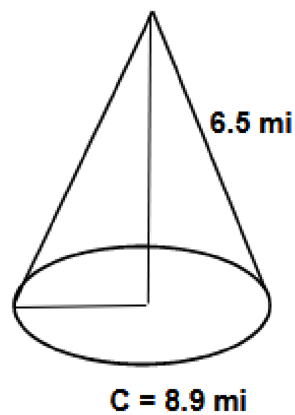
LA= 212.1 in^2

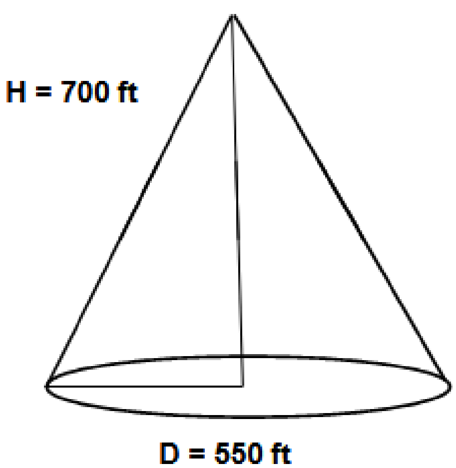
TA = 275.7 in^2

BA = 6.3 mi^2

LA= 28.9 mi^2

TA = 35.2 in^2

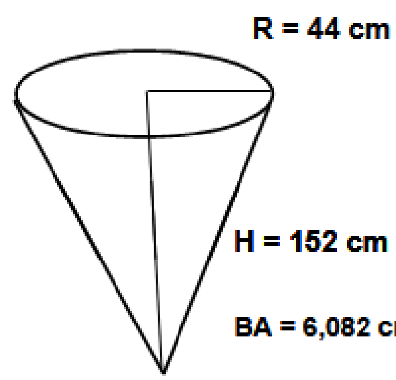




BA = 159,043 ft^2

LA= 635,229 ft^2

TA = 794,272 ft^2



BA = 6,082 cm^2

LA= 21,874 in^2

TA = 27,956 cm^2

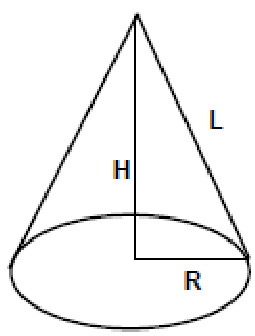
G16 LESSON: VOLUME CONES

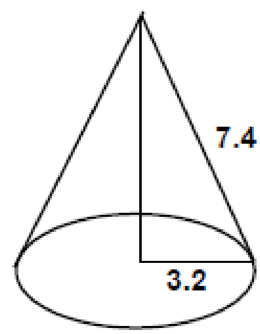
If a **Cone** has **Radius**, **R**, for its **Base** and has **Height**, **H**, and **Length**, **L**, then its **Volume**, **V**, is:

Base Area = πR^2 and **V** = $(1/3)\pi R^2 H = (1/3)\pi R^2\sqrt{L^2 - R^2}$

Volume is measured in **Cubic Units**, U^3, where **U** is a **linear measure**.

For example: **cubic inches**, in^3



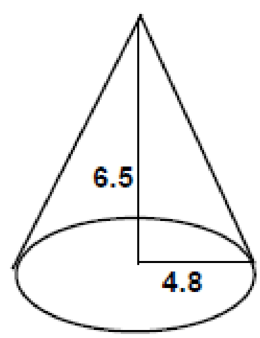


1

Find Volume

Volume = $(1/3)\pi R^2 H$

Volume = $(1/3)\pi R^2\sqrt{L^2 - R^2}$



2

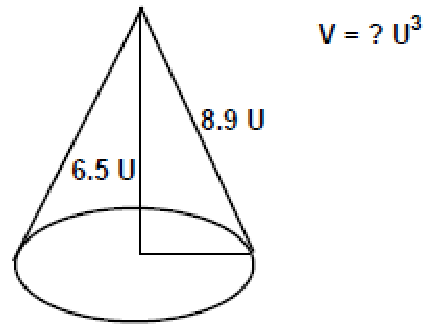
Find Volume

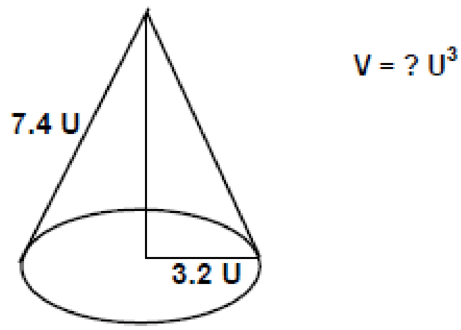
Answers: 1. 71.5 U3 **2. 156.8 cubic units or U^3**

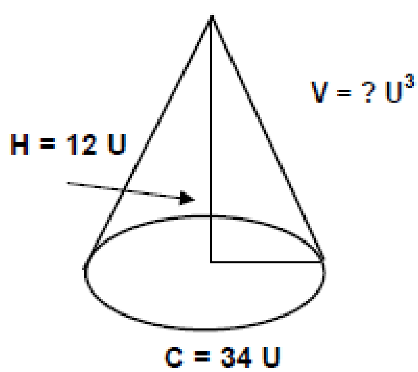
G16E

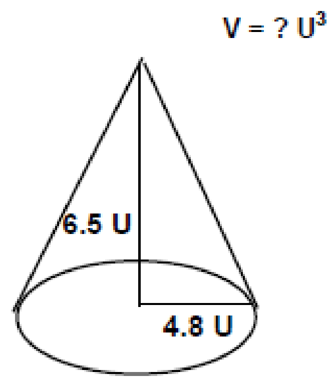
VOLUMES CONES

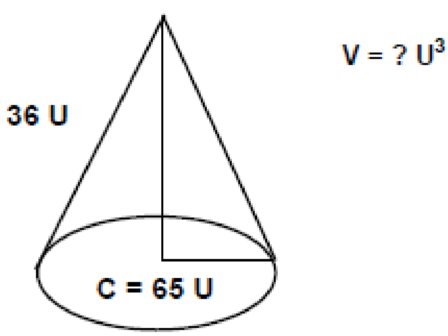
Find the **Volume**, in U^3

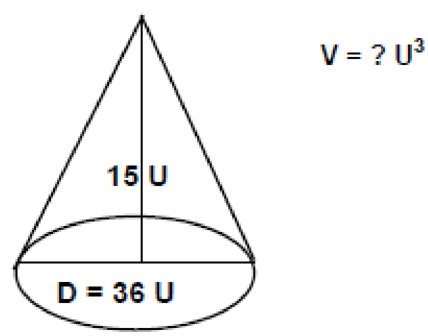








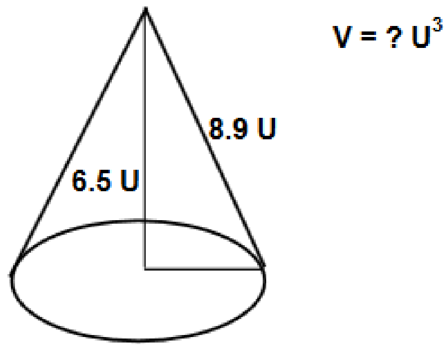


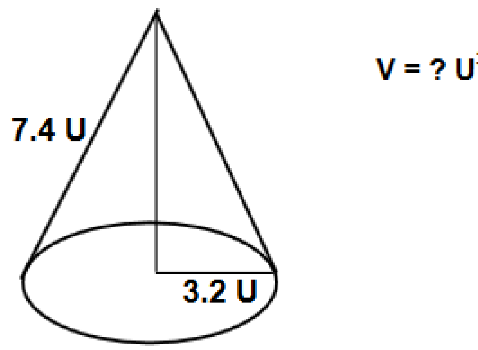


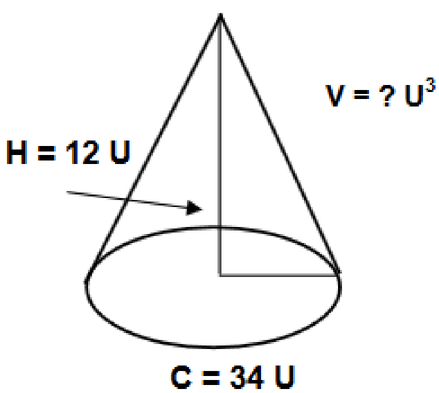
G16EA

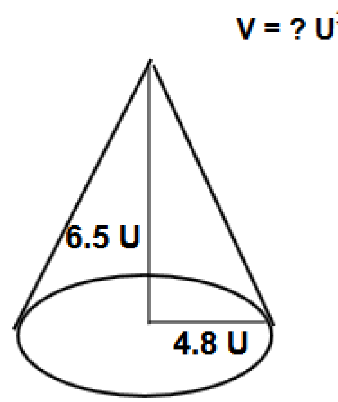
VOLUMES CONES

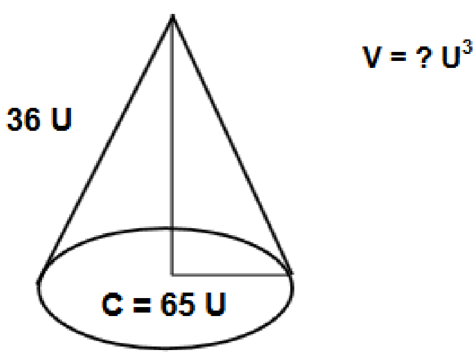
Find the **Volume**, in U^3

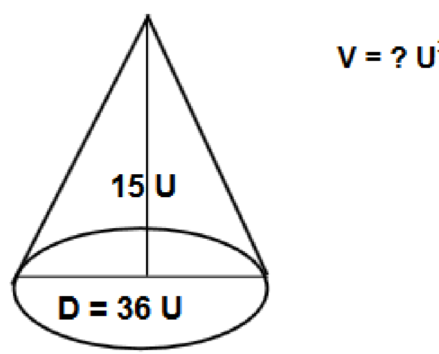








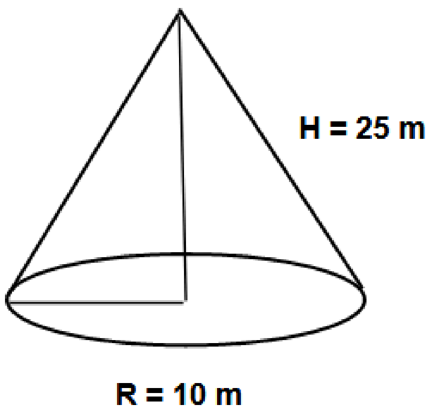


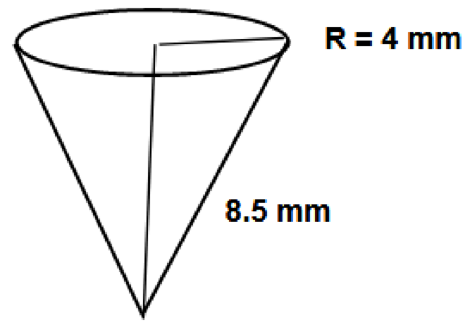


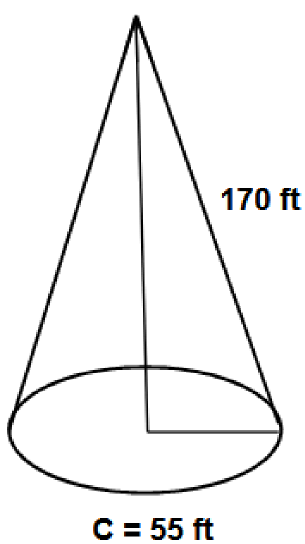
G16ES

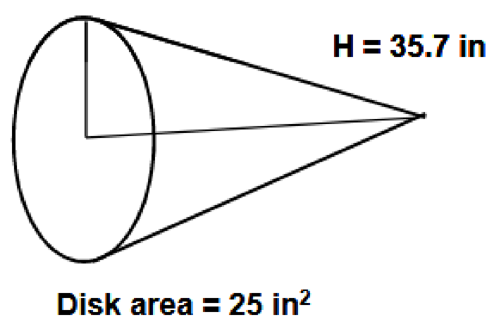
VOLUMES CONES

Find the volume.





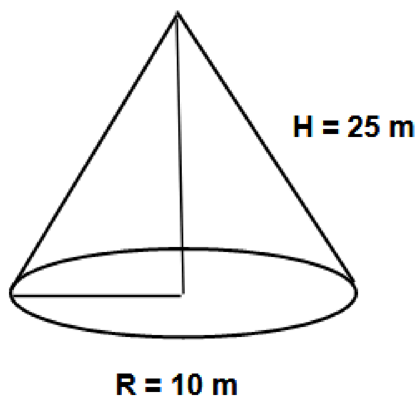




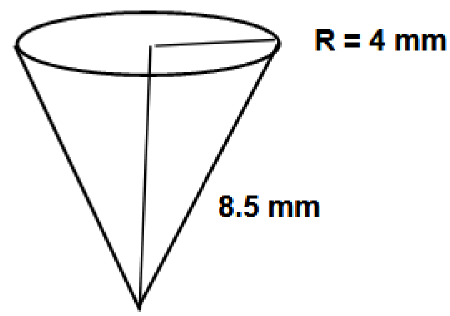
G16ESA

VOLUMES CONES

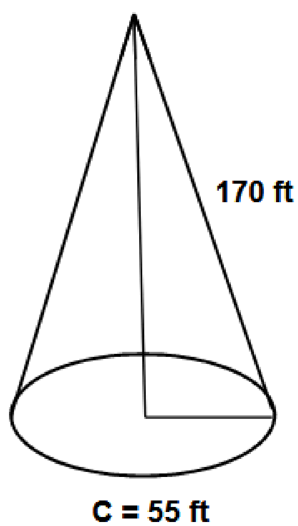
Find the volume.



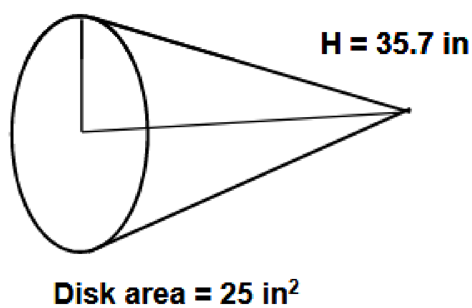
V = 2618 m^3



V = 125.7 mm^3



V = 13,623 ft^3



V = 297.5 in^3

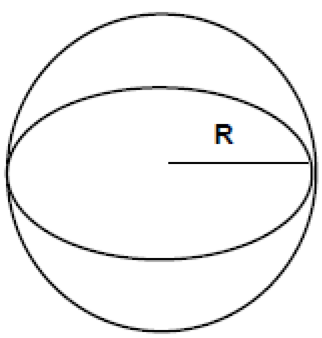
G17 LESSON: SURFACE AREA BALL OR SPHERE

The **Surface Area** of a **Sphere** with **Radius**, **R**, in **Linear Units, U**, is:

$$A = 4\pi R^2 \text{ Square Units, } U^2$$

This is very difficult to prove and we will defer that proof until Tier 4. However, I remember it as follows:

The **Area** of the **circle** of the **cross section** of the **Sphere** through its center is πR^2. I imagine it is rubber and we blow it up like a domed tent. Then its **Area** doubles and that is a **hemisphere** of **Surface Area** $2\pi R^2$. So, the whole **Sphere** is double this, or $4\pi R^2$ U^2.



$A = 4\pi R^2$

Problems

R = 5.2 ft A = 1

R = 150 mi A = 2

R = .035 cm A = 3

R = 1 3/4 ft A = 4

If the Surface Area of a Ball is to be 36 sq. in., what should its Radius be?

$4\pi R^2 = 36\ in^2$, then $R = \sqrt{36/4\pi} = 1.7$ inches

Answers: 1. 340 ft2 2. 282,750 mi^2 3. .0154 cm^2 4. 38.5 ft^2

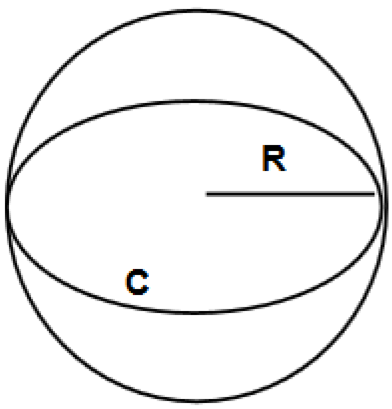
G17E

SURFACE AREA BALL OR SPHERE

Find the **Surface Area** of the **Spheres** or **Balls**.

What is the formula for the **Surface Area** of a **Sphere** with **Radius** R?

How do you remember it?



R = Radius
D = Diameter
C = Circumference

Exercises

R = 5.2 ft A = ?

R = 150 mi A = ?

R = .035 cm A = ?

R = 1 3/4 ft A = ?

C = 36 ft A = ?

C = 120 mi A = ?

C = 45/8 in A = ?

D = .025 cm A = ?

D = 68 in A = ?

If the Surface Area of a Ball is to be 36 sq. in., what should its Radius be?

G17 EA

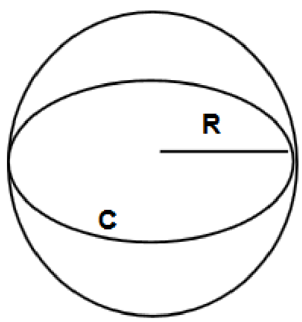
SURFACE AREA BALL OR SPHERE Answers: []

Find the **Surface Area** of the **Spheres** or **Balls**.

What is the formula for the Surface Area of a Sphere with Radius R? [**$4\pi R^2$**]

What's one way you can remember it? [**The Cross Section of the Ball is a circle of Radius R and Area πR^2.**]

Now, imagine blowing this up like it's rubber until each point is R from the center. [**Turns out the surface area is exactly …thus, Hemisphere area is $2\pi R^2$**]



R = Radius
D = Diameter
C = Circumference

Exercises

R = 5.2 ft	**A = 340 ft^2**
R = 150 mi	**A = 282,743 mi^2**
R = .035 cm	**A = .015 cm^2**
R = 1 3/4 ft	**A = 38.5 ft^2**
C = 36 ft	**A = 412.5 ft^2**
C = 120 mi	**A = 4,584 mi^2**
C = 45/8 in	**A = 6.8 in^2**
D = .025 cm	**A =002 cm^2**
D = 68 in	**A = 14,527 in^2**

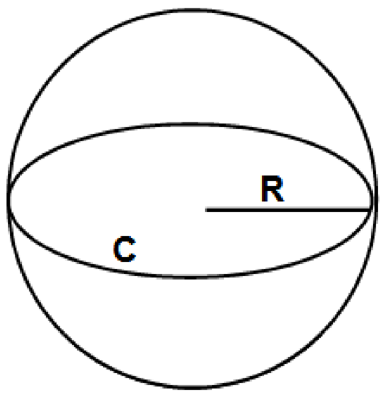
If the Surface Area of a Ball is to be 36 sq. in., what should its Radius be? 1.7 in

G17 ES

SURFACE AREA BALL OR SPHERE

Find the surface area for the spheres with the dimensions below.

1.) Recall the formula for surface area for a sphere, $SA = 4\pi R^2$. If the radious doubled, how much would the SA change? What about if the radius was halved?



2.) R = 35 cm

3.) R = 389 mi

4.) D = 12.6 mm

5.) C = 200,209 km

6.) C = 4π ft

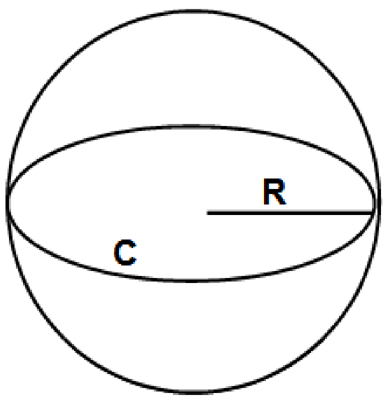
G17 ESA

SURFACE AREA BALL OR SPHERE

Find the surface area for the spheres with the dimensions below.

1.) Recall the formula for surface area for a sphere, $SA = 4\pi R^2$. If the radious doubled, how much would the SA change? What about if the radius was halved?

Answer: Because the radius is squared, doubling it would cause a 4x increase in surface area. Conversely, halving the radius would result in 4x less surface area.



2.) R = 35 cm
$SA = 15{,}394\ cm^2$

3.) R = 389 mi
$SA = 1{,}901{,}556\ mi^2$

4.) D = 12.6 mm
$SA = 498.8\ mm^2$

5.) C = 200,209 km
$SA = 12{,}759{,}020{,}060\ km^2$

6.) $C = 4\pi$ ft
$SA = 50.3\ ft^2$

G18 LESSON: VOLUME BALL OR SPHERE ARCHIMEDE TOMBSTONE

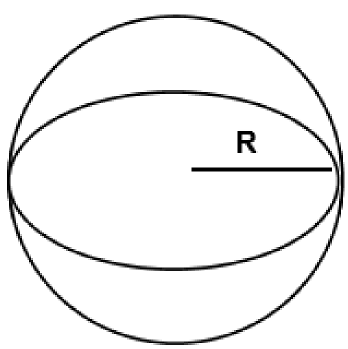
The **Volume** of a **Sphere** with **Radius**, **R**, in **linear units U**, is:

$$V = (4/3)\pi R^3 \text{ Cubic Units, } U^3$$

This is very difficult to prove and we will defer that proof until Tier 4. However, I remember it as follows:

> **Archimedes Tombstone:** Imagine a **Sphere** inscribed inside a **Cylinder**. The **Ratio** of the **Volume** or the **Sphere** to the **Volume** of the **Cylinder** is 2:3
>
> The **Cylinder** will have **Base Radius R** and Height **2R**.
> Thus, its **Volume** will be $\pi R^2 \times 2R = 2\pi R^3$
> The **Volume** of the **Sphere** is thus, $(2/3)\times 2\pi R^3 = (4/3)\pi R^3$

Note: I say "**triangle**" three times instead of "**tombstone.**"



$A = 4\pi R^2$

$V = (4/3)\pi R^3$

Problems

R = 5.2 ft	V =	1
R = 150 mi	V =	2
R = .035 cm	V =	3
R = 1 3/4 ft	V =	4

If the Volume of a Ball is to be 36 cu. in., what should its Radius be?

(4/3)πR3 = 36 in3, then R = $\sqrt[3]{36 \times 3/4\pi}$ = 2.05 inches

Answers 1. 589 ft^3 2. 14,137,000 mi^3 3. .00018 cm^3 4. 22.4 ft^3

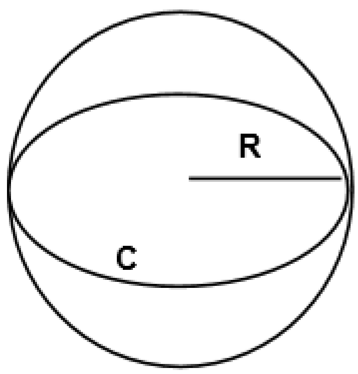
G18E

VOLUME   BALL OR SPHERE

Find the **Volume** of the **Spheres** or **Balls**.

What is the formula for the **Volume** of a **Sphere** with **Radius R**?

What's one way you can remember it?



R = Radius
D = Diameter
C = Circumference

Exercises

R = 5.2 ft   V = ?

R = 150 mi   V = ?

R = .035 cm   V = ?

R = 1 3/4 ft   V = ?

C = 36 ft   V = ?

C = 120 mi   V = ?

C = 45/8 in   V = ?

D = .025 cm   V = ?

D = 68 in   V = ?

If the Volume of a Ball is to be 100 cu. in., what should its Radius be?

G18 EA

VOLUME BALL OR SPHERE Answers: []

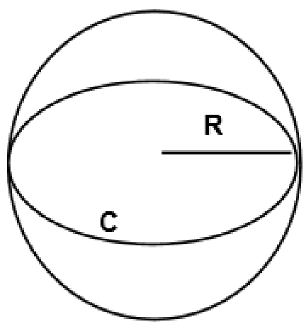
Find the **Volume** of the **Spheres** or **Balls**.

What is the formula for the **Volume** of a **Sphere** with **Radius R**?

[(4/3) πR^3]

What's one way you can remember it?

[Archimedes Tombstone formula whereby the Volume of the Sphere is 2/3 the Volume of a Cylinder the Sphere is inscribed in $(2/3) \times \pi R^2 \times 2R = (4/3)\,\pi R^3$]



R = Radius
D = Diameter
C = Circumference

Exercises

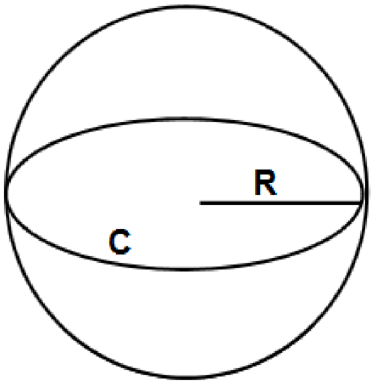
R = 5.2 ft	V = 589 ft^3
R = 150 mi	V = 14,137,167 mi^3
R = .035 cm	V = .00018 cm^3
R = 1 3/4 ft	V = 22.4 ft^3
C = 36 ft	V = 788 ft^3
C = 120 mi	V = 29,181 mi^2
C = 45/8 in	V = 1.67 in^3
D = .025 cm	V = .0000082 cm^3
D = 68 in	V = 164,636 in^3

If the Volume of a Ball is to be 100 cu. in., what should its Radius be? 2.88 in

G18 ES

VOLUME   BALL OR SPHERE

1.) Recall the formula for volume of a sphere, $V = (4/3)\pi R^3$. If the radious doubled, how much would the V change? What about if the radius was halved?



2.) R = 17 in

3.) R = 2.5 mm

4.) D = 25000 mi

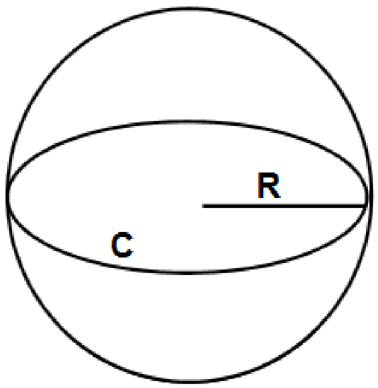
5.) C = 40 km

6.) C = 2π

VOLUME   BALL OR SPHERE

1.) Recall the formula for volume of a sphere, $V = (4/3)\pi R^3$. If the radious doubled, how much would the V change? What about if the radius was halved?

Answer: Because the radius is cubed, increasing it by a factor of 2 would increase the volume by a factor of 8. Conversely, halving the radius would reduce the volume by a factor of 8.



2.) R = 17 in
$V = 20{,}579.5\ in^3$

3.) R = 2.5 mm
$V = 65.4\ mm^3$

4.) D = 300 mi
$V = 113{,}097{,}336\ mi^3$

5.) C = 40 km
$V = 1{,}039{,}030\ km^3$

6.) $C = 2\pi\ U$
$V = (4/3)\pi\ U^3 = 4.19\ U^3$

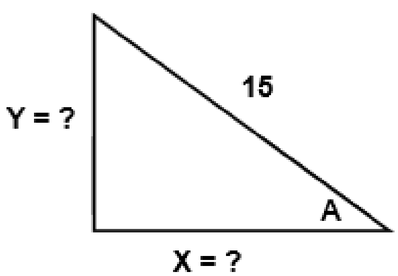
G19 LESSON: WHEN GEOMETRY IS NOT ENOUGH FOR TRIANGLES

We have learned to solve many practical problems using a combination of geometry and algebra. **Triangles** are the most common geometric figure we use in our models.

Yet, there are many practical problems involving **triangles** we still cannot solve with our current knowledge. This Lesson will point out some of these.

That's the "bad news." The "good news" is that we will be able to solve all of these problems using the tools we will learn in the last Section of the Foundation, Trigonometry.

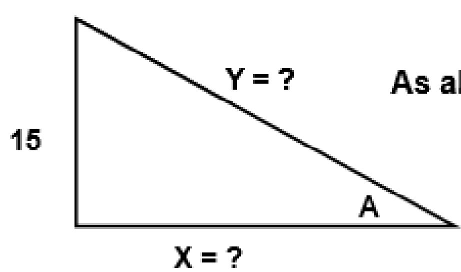
NOTE: See if you can catch the three times I use the word triangle instead of tombstone.



Problem: Find X and Y

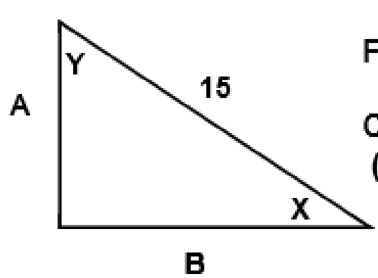
If A = 30^o or 45^o or 60^o we can solve this

With the tool of Trig, we can solve for any angle A.



As above, we can solve for A = 30^o or 45^o or 60^o

Trig will solve for all other angle A's

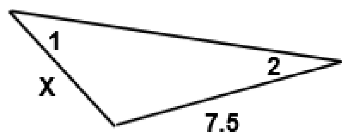


Find Angles X and Y given values of A or B.

Can Solve if A or B equals15 times (1/2) or (√2.2) of (√3/2), not otherwise, so far,

Trig solves for any A or B

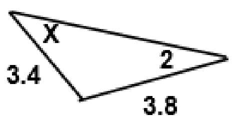
G19 When Geometry is Not Enough Problems



Find X No Similar Triangle available

Currently can only do for special values of 1 and 2

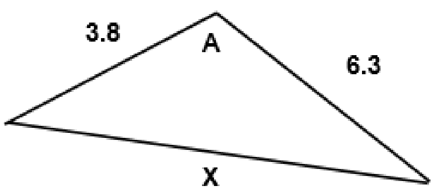
With Trig Tools can do for any angles 1 and 2



Can find X IF we have a similar triangle
with known corresponding sides

With Trig Tools Do Not need the similar triangle

Law of Sines



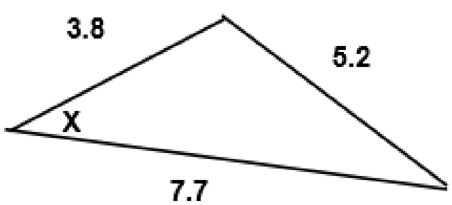
Find X

If A = 90^{o}, OK with Pythagorean Theorem

Trig Tool for any angle A.

Generalized Pythogorean Theorem

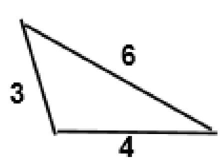
Law of Cosines



Find Angle X

Same Trig Tool as above

Useful in finding area of this triangle



Find the Area of this Triangle

Trigonometry has many profound applications beyond practical math.

G19E

WHEN GEOMETRY IS NOT ENOUGH FOR TRIANGLES

Give four examples of **triangle** "problems" we cannot yet solve with just the geometry and algebra we have learned, but which we will be able to solve with Trig.

This is an Optional Exercise.

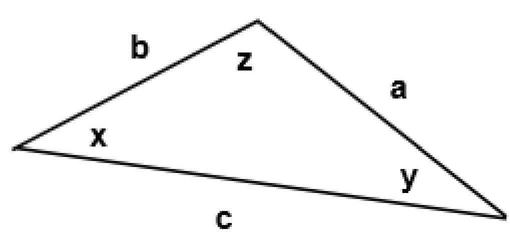
It is designed to help you appreciate the value the powerful Tool of Trigonometry will be for practical problem solving.

Before the scientific calculator was invented, Trig was pretty difficult to learn and apply to practical math.

Now, it is breeze. Aren't Power Tools wonderful?

HINT: Just imagine you know three of the variables below. Then can you find the others? With what you know now?

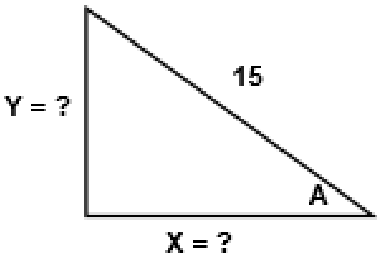
In many cases the answer will be NO. But, with Trig you will be able to solve any solvable triangle problem!



G19EA

WHEN GEOMETRY IS NOT ENOUGH FOR TRIANGLES

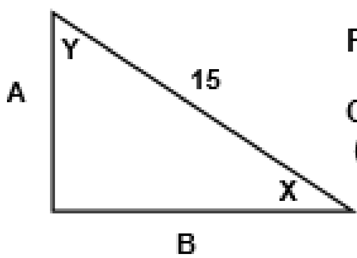
Give four examples of **triangle** "problems" we cannot yet solve with just geometry and algebra we have learned; but, which we will be able to solve with Trig.



Problem: Find X and Y

If A = 30° or 45° or 60° we can solve this now

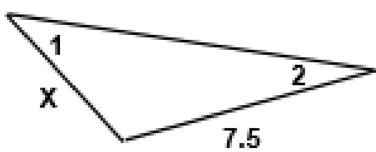
With the tool of Trig, we can solve for any angle A.



Find Angles X and Y given values of A or B.

Can Solve if A or B equals15 times
(1/2) or ($\sqrt{}$2.2) of ($\sqrt{}$3/2), not otherwise, so far.

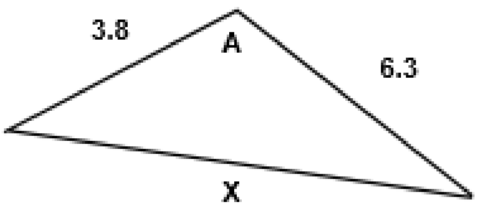
Trig solves for any A or B



Find X No Similar Triangle available

Currently can only do for special values of 1 and 2

With Trig Tools can do for any angles 1 and 2
Law of Sines



Find X or A given the other.

If A = 90°, OK with Pythagorean Theorem

Trig Tool for any angle A.

Generalized Pythogorean Theorem
Law of Cosines