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Workforce Development:

Basic, Intermediate, and Advanced Math for Industry

1.1 Lessons Abbreviation Key Table

C = Calculator Lesson
P = Pre-algebra Lesson
A = Algebra Lesson
G = Geometry Lesson
T = Trigonometry Lesson
S = Special Topics

The number following the letter is the Lesson Number.

E = Exercises with Answers: Answers are in brackets **[ ]**.
EA = Exercises Answers: (only used when answers are not on the same page as the exercises.)
ES = Exercises Supplemental: Complete if you feel you need additional problems to work.

1.2 Exercises Introduction

Why do the Exercises?

Mathematics is like a "game." The more you practice and play the game the better you will understand and play it.

The Foundation's Exercises, which accompany each lesson, are designed to reinforce the ideas presented to you in that lesson's video.

It is unlikely you will learn math very well by simply reading about it or listening to Dr. Del, or anyone else, or watching someone else doing it.

You WILL learn math by "doing math."

It is like learning to play a musical instrument, or write a book, or play a sport, or play chess, or cooking.

You will learn by practice.

Repetition is the key to mastery.

You will make mistakes. You will sometimes struggle to master a concept or technique. You may feel frustration sometimes "**WE ALL DO**."

But, as you learn and do math, you will begin to find pleasure and enjoyment in it as you would in any worthwhile endeavor. Treat it like a sport or game.

These exercises are the KEY to your SUCCESS!

ENJOY!

TI-30Xa INTRODUCTION

The TI-30Xa Scientific Calculator is very good for Practical Mathematics. We have chosen this model for its ease of use and low cost. You may use another calculator, but be aware that they all have different key positions and work somewhat differently.

This series of lessons will explain the various basic functions and processes we will be using in the Fundamentals Course.

Each lesson will consist of a video explanation of the lesson's topic and homework to reinforce the lesson.

After you have mastered the topic you may take a quiz to prove your mastery of the topic. It is best to master each topic sequentially since later topics may depend on previous topics.

IMPORTANT: Mathematics is like a "contact sport." You must play and practice to master the necessary skills and knowledge.

Most people find mathematics like a game whereby knowledge and skills are acquired over time with practice and study.

Treat it like a game. Have fun! Do not be discouraged by mistakes or setbacks. That is part of the game.

Your learning will be cumulative. You will notice that things that seem difficult today will become easy tomorrow.

C1 LESSON: ON/OFF FIX DEG M1 M2 M3

TI-30Xa is the "Power Tool" we will be using.

Keys will be underlined. There are 40 Keys.

36 of these Keys have a dual function indicated in yellow above the key, and reached by the Yellow 2nd Key

On/C is the On and Clear Key: Upper Right

OFF is the Off Key: Row 1 Column 5

In the Display at top of calculator:

M1 M2 M3 are the memory indicators (top left - Lesson C5)

DEG is angle indicator (Lesson C12)

FIX indicates you have fixed the number of digits that appear after the decimal point. It is located above the decimal point at bottom.

Nine digits is the default when you turn on the calculator.

A good practice is to turn the calculator OFF between calculations. Numbers stored in Memory, M1, M2, M3 will not be lost.

Take the C1 Quiz when you are ready.

C1E

ON/OFF FIX DEG M1 M2 M3

1. TI-30Xa is a P _ _ _ _ T _ _ _ of math?
2. The TI-30Xa has how many keys?
3. How many of these keys are dual function?
4. You activate a dual function with which key?
5. The ON/C key does what?
6. Where is the ON/C key?
7. Where is the OFF key?
8. How many Memory registers are there in the TI-30Xa?
9. Where is their indicator in the Display?
10. What does the DEG indicate in the display?
11. Where is the FIX function, and what does it do?
12. How do you display “n” digits after the decimal point?

Answers are on C1EA, page 9.

C1EA

ON/OFF FIX DEG M1 M2 Answers: [ ]'s

1. TI-30Xa is a P _ _ _ _ T _ _ _ of math? [**Power Tool**]

2. The TI-30Xa has how many keys? [**40**]

3. How many of these keys are dual function? [**36**]

4. You activate a dual function with which key?
   [**Yellow "2ND" Key in upper left corner.**]

5. The ON/C key does what?
   [**Turns TI-30Xa on and Clears the registers, and sets DEG. It does not change memory.**]

6. Where is the ON/C key? [**Upper Right Corner**]

7. Where is the OFF key? [**Below the ON/C Key**]

8. How many Memory registers are there in the TI-30Xa?
   [**Three, M1, M2, M3**]

9. Where is their indicator in the Display? [**Upper Left**]

10. What does the DEG indicate in the display?
    [**Angles will be entered in degrees**]

11. Where is the FIX function, and what does it do?
    [**Above the decimal point at bottom. It fixes the number of digits displayed after the decimal point.**]

12. How do you display n digits after the decimal point? [**2nd FIX n**]

C2 LESSON: REAL NUMBERS: ADD + SUBTRACT - EQUAL =

We assume you know basic arithmetic operations and rules. If not, you will need some more basic training.

Key k is indicated by **k** the underline.

The = Key is used to complete a calculation.

Addition + Key adds two numbers 3 + 4 = 7

Subtraction - Key subtracts numbers 7 - 2 = 5

Negative numbers will be discussed in Lesson 3

The TI-30Xa will take care of decimal locations.

12.3 + 7.5 = 19.8   12.3 + 7.05 = 19.35

Practice makes perfect!

The calculator is also a very good tool to help you learn the addition or multiplication tables.

And also, to help you learn to do approximate calculations which are a good idea to do a "quick check" for mistakes.

The more you "play" with it…the better you'll get!

C2E

ADD + SUBTRACT - EQUAL = Answers: [ ]'s

1. What key completes a calculation? [**=**]
2. Which key adds two numbers? [**+**]
3. Which key subtracts two numbers? [**-**]
4. 12.3 **+** 4.8 **=** ? [**17.1**]
5. 375 **+** 897 **=** ? [**1272**]
6. 0.075 **+** 0.0345 **=** ? [**0.1095**]
7. 87 **-** 39 **=** ? [**48**]
8. 12.34 - 7.05 = ? [**5.29**]
9. 0.0087 **-** 0.00032 **=** ? [**0.00838**]
10. 12 **+** 56 **+** 32 **+** 89 **=** ? [**189**]
11. 37 **-** 48 **=** ? [**-11**] (See C3 for Negative Numbers)
12. 3,879 **+** 7,425 **=** ? [**11,304**] (You supply commas)
13. 2.32 **+** 0.073 **=** ? [**2.39**]

Take the C2 Quiz if you are ready,
or do some more exercises, C2ES.

C2ES

ADD + SUBTRACT - EQUAL = Answers: [ ]'s

1. 17.3 + 234.8 + 3.7 = ? [**255.8**]
2. 37.5 + 8.97 = ? [**46.47 or 46.5**]
3. 0.175 + 0.0385 = ? [**0.214**]
4. 97 - 19 = ? [**78**]
5. 12.74 - 9.05 = ? [**3.69**]
6. 0.087 - 0.032 = ? [**0.055**]
7. 12 + 96 + 52 + 29 = ? [**189**]
8. 57 - 98 = ? [**-41**]
9. 3,979 + 4,425 = ? [**8404**]
10. 28 - 12 - 17 = ? [**-1**]
11. 2.72 + 0.773 = ? [**3.493**]
12. 54321 - 12345 = ? [**41976**]
13. 9999 - 7654 = ? [**2345**]

Take the C2 Quiz or review.

C3 LESSON: NEGATIVE NUMBERS + ≈ -

For every positive number N there is a corresponding N negative number -N, and vice versa.

$N + (-N) = 0$ $7 + (-7) = 0$

$-(-N) = N$ $-(-6) = 6$

You may create -N from N with the <u>+ ↔ -</u> Key located just left of the <u>=</u> Key

N <u>+ ↔ -</u> yields -N 17 <u>+ ↔ -</u> yields -17

-17 <u>+ ↔ -</u> yields 17

Subtraction is the same as adding a negative number.

$N - M = N + (-M)$ $8 - 3 = 8 + (-3) = 5$

-5 - 6 = -5 + (-6) = 5 <u>+ ↔ -</u> + 6 <u>+ ↔ -</u> = -11
-5 + -6 = -11

Play with this until you are comfortable with it. It's really easy once you catch on to it. Homework will really help here.

When you have mastered it, take the C3 Quiz.

C3E

NEGATIVE NUMBERS   Answers: [ ]'s

1. Where is the key that creates the negative of any number in the calculator's display? [**Bottom, Left of =**]

2. Create -7 in your calculator [**7 +=-**]

3. 8 **+** (-8) **=** ? [**0**]

4. 9 **+** (-4) **=** ? [**5**]

5. **-** (-5) = ? [**5**]

6. -7 + (-8) = ? [**-15**]

7. 18 - 11 = ? [**7**]

8. 18 + (-11) = ? [**7**]

9. 327 - 568 = ? [**-241**]

10. -13.7 + 8.5 = ? [**-5.2**]

11. -(-(- 7)) = ? [**-7**]

12. -3 + (-4) + (-5) = ? [**-12**]

Take the C3 Quiz if you are ready, or do some more exercises, C3ES.

C3ES

NEGATIVE NUMBERS Answers: [ ]'s

1. -(-(-6)) = ? [**-6**]
2. Create -27 in your calculator [**27+=-**]
3. 18 + (-18) = ? [**0**]
4. 19 + (-8) = ? [**11**]
5. - (-8) = ? [**8**]
6. -9 + (-4) = ? [**-13**]
7. 18 - 61 = ? [**-43**]
8. 18 + (-61) = ? [**-43**]
9. 3827 - 968 = ? [**2859**]
10. -18.7 + 7.5 = ? [**-11.2**]
11. -(-(- 2.7)) = ? [**-2.7**]
12. -7 + (-4) + (-2) = ? [**-13**]

Take the C3 Quiz or review.

C4 LESSON: MULTIPLY x DIVIDE ÷

We assume you know basic arithmetic operations and rules.
If not, you will need some more basic training.

Key **k** is indicated by **<u>k</u>** the underline.

The **=** Key is used to complete a calculation.

Multiplication <u>x</u> Key multiplies two numbers

3 <u>x</u> 4 = 12   12.5 <u>x</u> 7.8 = 97.5

(3/8) <u>x</u> (5/6) = 5/16 (See Lesson 10 x on fractions.)

Rules:

(-A)**x**B = -(AxB) or (-A)B = -(AB)
(-A)x(-B) = AxB or (-A)(-B) = AB

Division <u>÷</u> Key Divides two numbers

A/B means A <u>÷</u> B

12/4 = 12 ÷ 4 = 3	15.7 ÷ 2.8 = 5.6
A/(-B) = -(A/B) = (-A)/B	18 ÷ -6 = -3
(-A)/(-B) = A/B	(-15)/(-5) = 3

Again, practice is the key to mastery.
Have fun with the exercises. Then take the C4 Quiz.

C4E

MULTIPLY x / DIVIDE ÷ Answers: [ ]'s

1. 3.5 x 7.4 = ? [**25.9**]
2. 154 x 896 = ? [**137,984**] (You put in the comma.)
3. 0.0075 x 0.02 = ? [**0.00015**]
4. -54 x 87 = ? [**-4698**]
5. (-32) x (-76) = ? [**2432**]
6. 79 ÷ 3 = [**26.3**]
7. 859 ÷ 54 = ? [**15.9**]
8. 86 ÷ (-3) = ? [**-28.7**]
9. (-45) ÷ (-2.5) = ? [**18.0**]
10. (87 x 34) ÷ 5 = ? [**591.6**]
11. (5.4 x 7.1)x2.3 = ? [**88.2**]
12. 8754 ÷ (-23) = ? [**-381**]
13. (54.2 ÷ 3.4) x (8.7 ÷ (-4.3)) = ? [**-32.3**]

Take the C4 Quiz or do some more exercises, C4ES.

C4ES

MULTIPLY x DIVIDE ÷ Answers: [ ]'s

1. 3.8 x 9.4 = ? [35.7]
2. 74 x 396 = ? [29304]
3. 0.0035 x 0.08 = ? [0.00028]
4. -59 x 27 = ? [-1593]
5. (-36) x (-82) = ? [2952]
6. 89 ÷ 4 = ? [22.25]
7. 869 ÷ 34 = ? [25.6]
8. 88 ÷ (-3) + ? [-29.3]
9. (-47) ÷ (-6.5) = ? [7.2]
10. [47 x 74] ÷ 6 = ? [580 or 579.67]
11. (5.6 x 7.3) x 2.9 = ? [118.6]
12. 8954 ÷ (-32) = ? [-280 or -279.8]
13. (56.2 ÷ 3.2) x (9.7 ÷ (-2.3)) = ? [-74.1]

Take the C4 Quiz or review.

C5 LESSON: PERCENTAGE %

We say X% (X Percent) of A is: (X/100)x A

30% of 100 is (30/100)x100 = .30 x 100 = 30

45% of 156 is (45/100)x156 = .45 x 156 = 70.2

There is a **%** Key on the TI-30Xa.

It is above the **2** Key. Select **2nd** then the number **2** to get it.

45 **2nd** **2** yields .45

So: 45 **2nd** **2** x 156 = 70.2

To add **X%** of A to A: A + **X** **2nd** **2** = (1 **+** X/100)A

Add 35% of 256 to itself: 256 + 35 **2nd** **2** = 345.6

There will be a deeper Lesson on Percentages, Discounts and Mark-ups in Tier 3 which goes into more detail on percentages.

This is just to show you how the % Key works.

C5E

PERCENTAGE % Answers: [ ]'s

1. Where is the % key on the TI-30Xa? [**Above the 2**]

2. How do you activate the % Function? [**Press the yellow <u>2nd</u> Then the <u>2</u> key.**

3. What is 45% of 156? [**70.2**]

4. Enter 45 Display is ? [**45**]
   Press **2nd 2** Key Display is ? [**0.45**]
   Press the **x** Key Display is ? [**0.45**]
   Enter 156 Display is ? [**156**]
   Press **=** Key Display is ? [**70.2**]

5. What is 87% of 835? [**726.45**]

6. Add 35% or 287 to itself. [**387.45**]

7. Enter 287 Display is ? [**287**]
   Press **+** Key Display is ? [**287**]
   Enter 35 Display is ? [**35**]
   Press **2nd 2** Key Display is ? [**100.45**]
   Press **=** Key Display is ? [**387.45**]

8. 165% of 200 is? [**330**]

9. Add 80% of 125 to itself and get? [**225**]

10. 4% of 1000 is? [**40**]

Take the C5 Quiz if ready,
or do more exercises, C5ES.

C5ES

PERCENTAGE % Answers: [ ]'s

1. What is 145% of 156? [**226.2**]

2. Enter 145 Display is ? [**145**]
   Press **2nd 2** Key Display is ? [**1.45**]
   Press the **x** Key Display is ? [**1.45**]
   Enter 156 Display is ? [**156**]
   Press **=** Key Display is ? [**226.2**]

3. Enter 156 Display is ? [156]
   Press **x** Key Display is ? [156]
   Enter 145 Display is ? [145]
   Press **2nd 2** Key Display is ? [1.45]
   Press **=** Key Display is ? [226.2]

Do you see the two different ways?

4. What is 37**%** of 835? [**309**]

5. What is 137**%** of 835 [**1144 = 835 + 309**]

6. Add 55**%** of 287 to itself. [**444.85**]

7. Enter 287 Display is ? [**287**]
   Press **+** Key Display is ? [**287**]
   Enter 55% Display is ? [**55**]
   Press **2nd 2** Key Display is ? [**157.85**]
   Press **=** Key Display is ? [**444.85**]

Make up some problems for yourself and take the C5 Quiz.

C6 LESSON: MEMORY M1, M2, M3 STO RCL ( )

Sometimes you may need to store a number in the calculator to be recalled later.

STO and RCL do this.

There are three memory registers, **M1**, **M2**, and **M3**.

To store a number N in memory register 1 do this:

Enter N, then STO 1 and **N** is stored in **M1**

Later to recall N: RCL 1 will restore N.

Example: (3 x 4) + (5 x 7) + (4 x 8)

3 x 4 = 12 STO 1, 5 x 7 = 35 STO 2, 4 x 8 = 32

Now 32 + RCL **1** + RCL 2 **=** 79

Or use the **( )** keys: Simply duplicate the above.

Memory is used when you need to store a number for later use. Parenthesis are used for shorter term storage in a calculation.

For example, if you need to store someone's phone number; say, 5013452314, simply enter this and STO 1.

Now RCL 1 will recall it anytime in the future even if you turn the calculator OFF. Only storing another number in **M1** will erase it.

C6E

MEMORY M1, M2, M3 STO RCL (   )

1. How many memory registers does the TI-30Xa have?

2. Where do you see the **M1, M2,** and **M3** displayed?

3. Which keys do you use to store a number in memory **M2**?

4. Store 235 in **M2**.

5. How do you recall the number in stored in **M2**?

6. What number is in **M2**?

7. Do you lose the numbers stored in memory when you turn the calculator off?

8. How do you "clear" the memory register **M3**?

9. What can you also use for temporary memory storage when doing a calculation?

10. (12.3 + 87) x (34 + 56) = ?

Answers are on C6EA, page 24.

"Play" with the memory and ( ) until you are comfortable with them…then take the C6 Quiz.

C6EA

MEMORY M1, M2, M3 STO RCL ( ) Answers: [ ]'s

1. How many memory registers does the TI-30Xa have? [**3**]
2. Where are the **M1**, **M2** and **M3** displayed? [**Upper Left**]
3. Which keys do you use to store a number in memory **M2**? [**STO 2**]
4. Store 235 in **M2**. [**Enter 235 press STO 2**]
5. How do you recall the number in stored in **M2**? [**RCL 2**]
6. What number is in **M2**? [**235**]
7. Do you lose the numbers stored in Memory when you turn the calculator off? [**No**]
8. How do you "clear" the memory register M3? [**Enter 0 Press STO 3**]
9. What can you also use for temporary memory storage when doing a calculation? [**( )**]
10. (12.3 **+** 87) **x** (34 **+** 56) **=** ? [**8937**]

"Play" with the memory and ( ) until you are comfortable with them…then take the C6 Quiz.

C7 LESSON: X^2 SQUARE

Definition: **A^2** = AxA…we say: **A squared**

$5^2 = 5x5 = 25$ $(7.4)^2 = 7.4x7.4 = 54.8$

An easier way to get this is the **<u>x</u>^{2}** key

7.4x2 yields 54.8 (or 54.76 depending on the **FIX**.)

This is handy for larger numbers.

543.7 squared is simply 543.7 **<u>x</u>^{2}** = 295609.69

You must supply the commas: 295,609.69

Very quick and easy and used a lot in practical math.

NOTE: $(-A)^2 = A^2$ -5 **<u>x</u>^{2}** = 25 So **<u>x</u>^{2}** result is always positive.

As usual, exercises and C7 Quiz.

C7E

X^2 SQUARE Answers: [ ]'s

1. What is the definition of A^2? [**AxA**]

2. Where is the x^2 key on the TI-30Xa? [**3 down middle**]

3. $(137.4)^2 = ?$ [**18878.76 or 18,878.76**]

4. $(6.2)^2 = ?$ [**38.44**]

5. $(-8.7)^2 = ?$ [**75.69**]

6. $(3.4 + 8.7)^2 = ?$ [**146.41**]

7. $(5^2)^2 = ?$ [**625**]

8. $(78 \div 3.3)^2 = ?$ [**558.7**]

9. Can A^2 be negative? [**No**]

10. $7^2 - 3^2 = ?$ [**40**]

11. $(((((2)^2)^2)^2)^2)^2 = ?$ [**4,294,967,296**]

Play with x^2 Key until you have mastered it.

Take the C7 Quiz or practice some more with C7ES.

C7ES

X^2 SQUARE Answers: [ ]'s

1. $(92.56)^2 = ?$ **[8567.35]**

2. $(16.2)^2 = ?$ **[262.4]**

3. $(-75.7)^2 = ?$ **[5730.5]**

4. $(4.3 + 6.7)^2 = ?$ **[121]**

5. $(7^2)^2 = ?$ **[2401]**

6. $(478 \div 23.3)^2 = ?$ **[420.9]**

7. Can A^2 be 0? **[Yes, $0^2 = 0$]**

8. $8^2 - 12^2 = ?$ **[-80]**

9. $(((((2.05)^2)^2)^2)^2)^2 = ?$ **[9,465,063,976]**

   Compare to #11 on previous page!

10. $(2\ 3/4)^2 = ?$ **[7 9/16 = 121/16 = 7.5625]**

Play with x^2 key until you have mastered it.
Take the C7 Quiz or review.

C8 LESSON: √X SQUARE ROOT

Definition: $(\sqrt{A})^2 = A$

$\sqrt{25} = 5$ since 52 = 25

The "problem" is given A, what is √A?

In the old days, this was a difficult problem and there was not an easy way to determine it. But, today thanks to the power tool of math, the calculator, it is very easy.

Just use the <u>**√x**</u> key.

346 <u>**√x**</u> yields the answer 18.6

Also, note $\mathbf{x^2}$ and **√x** are "**inverses.**"

This was revolutionary in the 1970's. It changed many ways we taught engineering and science subjects along with the trig functions.

NOTE: You may not take the square root of a negative number with this calculator. The square root of a negative number exists, but it is not a real number. It is called a complex or imaginary number and will require a more sophisticated power tool. See Tier 4.

For now, **-**7 √**x** yields an "Error" message.

As usual, Exercises and the C8 Quiz.

C8E

$\sqrt{X}$ SQUARE ROOT   Answers: [ ]'s

1. Define $\sqrt{A}$	[$(\sqrt{A})^2 = A$]
2. $\sqrt{36}$ = ?	[6]
3. $\sqrt{137}$ = ?	[11.7]
4. $\sqrt{19.4}$ = ?	[4.4]
5. $\sqrt{(5.4 + 87.2)}$ = ?	[9.6]
6. $(\sqrt{76})^2$ = ?	[76]
7. $\sqrt{(35)^2}$ = ?	[35]
8. $\sqrt{-73}$ = ?	[Error] Why?
9. $\sqrt{(\sqrt{98})}$ = ?	[3.15]
10. $\sqrt{98765432}$ = ?	[9938]

Play with √ until you are comfortable with it.

Take the C8 Quiz or do some more exercises, C8ES.

C8ES

√X **SQUARE ROOT** Answers: [ ]'s

1. Define √A	[√A x √A = A]
2. √256 = ?	[16]
3. √1,000,000 = ?	[1,000]
4. √1000 = ?	[31.6]
5. √1024 = ?	[32]
6. $(\sqrt{1776})^2$ = ?	[1776]
7. √(√(√(√(√4,294,967,296)))) = ?	[2]
8. √-(-81) = ?	[9]
9. √(√81) = ?	[3]
10. √987654321 = ?	[31427 ~ 10, 000π]

Play with √ until you are comfortable with it.

Take the C8 Quiz or review.

C9 LESSON: 1/X RECIPROCAL "FLIP IT"

1 ÷ x is called the "reciprocal." Thus, 1/5 = .2.

Now the 1/x Key makes calculating it easy.

5 1/x yields .2

7 1/x yields .142857143 or .143 or .14 (**FIX**)

NOTE: 1/x is its own inverse; N **1/x 1/x** yields N…You try it!

To recap our progress so far:

+ - x ÷ x^2 √x 1/x = are the eight "**work horse**" keys of practical math.

Learn them well. They are your friends.

The **( )** and **RCL** and **STO** will help sometimes.

So far, we have dealt only with real numbers expressed as base ten decimal numbers. This is often all you will ever need. But; sometimes, we express numbers as fractions. There are some wonderful keys that will help here too. (**See C10, C11, and C12**)

C9E

1/X RECIPROCAL "FLIP IT" Answers: [ ]'s

1. Define 1/x [**1 ÷ x**]
2. 1/89 = ? [**0.011**]
3. 1 ÷ 89 = ? [**0.011**]
4. The reciprocal of 3 is ? [**1/3 = 0.33**]
5. 1/1/79 = ? [**79**]
6. 1/1/S = ? [**S**]
7. 1/0.7 = ? [**1.429**]
8. 1/0.07 = ? [**14.29**]
9. 1/0.007 = ? [**142.9**]
10. $1/(3^2 + 4^2) = ?$ [**0.04 or 1/25**]
11. √[1/25] = ? [**0.2**]
12. $(1/25)^2 = ?$ [**0.0016**]

Play with 1/x

Take the C9 Quiz or do more exercises, C9ES.

C9ES

1/X RECIPROCAL "FLIP IT" Answers: [ ]'s

1. 1/0 = ? [**Error**]
2. 1/1 = ? [**1**]
3. 1/0.5 = ? [**2**]
4. 1/(1/2) = ? [**2**]
5. 1/1/9 = ? [**9**]
6. 1/1/A = ? [**A**]
7. 1/(3 + 4) = ? [**0.14**]
8. 1/√16 = ? [**0.25**]
9. 1/(1 + 2 + 3) = ? [**1/6 = 0.166667**]
10. 1/1/1/1/1/3 = ? [**0.3333**]
11. 1/1/1/1/1/1/3 = ? [**3**]
12. $(1/7)^2$ = ? [**0.02**]

Play with 1/x
Take the C9 Quiz or review.

C10 LESSON: FRACTIONS $\underline{A^{B/C}}$ + - X ÷ 1/X

Let's quickly review fractions. Let A and B be two **integers**. Then, A/B is called a **fraction**. If A > B then this fraction is greater than 1 and called **improper**. There are four rules for adding, subtracting, multiplying and dividing fractions you should know.

A/B + C/D = (AD +BC)/BD
A/B - C/D = (AD -BC)/BD
A/BxC/D = AC/BD
(A/B)/(C/D) = A/BxD/C

2/3 + 4/5 = (2x5 + 3x4)/3x5 = (10+12)/15 =
22/15 = 1 7/15

The $\underline{a^{b/c}}$ lets you enter the two fractions and add them. Watch the video to see how.

Similarly, you can subtract, multiply, and divide two fractions. See the video. **Do the exercises**.

The largest denominator you may enter is 999. So, if you should multiply two fractions resulting in a denominator greater than 999, the answer will be in decimal form.

Also, you may apply the other function keys to fractions just like any other number.

C10E

FRACTIONS $a^{b/c}$ + - X ÷ 1/X Answers: [ ]'s

1. 3/4 + 7/8 = ? [1 5/8 = 13/8 = 1.625]

2. 7/8 - 2/3 = ? [5/24]

3. 2/3x4/5 = ? [8/15]

4. 5/6 ÷ 2/3 = ? [1 1/4 = 5/4 = 1.25]

5. -5/6 x 2/3 = ? [-5/9]

6. -3/4 x -2/3 = ? [1/2]

7. 1/(2/3) = ? [1.5 = 1 1/2 = 3/2]

8. $(6/7)^2$ = ? [0.734693878 = 36/49]

9. √(5/6) = ? [0.91287]

10. What is largest denominator you can enter for a fraction with the TI-30Xa? [999]

11. 17/8 + 13/3 = ? [6 11/24 = 155/24 = 6.46]

12. 5/6 ÷ 7/9 = ? [1 1/14 = 15/14 = 1.07]

Play with fractions.

Take the C10 Quiz or do more exercises, C10ES.

C10ES

FRACTIONS $a^{b/c}$ + - X ÷ 1/X Answers: [ ]'s

1. 3/7 + 7/8 = ? [**1 17/56 = 73/56 = 1.30**]

2. 7/8 - 5/6 = ? [**1/24**]

3. 2/3 x 2/5 = ? [**4/15**]

4. 5/9 ÷ 2/3 = ? [**5/6**]

5. -5/6 x 4/3 = ? [**-1 1/9 = -10/9 = -1.11**]

6. -5/4 x -2/3 = ? [**5/6**]

7. 1/(2/7) = ? [**3.5 = 3 1/2 = 7/2**]

8. $(5/7)^2$ = ? [**25/49 = 0.51**]

9. √(4/7) = ? [**0.756**]

10. What is largest denominator you can enter for a fraction with the TI-30Xa? [**999**]

11. 1 7/8 + 2 3/4 = ? [**4 5/8**]

12. 5/8 ÷ 7/12 = ? [**1.07 = 15/14 = 11/14**]

Play with fractions.

C11 LESSON: D/C PROPER / IMPROPER FRACTION

"**d/c**" is a yellow "key" seen above the **$a^{b/c}$ key.** You get to it by selecting **2nd** **$a^{b/c}$**.

If A **<** B, A/B is called a proper fraction. (6/8)

If A **>** B, A/B is called an improper fraction. (8/6)

A Mixed Fraction is an integer plus a fraction like 2 3/4.

If A and B share no common factor we say A/B is reduced to lowest terms. 6/8 = 3/4 in lowest terms.

The d/c Key does this plus more. It is **2nd** **$a^{b/c}$**.

Enter 2 3/6 as a mixed fraction (watch video) .

Hit the d/c Key and get 15/6…**again**…2 1/2…**again**…5/2.

So you first get an improper, then mixed lowest terms and then improper lowest.

Play with it. Do some exercises. Have fun.

Remember…largest denominator is 999, otherwise it will convert automatically to decimal. (See next Lesson, C12.)

C11E

D/C PROPER/IMPROPER FRACTION

Answers: [ ]'s

1. Where is the "d/c" Key or Function? [2nd ab/c]
   Express the answer as an improper fraction and a mixed fraction.

2. 3/4 + 4/5 = ? **[31/20 = 1 11/20]**

3. 2/3 ÷ 4/7 = ? **[7/6 = 1 1/6]**

4. 1 2/3 + 3 3/4 = ? **[65/12 = 5 5/12]**

5. 6 7/8 – 2 2/3 = ? **[101/24 = 4 5/24]**

6. $(2\ 3/4)^2$ = ? **[121/16 = 7 9/16 = 7.5625]**

7. -(6/7) x 13/8 **[-39/28 = -1 11/28]**

8. 2 x 4 3/4 = ? **[19/2 = 9 1/2]**

9. 15/7 + 2 3/4 + 12/5 = ? **[1021/140 = 7 41/140]**

10. 2 3/4 ÷ 15/7 = ? **[77/60 = 1 17/60]**

11. √(7/4 - 5/13) = ? **[1.17]**

12. $\sqrt{(3^2 + 4^2)}$ = ? [5]

NOTE: In Question 6, the answer is 7.5625. In Lesson 12, you will learn how to convert 79/16 to a decimal.

Take the C11 Quiz or do more exercises, C11ES

C11ES

D/C PROPER/IMPROPER FRACTION

Answers: [ ]'s

1. Where is the "F <--> D" Key or Function?

   [**2nd <-----**] [**Lower Left Corner**]

   Express the answer as an improper fraction and a mixed fraction.

2. 3/7 + 17/21 = ? [**1 5/21 = 26/21**]

3. 2/3 ÷ 2/7 = ? [**2 1/3 = 7/3**]

4. 2 2/3 + 5 3/4 = ? [**8 5/12 = 101/12**]

5. 4 7/8 – 2 2/5 = ? [**2 19/40 = 99/40**]

6. $(2\ 3/5)^2$ = ? [**6 19/25 = 169/25 = 6.76**]

7. -(6/7) x 1 3/8 [**-1 5/28 = -33/28**]

8. 3.5 x 3 3/5 = ? [**12 3/5 = 63/5**]

9. 1 5/7 + 2 3/4 + 12/5 = ? [**6 121/140 = 961/140**]

10. 3 3/4 ÷ 13/7 = ? [**2 1/52 = 105/52**]

11. √(17/5 - 2/13) = ? [**1.8**]

12. $\sqrt{\{(1/3)^2 + (1/4)^2)\}}$ = ? [**0.417 = 5/12**]

NOTE: In Question 6, the answer is 6.76. In Lesson 12, you will learn how to convert 169/25 to a decimal.

Take the C11 Quiz or review.

C12 LESSON: F ↔ D FRACTION TO DECIMAL CONVERSION

Any fraction can be converted to a decimal, although sometimes it will only be an approximation.

1/2 = .5 exactly, 1/3 = .3333 approximately.

This can be accomplished automatically with the **F ↔ D** yellow "Key" via **2nd** ← .

2/3 **F ↔ D** .66667 depending on the **FIX**.

F ↔ D again and you get 2/3 back.

Warning. If you enter .66667 and then **F ↔ D**, nothing will happen…no fraction. **F ↔ D** only works when you **start** with a fraction.

So, it is convenient when you want to end up with a decimal.

Ex: 8/15 + 9/17 = 116/255… you want the decimal equivalent.

Just **F ↔ D** and get 1.06275 (depending on **FIX**)

Also, you can go back, and then use **d/c** to get 271/255.

Again, **have fun** with some exercises and it will soon be very easy to use these three keys. Even if you can "do" fractions manually, this will be much faster and more error free. That's the point of a "power tool."

C12E

F ↔ D FRACTION TO DECIMAL CONVERSION

Answers: [ ]’s

1. Where is the **F ↔ D** "Key" or Function?
   [**2nd ← Bottom Left of Keypad**]

2. Convert 3/7 to decimal [**0.4286**]

3. Convert .375 to fraction [**3/8**]

4. Convert 1/3 to decimal [**0.33333333**]

5. Convert 0.33 to fraction [33/100]

6. Convert 0.333 to fraction [**0.333**]

7. What Happened? Why not 1/3?
   [**Denominator would be 1000, larger than 999**]

8. What is largest denominator you can enter? [**999**]

9. Can you get 3/250 + 4/7 in fraction form?
   [**No, not with the TI-30Xa.**
   (3x7 + 4x250)/1750 = 1021/1750 = 0.5834]

10. Convert 568/126 to improper fraction in lowest terms.
    [**284/63**]

Take C12 Quiz or do some more exercises, C12ES.

F ↔ D FRACTION TO DECIMAL CONVERSION

Answers: [ ]'s

1. Convert 7/3 to decimal [**2.33**]
2. Convert 3/8 to decimal [**0.375**]
3. Convert 0.385 to fraction [**77/200**]
4. Convert 2 1/3 to decimal [**2.333**]
5. Convert 3 1/7 to decimal [**3.1428**]
6. Convert 0.044 to fraction [**11/250**]
7. Convert 0.0444 to fraction [**0.0444**]
8. What Happened? [**Too large a denominator**]
9. What is largest denominator for the TI-30Xa? [**999**]
10. Can you get 3/250 + 4/7 in fraction form? [**No**]
11. Convert 476/252 to improper fraction in lowest terms. [**17/9**]

Take the C12 Quiz or review.

PRE-ALGEBRA INTRODUCTION

In this Foundation course we will be dealing with what are commonly called "Real Numbers" which consist of:

Integers or Whole Numbers, both positive and negative.

Fractions, or quotients, or ratios of integers.

We will usually express numbers in the standard decimal format such as:

327.45 = 3x100 + 2x10 + 7 + .4 + .05 where

.4 = 3/10 and .5 = 5/100

The Real Numbers correspond to points on a straight line.

There are four basic arithmetic operations: + - x ÷ and a few higher level operations such as: x^2 1/x √x

There are several "Rules" or "Laws" of arithmetic.

We assume you already know most of this and will review it briefly in the following Pre-algebra lessons. See the Table of Contents for a listing of the lessons.

We will use the TI-30Xa calculator for most of the calculations we perform in this Foundations Course since it accelerates the learning and application of what you will be learning significantly.

The Keys to perform these operations have been discussed in the Lessons on the use of the TI-30Xa calculator.

P1 LESSON: REAL NUMBERS, INTEGERS AND RATIONALS

First, there are the "counting numbers," 1, 2, 3, 4…also called Natural Numbers and Positive Integers.

We count with the usual decimal system which you should know.

Then we have the number Zero (0) which signifies the absence of something.

Then there are the "negative integers." These are just like the integers; but, have a - sign in front of them, e.g., -5, -6 . . .

Then there are the "fractions" or "rational" numbers which are the ratios or quotients of integers, 3/4, -7/8, 15/7, etc.

We will usually express numbers in the standard decimal format such as:

$$327.45 = 3x100 + 2x10 + 7 + .4 + .05 \text{ where}$$

$$.4 = 4/10 \text{ and } .05 = 5/100$$

It is sometimes easiest to understand these numbers when they are corresponded to points on a straight line, see the next lesson P2.

Later we will review the various operations and "rules" of arithmetic.

Always use the calculator to help yourself understand the various things we are discussing.

We assume this is essentially a review for things you already have learned.

P1E

ARITHMETIC REVIEW

1. What kind of numbers will we deal with in the Foundation Course?

2. What are Integers?

3. What are Rational Numbers?

4. What number is 3x100 + 2x10 + 7 + 0.4 + 0.05?

5. What do the Real Numbers correspond to?

6. What are the four basic operations?

7. What calculator will we use in the Foundation Course?

8. What other three subjects will we learn about in the Foundations Course after Pre-Algebra?

Answers on P1EA, page 8.

Take the Quiz

P1EA

ARITHMETIC REVIEW   Answers: [ ]'s

1. What kind of numbers will we deal with in the Foundation Course? [**Real Numbers**]

2. What are Integers? [**Whole or counting numbers both positive and negative**]

3. What are Rational Numbers? [**Fractions, a/b where a and b are integers, b ≠ 0**]

4. What number is 3x100 + 2x10 + 7 + 0.4 + 0.05? [**327.45**]

5. What do the Real Numbers correspond to? [**Points on a straight line**]

6. What are the four basic operations? **[+, -, x, ÷]**

7. What calculator will we use in the Foundation Course?

   [**TI-30Xa**]

8. What other three subjects will we learn about in the Foundations Course after Pre-Algebra?

   [**Algebra, Geometry, Trigonometry**]

P2 LESSON: THE NUMBER LINE, NEGATIVE NUMBERS

The Real Numbers we will be using in this Foundation Course can be corresponded to the points on a straight line called the Number Line.

We select a point to call Zero, 0.

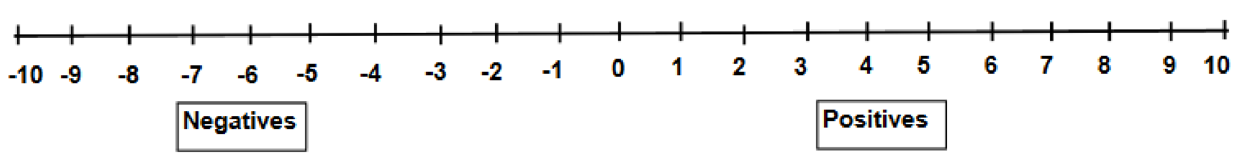
We then select a point to the right of 0 and label it 1.

This establishes a "scale" and all numbers now correspond to one unique point on the line. (See below)

Positive numbers are to the right of 0, and Negative numbers are to the left of 0. The Negatives are a sort of "mirror" image of the Positives.

> a < b means a is to the left of b on the number line.
> a > b means a is to the right of b on the number line.
> a = b means a and b correspond to the same point.

You should be able to find the appropriate point on the line for any number, and vice versa.

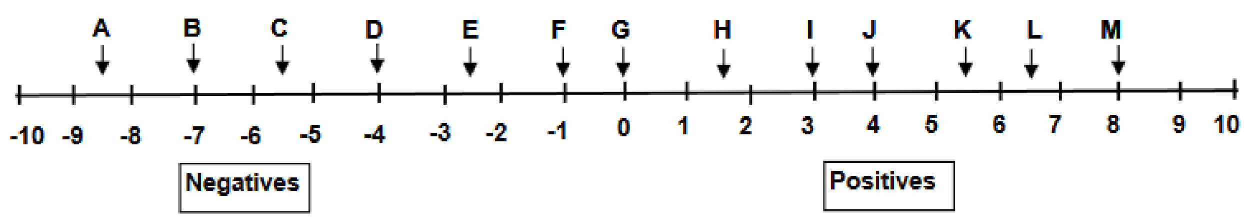


P2E

THE NUMBER LINE, NEGATIVE NUMBERS

Answers: [ ]'s

1. Which letter is above 5.5? [**K**]
2. Which letter is above 3? [**I**]
3. Which letter is above -7? [**B**]
4. Which letter is above -2.5? [**E**]
5. What number is C above? [**-5.5**]
6. What number is L above? [**6.5**]
7. What number is G above? [**0**]
8. Is -3 > -6? [**Yes**]
9. Is -3 < 1? [**Yes**]
10. Is -6 > 0? [**No**]



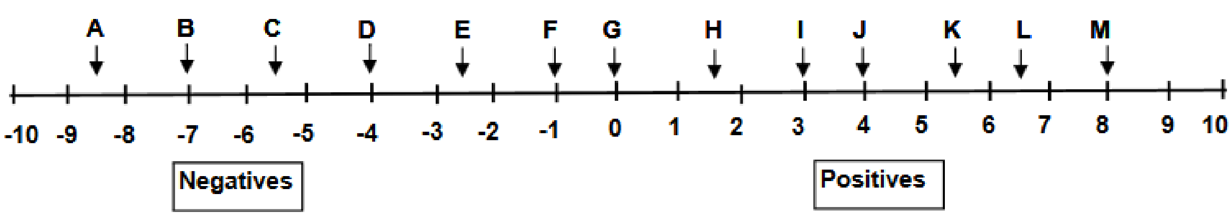
Problem: Given a number, find its location on the number line.
Problem: Give a point on the number line, estimate its value.

P2ES

THE NUMBER LINE, NEGATIVE NUMBERS

Answers: [ ]'s

1. Which letter is above -4? [**D**]
2. Which letter is above 1.6? [**H**]
3. Which letter is above -5.5? [**C**]
4. Which letter is above 6.5? [**L**]
5. What number is E above? [**-2.5**]
6. What number is K above? [**5.5**]
7. What number is C above? [**-5.5**]
8. Is -1 > -3? [**Yes**]
9. Is -3 < -1? [**Yes**]
10. Is -6 > -7? [**Yes**]



Problem: Given a number, find its location on the number line.
Problem: Give a point on the number line, estimate its value.

P3 LESSON: RULES OF ADDITION + -

Rules of Addition: a, b, c represent an arbitrary real numbers

1. $a + 0 = a$ $\quad$ $7 + 0 = 7$

2. $a + b = b + a$ $\quad$ $15 + 6 = 6 + 15 = 21$

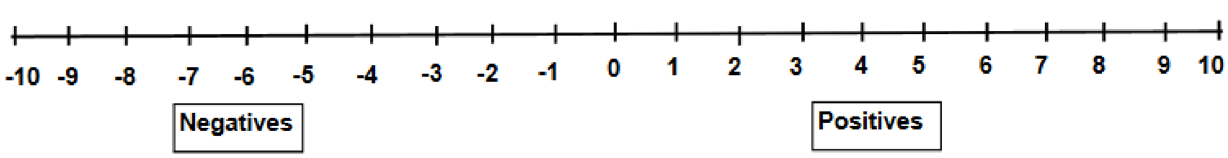
3. $(a + b) + c = a + (b + c)$ $\quad$ $(4 + 7) + 5 = 4 + (7 + 5) = 16$

4. $-(-a) = +a = a$ $\quad$ $-(-8) = 8$

5. $b - a = b + (-a)$ $\quad$ $7 - 3 = 7 + (-3) = 4$ $\quad$ $4 - 9 = 4 + (-9) = -5$

6. $a - a = a + (-a) = 0$ $\quad$ $8 - 8 = 0 = 8 + (-8)$

Note how addition works on the Number Line. Watch the video lesson that accompanies this lesson.



Problem: Given two numbers, find their sum's location on the number line.

Problem: Given two numbers, find their difference's location on the number line.

P3E

RULES OF ADDITION + -   Answers: [ ]'s

1. 3 + 9 = ? [**12**]

2. 126 + 879 + 438 = ? [**1443**]

3. 15.4 + 85.9 + 34.7 = ? [**136.0**]

4. 56.4 - 87.2 = ? [**-30.8**]

5. 0.078 + 0.048 = ? [**0.126**]

6. 87 - 341 = ? [**-254**]

7. 98 - (-34) = ? [**132**]

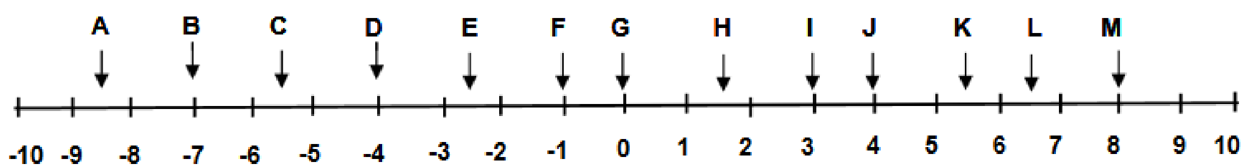
8. Where is D + I on the number line? [**F**]

9. Where is K - H on the number line? [**J**]

10. -17.2 - 34.8 + 12.5 = ? [**-39.5**]

11. 245,400 + 782,900 = ? [**1,028,300**]

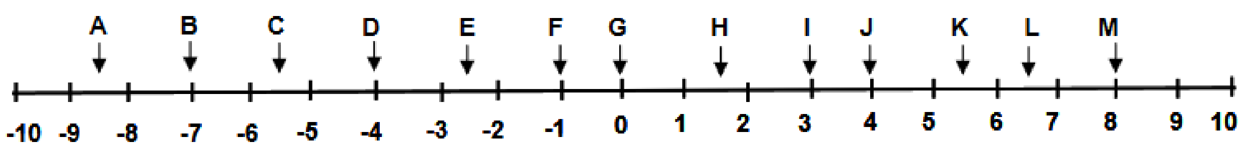
12. -(-34) -(-23) = ? [**57**]



P3ES

RULES OF ADDITION +, -   Answers: [ ]'s

1. 13 +29 = ?   [**42**]
2. 176 + 839 +538 = ?   [**1553**]
3. 17.4 + 35.3 + 34.9 = ?   [**87.6**]
4. 57.4 - 89.2 = ?   [**-31.8**]
5. 0.068 + 0.036 = ?   [**0.104**]
6. 83 - 345 = ?   [**-262**]
7. 92 - (-34) = ?   [**126**]
8. Where is J + F on the number line?   [**I**]
9. Where is K - F on the number line?   [**L**]
10. Where is 7.7 - 2.2 on the number line?   [**K**]
11. -(-37) + (-23) = ?   [**14**]
12. -(-37) -(-23) = ?   [**60**]



Take Quiz or review

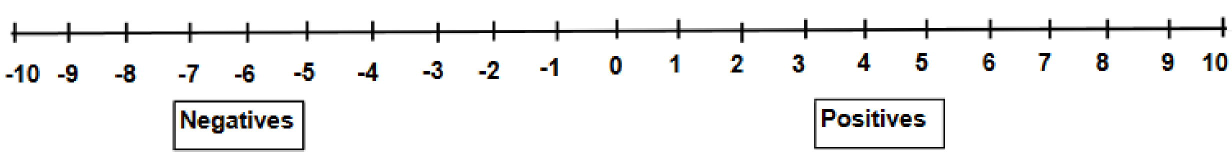
P4 LESSON: RULES OF MULTIPLICATION X ÷

Multiplication of Real Numbers $a x b$ or ab or $a \cdot b$

a, b, c represents arbitrary real numbers

1. $a \times 0 = 0$	$7 \times 0 = 0$
2. $a \times 1 = a$	$13 \times 1 = 13$
3. $a \times b = b \times a$ [ab = bc]	$15 \times 6 = 6 \times 15 = 90$
4. $(ab)c = a(bc)$	$(4 \times 7) \times 5 = 4 \times (7 \times 5) = 140$
5. $(-a) \times b = -(a \times b)$	$(-13) \times 12 = -156$
6. $(-a) \times (-b) = a \times b$	$-5 \times (-6) = 30$
7. $a \times (1/a) = 1$ $(a \neq 0)$	$7 \times (1/7) = 1$
8. $a \div b = a \times (1/b)$ $(b \neq 0)$	$12 \div 4 = 3 = 12 \times (1/4)$

Note how Multiplication works on the Number Line. Watch the Video lesson that accompanies this lesson.



Problem: Given two numbers, find their product's location on the number line.

Problem: Given a number, find its reciprocal location on the number line.

RULES OF MULTIPLICATION X ÷ Answers: [ ]'s

Multiplication of Real Numbers axb or ab or a·b

1. 4 x 5 = ? [**20**]

2. 12.4 x 13.8 = ? [**171.1**]

3. 739 x 546 = ? [**403,494**]

4. 3.2 x 7.8 x 5.4 = ? [**134.8**]

5. -34 x 27 = ? [**-918**]

6. 0.0034 x 0.056 = ? [**0.00019**]

7. -87 x (-23) = ? [**2001**]

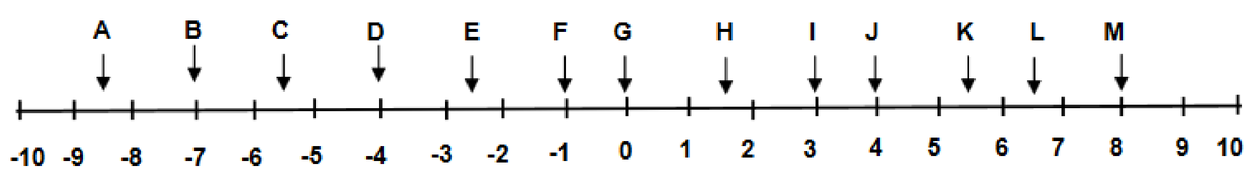
8. Where is J x F on the number line? [**D**]

9. 43.5 ÷ 6.9 = ? [**6.3**]

10. 198 ÷ 5,748 = ? [**0.034**]

11. 78 ÷ (-.03) = ? [**-2600**]

12. -45 ÷ -2.3 = ? [**19.6**]



Take the Quiz or do more exercises on P4ES

P4ES

RULES OF MULTIPLICATION X ÷ Answers: [ ]'s

Multiplication of Real Numbers axb or ab or a·b

1. 0.4 x 0.5 = ? [**0.2**]

2. 17.4 x 54.8 = ? [**953.5**]

3. -0.4 x 0.5 = ? [**-0.2**]

4. 3.4 x 7.8 x 5.7 = ? [**151.2**]

5. -0.4 x (-0.5) = ? [**0.2**]

6. 0.0037 x 0.046 = ? [**0.00017**]

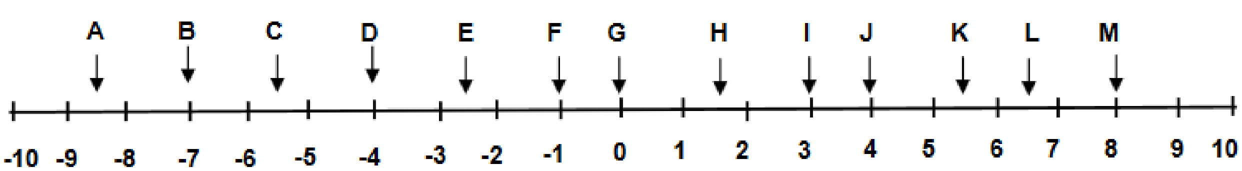
7. 2 2/3 x 1 1/4 = ? [**3 1/3 = 10/3 = 3.3**]

8. Where is -0.5 x 8 on the number line? [**D**]

9. 4 3/5 ÷ 2 3/5 = ? [**1 10/13 = 23/13 = 1.77**]

10. 0.198 ÷ 0.058 = ? [**3.41**]

11. 78 ÷ (-0.3) = ? [**-260**]



Take Quiz or review

P5 LESSON: DISTRIBUTIVE LAW + AND X COMBINED

Distributive Law and Factoring Real Numbers

a, b, c represents arbitrary real numbers

1. ax(b + c) = axb + axc or a(b +c) = ab + ac Simplifying

2. axb + axc = ax(b +c) or ab + ac = a(b + c) Factoring

x, y, z represent arbitrary numbers

3. (x + y)z = xz + yz Simplifying

4. xz + yz = (x + y)z = z(x + y) Factoring

Note how the Distributive Law works on the Number Line. Watch the Video lesson that accompanies this lesson.

P5E

DISTRIBUTIVE LAW + AND X COMBINED

Answers: [ ]'s

Distributive Law and Factoring Real Numbers

1. 12x(34 + 23) = ? [**684**]
2. (2.5 - 3.7)x6.9 = ? [**-8.3**]
3. (78.9 + 43.7)x(34.1 + 13.4) = ? [**5823.5**]
4. 45x67 + 45x82 = 45x(?) [**67 + 82 = 149**]
5. 576x4 - 576x3 = ? [**576x(4-3) = 576**]
6. ab + ad = ax(?) **[(b + d)]**
7. tu + vt = t(?) **[(u + v)]**
8. $ab^2 - ac^2 = ?x(b^2 - c^2)$ [**a**]
9. 5.4x2 + 5.4x3 + 5.4x4 + 5.4x5 + 5.4x6 = ? [**5.4x20 = 108**]
10. $z^3v + t^2v = (?)v$ [$z^3 + t^2$]
11. -3.4x(7.8 - 9.4) = ? [**5.4**]
12. (123 + 876 - 276)x0 = ? [**0**]
13. 54.5(21.4 + 87.3 - 17.4) [**4975.9**]
14. 0.02x(0.003 + 0.015) = ? [**0.00036**]
15. -17x(-6 - 9) = ? [**255**]

Take Quiz or do more exercises on P5ES

P5ES

DISTRIBUTIVE LAW + AND X COMBINED

Answers: [ ]'s

Distributive Law and Factoring

1. 13x(35 + 43) = ? [**1014**]
2. (3.5 - 4.9)x6.2 = ? [**-8.7**]
3. (7.9 + 43.7)x(4.1 + 13.4) = ? [**903**]
4. 42x69 + 42x82 = 42x(?) [**69 + 82 = 151**]
5. 579x7 - 579x6 = ? [**579**]
6. as + ad = ax(?) **[(s + d)]**
7. ta + bt = t(?) **[(a + b)]**
8. $ab^3 - abc^2 = ?x(b^2 - c^2)$ [**ab**]
9. abc - ac = acx? **[(b - 1)]**
10. $z^2v - vt^2 = (?)v$ [$z^2 - t^2$]
11. 33/4x(7/8 - 9/4) = ? [**-55/32 = -165/32 = -5.16**]
12. (12.3 + 886 - 276)x0 = ? [**0**]
13. 56.5(27.4 + 7.3 - 17.4) = ? [**977.5**]
14. 0.01x(0.008 + 0.015) = ? [**0.00023**]

Take Quiz or review

P6 LESSON: FRACTIONS, A/B AND C/D, RULES

Rules for adding and multiplying and dividing fractions

a, b, c, d represent arbitrary real numbers with $b \neq 0$, $d \neq 0$

1. a/b + c/d = (ad + bc)/bd
2. a/b - c/d = (ad - bc)/bd
3. (a/b)x(c/d) = (ac)/(bd)
4. (a/b) ÷ (c/d) = (a/b)x(d/c) = (ad)/(bc), now $c \neq 0$ also
5. Rules regarding - same as in multiplication. - ÷ - = +

You may learn to do this manually, or you can learn to use the TI-30Xa calculator. It does restrict denominators to be less than 1000.

****

Review the calculator lessons C10, C11, and C12, if necessary.

Work problems along with Dr. Del as he does them:

2/3 + 3/4 = 17/12 = 1 5/12

(-1/2) (x 2/3) = -1/3

(-1/2)x(-2/3) = 1/3

(3/4)x(7/8) = 1 5/8 = 13/8 = 1.625

P6E

FRACTIONS, A/B AND C/D, RULES   Answers: [ ]'s

1. 2/3 + 3/4 = ?   [1 5/12 = 17/12 = 1.42]
2. 5 6/7 + 3 8/9 = ?   [9 47/63 = 9.75]
3. 1 7/8 – 1 1/2 = ?   [3/8]
4. 7/8 - 3/5 = ?   [11/40]
5. 6/7 x 3/8 = ?   [9/28]
6. 6/7 ÷ 3/8 = ?   [2 2/7 = 16/7 = 2.29]
7. Express 18/5 as a mixed fraction.   [3 3/5]
8. Express 18/5 in decimal form.   [3.6]
9. Express 0.35 as a fraction.   [7/20]
10. Express 4 7/8 as an improper fraction.   [39/8]
11. Express 4 7/8 as a decimal.   [4.875]
12. 3/4x(1 2/3 + 2 1/2) = ?   [3 1/8 = 25/8 = 3.125]
13. 2 3/4 - 23/8 = ?   [3/8]
14. 3 5/8 x 3 5/8 = ?   [13 9/64 = 13.14]
15. Express 2/3 as a decimal Real Number. [0.6667]
16. 1/a + 1/b = ?   [(a + b)/ab]

Take Quiz or do more exercises on P6ES

P6ES

FRACTIONS, A/B AND C/D, RULES   Answers: [ ]'s

1. 2/5 + 3/8 = ? [**31/40**]
2. 2 6/7 + 1 2/3 = ? [**4 11/21 = 95/21 = 4.5**]
3. 1 5/6 – 1 1/2 = ? [**1/3**]
4. 5/8 - 4/5 = ? [**-7/40**]
5. 4/7 x 5/8 = ? [**5/14**]
6. 4/7 ÷ 5/8 = ? [**32/35**]
7. Express 19/7 as a mixed fraction. [**2 5/7**]
8. Express 18/5 in decimal form. [**3.6**]
9. Express 0.22 as a fraction. [**11/50**]
10. Express 3 5/9 as an improper fraction. [**32/9**]
11. Express 3 5/9 as a decimal. [**3.56**]
12. 3/4 x(2 2/3 + 3 1/2) = ? [**4 5/8 = 37/8 = 4.625**]
13. 2 3/5 – 2 3/4 = ? [**-3/20 = -0.15**]
14. $(3\ 5/8)^2$ = ? [**13 9/64 = 841/64 = 13.1**]
15. 1/ab + 1/cb = ? [**(c + a)/(abc)**]

Take Quiz or review

P7 LESSON: SQUARES X^2 X SQUARED

$A^2 = AxA$ and we say: A squared

1. $(AB)^2 = A^2B^2$ Commutative Law yields this.

2. $(1/A)^2 = 1/A^2$

3. $(A + B)^2 = A^2 + 2AB + B^2$ Distributive Law yields this.

4. $(A - B)^2 = A^2 - 2AB + B^2$ Distributive Law again.

The x^2 Key will automatically square any number.

Work problems along with Dr. Del as you watch the video:

$(3x4)^2 = 144 = 3^2x4^2$ or (3x4)^2 = 144 = (3^2) x (4^2)

$(1/7)^2 = 1/7^2$

$(25.3)^2$ = (25.3)^2 = 640.09

$(-8)^2$ = (-8)^2 = 64

$A^2 > 0$ A^2 is positive, if A is non zero

P7E

SQUARES X^2 X SQUARED Answers: [ ]'s

1. $(34.5)^2 = ?$ **[1190.25]**
2. $(87)^2 = ?$ **[7569]**
3. $(-23)^2 = ?$ **[529]**
4. $(2.4^2 + 3.5^2)^2 = ?$ **[324.4]**
5. $(65.9)^2 = ?$ **[4343]**
6. $(89 + 57 - 32)^2 = ?$ **[12996]**
7. $(12.3)^2/7.6$ **[19.9]**
8. $(15.4 \div 0.35)^2 = ?$ **[1936]**
9. $(1 + 0.08)^2 = ?$ **[1.167]**
10. $(X + Y)^2 - X^2 - Y^2 = ?$ **[2XY]**
11. $(A - B)^2 - A^2 - B^2 = ?$ **[-2AB]**
12. $(3/4)^2 = ?$ **[9/16 = 0.5625]**
13. $3^2 + 4^2 = ?$ **[$25 = 5^2$]**
14. $(0.25)^2 = ?$ **[0.0625]**

Take Quiz or do more exercises on P7ES.

P7ES

SQUARES X^2 X SQUARED Answers: [ ]'s

1. $(3\ 4/5)^2 = ?$ [**14.44 = 14 11/25**]

2. $(8.7)^2 = ?$ [**75.7**]

3. $(-2/3)^2 = ?$ [**0.444 = 4/9**]

4. $(1.4^2 + 2.5^2)^2 = ?$ [**67.4**]

5. $(1\ 2/3 - 2\ 3/4)^2 = ?$ [**1.17 = 1 25/144**]

6. $(8.9 + 5.7 - 3.2)^2 = ?$ [**130.0**]

7. $(3.3)^2/(2.6)^2 = ?$ [**1.6**]

8. $(12.4 \div 0.85)^2 = ?$ [**212.8**]

9. $[(1 + 0.05)^2]^2 = ?$ [**1.22**]

10. $X^2 + Y^2 + 2XY = ?$ [**$(X + Y)^2$**]

11. $(0.01)^2 = ?$ [**0.0001**]

12. $(2/3)^2 = ?$ [**4/9 = 0.444**]

13. $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = ?$ [**55**]

14. $(1.25)^2 = ?$ [**1.56**]

Take Quiz or review

P8 LESSON: SQUARE ROOTS √ X

√ A is a number whose square will equal A.

$(\sqrt{A})^2 = A$, √ A can be positive or negative

A must be positive or √ A will not be a real number.

The √ x Key will calculate the square root of any positive number and give you the positive square root.

√ x will return an Error message on the TI-30Xa if $x < 0$.

$\sqrt{a^2} = a$, √x and x^2 are inverses, i.e., undo each other.

Note: $\sqrt{(a + b)} \neq \sqrt{a} + \sqrt{b}$

****

$\sqrt{9} = 3$ $(-3)^2 = 3^2 = 9$

$\sqrt{16} = 4$ $4^2 = 16$

$\sqrt{89} = 9.4$ Note: $(9.4)^2 = 88.36$

$\sqrt{2} = 1.414213562....$ "Irrational" Number Infinite non-repeating decimal

Irrational means NO fraction will equal √2

Fractions a/b, where a, b are integers, are called "Rational numbers"

√-6 Error "Complex Number"

$\sqrt{(12 + 54)} = 8.12$ where $\sqrt{12} + \sqrt{54} = 3.46 + 7.39 = 10.8$

You DO IT! Then "play with" the √ function, key.

P8E

SQUARE ROOTS √ X Answers: [ ]'s

1. $\sqrt{81}$ = ? [**9**]
2. $\sqrt{56.9}$ = ? [**7.5**]
3. $\sqrt{745365}$ = ? [**863**]
4. $\sqrt{(87)^2}$ = ? [**87**]
5. $(\sqrt{95})^2$ = ? [**95**]
6. $\sqrt{(9 + 16)}$ = ? [**5**]
7. $(1 + \sqrt{32})^2$ = ? [**44.3**]
8. $\sqrt{0.25}$ = ? [**0.5**]
9. $\sqrt{0.0001}$ = ? [**0.01**]
10. $(\sqrt{16} + \sqrt{9})^2$ = ? [**49**]
11. √(1/4 = ? [**1/2**]
12. $\sqrt{(1/2)}$ = ? [**0.707**]
13. $\sqrt{(9/16)}$ = ? [**3/4**]
14. $\sqrt{(-9)}$ = ? [**Error**]

Take Quiz or do more exercises on P8ES.

P8ES

SQUARE ROOTS √ X   Answers: [ ]'s

1. $\sqrt{144} = ?$ [**12**]
2. $\sqrt{256} = ?$ [**16**]
3. $\sqrt{123456} = ?$ [**351.4**]
4. $\sqrt{(67)^2} = ?$ [**67**]
5. $(\sqrt{67})^2 = ?$ [**67**]
6. $\sqrt{(3^2 + 4^2)} = ?$ [**$\sqrt{5^2} = 5$**]
7. $(\sqrt{23} + \sqrt{32})^2 = ?$ [**109.3**]
8. $\sqrt{0.1111} = ?$ [**0.3333**]
9. $\sqrt{0.000001} = ?$ [**0.001**]
10. $\sqrt{0.00001} = ?$ [**0.0032**]
11. $\sqrt{(1/25)} = ?$ [**0.2 = 1/5**]
12. $\sqrt{(1+ 3 +5 + 7 +9)} = ?$ [**5**]
13. $\sqrt{(1+3+5+7+9+11+13+15)} = ?$ [**8**]
14. Do you see a pattern in the last two problems?

Take Quiz or review

P9 LESSON: RECIPROCAL 1/X X ≠ 0

1. 1/x = 1 ÷ x — 1/4 = 1 ÷ 4 = .25
2. 1/(1/x) = x — 1/x is its own inverse
3. 1/a + 1/b = (a + b)/ab — see fractions
4. $(1/x)^2 = 1/x^2$ — see rules of exponents (P10)
5. 1/√x = √(1/x) — see rules of exponents (P10)

****

1/0 is undefined   1/0 Error   Never divide by 0

1/1/4 = 4   1/x Key is its own inverse

1/9 = .1111111111…

$(1/3)^2 = 1/3^2$ = 1/9 = .1111111111...

1/√16 = 1/4 =√(1/16) = .25

√.5 = .707 and .5 < .707

P9E

RECIPROCAL 1/X, $X \neq 0$ Answers: [ ]'s

1. 1/7 = ? [**0.1429**]
2. 1/25 = ? [**0.04**]
3. 1/0.05 = ? [**20**]
4. 1/(0.1 + 0.2) = ? [**3.33**]
5. $(1/3.3)^2$ = ? [**0.0918**]
6. $1/(3.3)^2$ = ? [**0.0918**]
7. √(1/9) = ? [**1/3**]
8. $1/\sqrt{(3^2 + 4^2)}$ = ? [**0.2**]
9. 1/1/7 [**7**]
10. 1/0 [**Error**]
11. 1/(a + b) = ? **[1/(a + b)]**
12. 1/√9 = ? [**1/3**]
13. 1/(√16 + √25) [**0.111111111**]
14. $(1 + 1/10)^2$ = ? [**1.21**]
15. What operation is its own inverse? [**1/x**]

Take Quiz or do more exercises on P9ES

P9ES

RECIPROCAL 1/X, $X \neq 0$ Answers: [ ]'s

1. 1/4 = ? [**0.25**]
2. 1/0.5 = ? [**2**]
3. 1/0.01 = ? [**100**]
4. 1/(0.3 + 0.4) = ? [**1.43**]
5. $(1/2.5)^2$= ? [**0.16**]
6. $1/(2.5)^2$ = ? [**0.16**]
7. √(1/25) = ? [**0.2 = 1/5**]
8. 1/(1 2/3) = ? [**0.6 =3/5**]
9. 1/1/(3.7) = ? [**3.7**]
10. $1/45^{0=}$? [**1**]
11. 1/1/a) = ? [**a**]
12. 1/√49 = ? [**1/7 = 0.143**]
13. 1/1/1/1/1/1/5 [**5**]
14. 1/1/1/1/1/1/1/5 = ? [**0.2 = 1/5**]

Take Quiz or review

P10 LESSON: EXPONENTS Y^X Y > 0, X CAN BE ANY NUMBER

Definitions $A^0 = 1$ y^x is sometimes used for y^x

1. $A^n = AxAx \ldots xA$, n times when n positive integer
2. $A^{1/n}$ is number such that $(A^{1/n})^n = A$
3. $A^{-n} = 1/A^n$ Negative exponents.
4. $A^{n/m} = (A^{1/m})^n$ Exponents defined for any rational number.
5. A^x can be defined for any real number. A > 0.

Rules of Exponents

6. $A^{n+m} = A^n x A^m$
7. $(A^n)^m = A^{nm}$

****

y^x y times itself x times, y is base, x is exponent or power

$3^4 = 81 : 4^3 = 64 : 2^3 = 8$

Name	Digital Base 2		Metric Base 10	
Kilo K	2^{10} =	1024	10^3 = 1000	Thousand
Mega M	2^{20} =	1048576	10^6 = 1000000	Million
Giga G	2^{30} =	11073741824	10^9 = 1000000000	Billion
Tera T	2^{40} =	You do it.	10^{12} =12 Zeros	Trillion

$8^{1/3} = 2$ $(987)^{1/3} = 9.956$

$9^{-2} = .0123 = 1/9^2$ $(16)^{-1/2} = .25 = 1/4$

$(81)^{-1/4} = .3333\ldots = 1/81^{1/4}$

$177,147 = 3^{11} = 3^{(4+7)} = 3^4 x 3^7 = 81x2187$

$9^3 = (3^2)^3 = 3^6 = 729$ $5^{2.6} = 65.66$

P10E

EXPONENTS Y^X ; Y > 0, X ANY NUMBER

Answers: [ ]'s

1. 2^8 = ? [256]
2. 12^3 = ? [1728]
3. $(17.1)^4$ = ? [85504]
4. 10^9 = ? [1,000,000,000]
5. $(1 + 0.06)^{20}$ = ? [3.2]
6. $15^{2.7}$ = ? [1498]
7. $1/(0.5)^4$ = ? [16]
8. $25^{1/2}$ = ? [5]
9. $81^{1/4}$ = ? [3]
10. 5^{-2} = ? [0.04 = 1/25]
11. 2^{30} = ? [1,073,741,824 1 GIG]
12. $1000x(1.06)^{100}$ = ? [339,302]
13. $1000x(1.07)^{100}$ = ? [867,716]
14. $26x(1 + 0.06)^{400}$ = ? [$3.446x10^{11}$ = 344,600,000,000]

Take Quiz or do more exercises on P10ES.

P10ES

EXPONENTS Y^X ; Y > 0, X ANY NUMBER

Answers: [ ]'s

1. 2^{10} = ? [1,024 K]
2. 2^{20} = ? [1,048,576 M]
3. 2^{30} = ? [1,073,741,824 G]
4. 10^3 = ? [1,000 K]
5. 10^6 = ? [1,000,000 M]
6. 10^9 = ? [1,000,000,000 G]
7. $1/5^2$ = ? [0.04]
8. 5^{-2} = ? [0.04]
9. $1281^{1/4}$ = ? [5.98]
10. $(5.98)^4$ = ? [1279]
11. 2^{64} = ? [$1.845 \text{x} 10^{19}$]
12. $(1.02)^{2000}$ = ? [158,000,000,000,000,000]

[$1 invested at time of Christ's birth earning 2% per year compounded would be more money than in the world today. 1% would yield only 440 million.]

Take Quiz or review

INTRODUCTION TO ALGEBRA

Algebra is a "technology" for finding unknown numbers, X, Y, Z, etc., from known numbers A, B, C, etc. In our Foundation course, we will only deal with one unknown number, usually denoted X, but we could denote it with any symbol.

The Algebra technique is to create an Equation involving the unknown number X and the known numbers A, B, C, etc., based on their known relationships and then "solving" the equation for the unknown, and checking the answer.

Step 1 is to "**create**" the equation between X and the knowns.

Step 2 is to "**solve**" this equation by finding out what value of X makes the equation true when substituted for X.

Step 3 is to "**verify**" or "**check**" the solution by making the substitution.

Simple Example: [Word Problem] Three years from now Mary will be twice as old as Joe who is 7 years old today. How old is Mary now?

Step 1. Let X be Mary's age today. This is the unknown we want to find. In three years Mary will be X + 3 years old. In three years Joe will be 7 + 3 = 10 years old. So, we are given that in three years X + 3 = 2x10 = 20

Step 2. Solve the equation. By trial and error, it appears 17 might be the answer.

Step 3. Check. Substitute 17 for X. 17 + 3 = 20. So, 17 is the answer.

Now, in general, it is not too hard to do Step 1. Define what X stands for and then relate the given facts to X and create an equation.

Step 2 can be very easy; or, very difficult, to solve. In the Foundation course, we will deal with equations that arise in many common situations, and these are usually easy to solve.

Step 3 is quite easy with a calculator.

A1 LESSON: FOUR WAYS TO SOLVE AN ALGEBRA EQUATION

Suppose you have an equation with one unknown, **X**. How can you solve it?

There are essentially four ways.

1. <u>**Guess the answer**</u>. Check to see if you are right. This is a good way with really simple equations. It can be the best way with very complicated equations **IF** you have a computer to help. This is then called **Numerical Analysis**.

2. <u>**Apply a Formula**</u>. This is fine **IF** you know an appropriate formula. This is useful if you are solving the same type of equation frequently and have the formula available. However, it can be quite difficult to find or remember the correct formula. Formulas are often given in Handbooks for special situations.

3. <u>**Apply a Process**</u>. This is the best way for certain equations, and it is how we will solve most of our equations in this Foundation course, and in the real world.

4. <u>**Apply a Power Tool**</u>. This is the best way for complex equations. One great tool for this is Mathematica. This is how engineers solve most of their equations. But, you must learn to use this tool first. We will cover it extensively in the upper Tiers in our advanced training. If also applies to other types of equations.

In our Foundation course, we will learn to <u>**Apply a Process**</u>. This usually is easier than trying to find the correct formula. It is faster and more accurate for certain types of equations we will be dealing with.

A1E

Four Ways to Solve an Algebra Equation

Suppose you have an equation with one unknown, X. How can you solve it?

What are the Four Ways to solve an equation?

1.

2.

3.

4.

Which way will be utilized and learned in the Foundations Course? Why?

A1EA

Four Ways to Solve an Algebra Equation

Suppose you have an equation with one unknown, X. How can you solve it?

What are the Four Ways to solve an equation?

1. <u>Guess the answer</u>. Check to see if you are right. This is a good way with really simple equations. It can be the best way with very complicated equations IF you have a computer to help. This is then called Numerical Analysis.

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Which way will be utilized and learned in the Foundations Course? Why?

In our Foundation course, we will learn to <u>Apply a Process</u>. This usually is easier than trying to find the correct formula. It is faster and more accurate for certain types of equations we will be dealing with.

A1ES

Four Ways to Solve an Algebra Equation

1. In the PMF, what do we want to know about an Algebra Equation?

2. Why do we normally use Applying a Process instead of Applying a Formula to solve algebra equations?

A1ESA

Four Ways to Solve an Algebra Equation   Answers: [ ]

1. In the PMF, what do we want to know about an Algebra Equation?

[**We want to see if we can find the value of the unknown in the equation, most generally denoted by X!**]

2. Why do we normally use Applying a Process instead of Applying a Formula to solve algebra equations?

[**Applying a Formula only works for special types of problems and specific formulas, and requires a good deal of memorization. Applying a Process allows us to work with many types of equations with needing to memorize specific formulas!**]

A2 LESSON: THE RULE OF ALGEBRA

Suppose we have an equation to solve. It will have a Left Side (**LS**) and a Right Side (**RS**) either of which might contain the unknown, **X**, and other known numbers. (Any letter could be the unknown.)

Equation: **LS** = **RS** Can switch sides **RS** = **LS**

THE RULE of Equation Solving is: You may do the same thing to both sides of the equation and obtain a new equation:

1. **LS** + A = **RS** + A, **LS** - A = **RS** – A Add or Subtract a Number to both sides of the equation.
2. **LS**xA = **RS**xA, **LS**÷A = **RS**÷A Multiply or Divide a Number
3. 1/**LS** = 1/**RS** Invert both sides
4. $(\mathbf{LS})^2 = (\mathbf{RS})^2$ Square both sides
5. √**LS** = √**RS** Square Root Both Sides
6. SIN (**LS**) = SIN (**RS**) Take the SIN of both sides.
7. Any legitimate math operation to both sides.

The idea is to apply a sequence of operations or transformations to both sides until you arrive at:

X = Number "The Solution"

Then **<u>check your answer</u>** by substituting this Number into the Equation in place of **X** and see that both sides are equal.
We will see many examples of this in the lessons to follow. These are the kinds of Equations you will be solving in the "real world."

A2E

THE RULE OF ALGEBRA

Suppose we have an equation to solve. It will have a Left Side (**LS**) and a Right Side (**RS**) either of which might contain the unknown, **X**, and other known numbers. (Any letter could be the unknown.)

Equation: **LS** = **RS** Can switch sides **RS** = **LS**

1. What is **THE RULE** of Equation Solving?

2. Give examples of applying this Rule.

3. Describe the process you will use to solve an equation using this Rule.

4. After you have a solution: **X** = Number, what should you always do, especially if the answer is important?

THE RULE OF ALGEBRA

Suppose we have an equation to solve. It will have a Left Side (**LS**) and a Right Side (**RS**) either of which might contain the unknown, **X**, and other known numbers. (Any letter could be the unknown.)

Equation: **LS** = **RS** Can switch sides **RS** = **LS**

1. **THE RULE** of Equation Solving is: *You may do the same thing to both sides of the equation and obtain a new equation:*

2. Examples:

1) **LS** + **A** = **RS** + **A**, **LS** - **A** = **RS** - A (add or subtract a number to both sides of the equation)
2) **LS**x**A** = **RS**x**A**, **LS**÷**A** = **RS**÷**A** (multiply or divide a number)
3) 1/**LS** = 1/RS (invert both sides)
4) $(\mathbf{LS})^2 = (\mathbf{RS})^2$ (square both sides)
5) √**LS** = √**RS** (square root both sides)
6) SIN (**LS**) = SIN (**RS**) (take the SIN of both sides)
7) Any legitimate math operation to both sides.

3. The idea is to apply a sequence of operations or transformations to both sides until you arrive at:

X = Number "**The Solution**"

4. Then <u>check your answer</u> by substituting this Number into the Equation in place of **X** and see that both sides are equal.

We will see many examples of this in the lessons to follow. These are the kinds of Equations you will be solving in the "real world."

THE RULE OF ALGEBRA

1. When solving an equation, any math operation (adding, subtracting, multiplying, etc.) done to one side of the equation must be ___________________________. *fill in the blank*

2. If we solved the equation X + 3 = 8, and got X = 6, what IMPORTANT STEP would help us realize we made a mistake?

THE RULE OF ALGEBRA Answers: [ ]

1. When solving an equation, any math operation (adding, subtracting, multiplying, etc.) done to one side of the equation must be [done to the other side of the equation].

2. If we solved the equation X + 3 = 8, and got X = 6, what IMPORTANT STEP would help us realize we made a mistake? [**If we checked our solution by plugging it back into the original equation we would see that X = 6 gives 9 = 8, which is obviously incorrect!**]

A3 LESSON: X + A = B THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

X + **A** - **A** = **B** - **A** [subtract **A** from both sides] [transpose **A**]

Thus: X = **B** – **A** since **A** - **A** = **0** and X + **0** = X

Example: X + 2 = 5 [subtract **2** from both sides]

Solution: X = X + 2 - 2 = 5 - 2 = 3

Example: X - 7 = -13 [add **7** to both sides]

Solution: X = X - 7 + 7 = -13 + 7 = -6 [we have transposed **7**]

Example: 8.13 = -7.19 + X

Same as: X - 7.19 = 8.13 [since can switch sides]

Solution: Add 7.19 to both sides. X = 15.32 (use calculator)

Example: X + (-18.4) = +√37.9

Same as: X - 18.4 = 6.16 [take square root +(-) = -]
X = X - 18.4 + 18.4 = 6.16 + 18.4 = 24.56 = 24.6
[add 18.4]

Example: X - **SIN**(37º) = [COS(68º)]2 [do not be intimidated

SIN(37º) = .6018 COS(68º) = .3746 $(.3746)^2$ = .1403

SO: X - .6018 = .1403 and

THUS: X = .7421

A3E

X + A = B THIS IS AN EASY LINEAR EQUATION

X + A - A = B - A [subtract A from both sides] [transpose A]

Thus: X = B – A **since** A - A = 0 **and** X + 0 = X

Solve for X, the Unknown

1. X + 42 = 59
2. X - 17 = -43
3. 8.13 = -17.19 + X
4. X + (-28.4) = +$\sqrt{87.9}$
5. 6.5 - X = 23.5
6. 5432 = X + 4375
7. X - $\sqrt{675}$ = $\sqrt{9876}$
8. X - 3/4 = 9/13
9. 6/7 = 8/11 – X
10. 0.00035 + X = 0.0017
11. X - **SIN**(37°) = $[\text{COS}(68^{\circ})]^2$
12. **COS**(48°) = **TAN**(78°) – X
13. $(13.4 + 9.7)^2 + X = 87.4^2$

A3EA

X + A = B   THIS IS AN EASY LINEAR EQUATION   Answers: [ ]

X + A - A = B - A   [subtract A from both sides]   [transpose A]

Thus: X = B – A   since   A - A = 0   and   X + 0 = X

1. X + 42 = 59 [**17**]
2. X - 17 = -43 [**-26**]
3. 8.13 = -17.19 + X [**25.32**]
4. X + (-28.4) = +√87.9 [**37.8**]
5. 6.5 - X = 23.5 [**-17**]
6. 5432 = X + 4375 [**1057**]
7. X - √675 = √9876 [**125.4**]
8. X - 3/4 = 9/13 [**75/52=123/52**]
9. 6/7 = 8/11 – X [**-10/77**]
10. 0.00035 + X = 0.0017 [**0.00135**]
11. X - SIN(37^{o}) = [COS(68^{o})]^2 [**0.742**]
12. COS(48^{o}) = TAN(78^{o}) – X [**4.035**]
13. (13.4 + 9.7)^2 + X = 87.4^2 [**7105.2**]

A3ES

$X + A = B$ THIS IS AN EASY LINEAR EQUATION

Answers: [ ]

1. $X + 54 = 100$	[$X = 46$]
2. $8.7 - X = 4.9$	[$X = 3.8$]
3. $X + (-0.567) = 3.14$	[$X = 3.707$]
4. $X + \sqrt{25} = 10$	[$X = 5$]
5. $17^2 - X = 100$	[$X = 189$]
6. $X - \sin(30°) = 1$	[$X = 1.5$]
7. $X - 5/6 = 4/5$	[$X = 1.633$]
8. $7/6 = 8/5 - X$	[$X = 0.433$]
9. $0.3017^4 + X = 0.0012^2$	[$X = -0.0083$]
10. $[\cos(180°)]^2 - X = \sin(270°)$	[$X = 2$]
11. $\pi - X = \pi/2$	[$X = \pi/2$]
12. $(2^3 + X) - 4 = (2^2 + 3^2)$	[$X = 9$]

A4 LESSON: AX = B THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

X = AX/A = B/A [divide both sides by **A**] **Note**: A/A = 1

Example: 3X = 12

Solution: X = 3X/3 = 12/3 = 4 [divide by **3** both sides always]

Example: 2.16X = -56.3

Solution: X = -56.3/2.16 = -26.0648 = -26.1

Example: -37.8 = -6.78X

Solution: -6.78X = -37.8 [switch sides]

Then: X = (-37.8)/(-6.78) = 5.6 [divide by -6.78]

Example: $(3.85)^2$X =√349/ **SIN**(79^0) [easy does it!]

$(3.85)^2$ = 14.8 √349 = 18.7 **SIN** (79^0) = .982

So: 14.8X = 18.7/.982 = 19.0 X = 1.29 [divide by 14.8]

Always simplify the numbers first, and then solve the equation. The calculator makes this easy. Also, always **CHECK** your answer by plugging it back into the equation and being sure both sides are equal.

$(3.85)^2$x1.29 = 19.1 √349/ **SIN**(79º) = 19.0 [round off error]

A4E

AX = B   THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

X = **AX/A** = **B/A**   [divide both sides by **A**]   **Note**: **A/A** = 1

Solve for **X**, the Unknown

1. $4X = 12$
2. $2.16X = -56.3$
3. $-37.8 = -6.78X$
4. $0.003X = 0.15$
5. $(4/5)X = 7/9$
6. $(1+3)^2X = \sqrt{65}$
7. $(3.85)^2X = \sqrt{349}/ \text{SIN}(79^0)$ {Easy does it!}
8. $(1 + 2/3) = (7/12)X$
9. $2345X = 9876$
10. $54.5 = -87.7X$
11. $\text{COS}(32^o)X = 3\text{SIN}(32^o)$
12. $X = 3\text{TAN}(32^o)$

A4EA

AX = B THIS IS AN EASY LINEAR EQUATION Answers: [ ]

What can you do to both sides to get closer to a solution?

X = **AX**/**A** = **B**/**A** [divide both sides by **A**] **Note**: **A**/**A** = 1

Solve for **X, the Unknown**

1. 4X = 12 [**3**]
2. 2.16X = -56.3 [**-26.1**]
3. -37.8 = -6.78X [**5.58**]
4. 0.003X = 0.15 [**50**]
5. (4/5)X = 7/9 [**35/36 = 0.97**]
6. $(1+3)^2$X = √65 [**0.5**]
7. $(3.85)^2$X =√349/ **SIN**(79^0) [**1.28**]
8. (1 + 2/3) = (7/12)X [**20/7 = 2 6/7 = 2.86**]
9. 2345X = 9876 [**4.2**]
10. 54.5 = -87.7X [**-0.62**]
11. **COS**(32^o)X = 3**SIN**(32^o) [**1.875**]
12. X = 3**TAN** (32^o) [**1.875**]

A4ES

AX = B THIS IS AN EASY LINEAR EQUATION Answers: [ ]

1. 5X = 27.25 [**X = 5.45**]
2. 67 - 2 = 13X [**X = 5**]
3. 5.1X - 3 = 2.1 [**X = 1**]
4. 9 = 3X + 17 [**X = - 2.6**]
5. $(5^2)X = 1000$ [**X = 40**]
6. TAN(30°)X = 18 [**X = 31.18**]
7. (√169)X = 26 [**X = 2**]
8. (-7/8) = (-8/5)X [**X = 0.5469**]
9. $[SIN(60°)]^2X = 3$ [**X = 4**]
10. **In the equation AX = B, when solving it we would divide B by A. Notice how dividing B by A is the same as MULTIPLYING B by (1/A).** In the equation, (2/3)X = 2, we would solve by dividing 2 by (2/3). If we want to think in terms of multiplication, what we would multiply 2 by instead?

    [**We would think of multiplying 2 by the reciprocal of 2/3, which is 3/2.**]

11. $(\sqrt{36})[COS(60°)]^2 = SIN(270°)X$ [**X = -1.5**]
12. 3X + 3X + 3X = -0.62612 [**X = -0.0696**]

A5 LESSON: AX+B = CX+D THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

Get all the X terms on one side and numbers on other side.

$AX - CX = D - B$ or $(A - C)X = D - B$ [distributive law]

$X = (D - B)/(A - C)$ [divide both sides by (A - C)]

Example: $3X + 7 = 5 - 7X$

Solution: $3X + 7X = 5 - 7$ **or** $10X = -2$ **or** $X = -2/10 = -.5$

Example: $-18.3X + 4.6X - 22.4 = 13.9X - 45.4 + 3.9X$

$-18.3X + 4.6X -13.9X - 3.9X = -45.4 + 22.4$

$(-18.3 + 4.6 -13.9 -3.9)X = -31.5X = -23.0$

$X = -23.0/-31.5 = .730$

Once again…always do the numerical calculations first.

Example: $(2.13)^2X - \mathbf{LOG}(345) = 1/\mathbf{COS}(12.5^o) + \sqrt{(5 + 1/.15)}X$

$(2.13)^2 = 4.54$ $\mathbf{LOG}\ (345) = 2.54$

$\mathbf{COS}(12.5^o) = .976$ $1/.976 = 1.024$

and: $\sqrt{(5 + 1/.15)} = \sqrt{(5 + 6.67)} = 3.42$ [easy w/calculator]

$4.54X - 2.54 = 1.024 + 3.42X$

or: $(4.54 - 3.42)X = 1.024 + 2.54$

$1.12X = 3.56$

$X = 3.56/1.12 = 3.18$ [you check the answer]

$(2.13)^2x3.18 - \mathbf{LOG}\ (345) = 11.9 = 1/\mathbf{COS}\ (12.5^o) +$

$\sqrt{}\ (5 + 1/.15)3.18$

A5E

AX + B = CX + D   THIS IS AN EASY LINEAR EQUATION.

Solve for **X**, the Unknown. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30Xa**.

1. $3X + 7 = 5 - 7X$
2. $3.2X - 9 = 4.1X + 7.8$
3. $-12X - 98 = 23X + 76$
4. $0.002X - 0.015 = 0.0087 - 0.005X$
5. $(3/4)X - 2/7 = (4/5)X + 3/8$
6. $\mathbf{SIN}(28^{o})X - 1.4 = \mathbf{COS}(28^{o})X + 2.3$
7. $-18.3X + 4.6X - 22.4 = 13.9X - 45.4 + 3.9X$
8. $(2.13)^2X - \mathbf{LOG}(345) = 1/\mathbf{COS}(12.5^{o}) + \sqrt{(5 + 1/0.15)}\ X$
9. $2\ 5/6X - 7.1 = 7\ 2/3X + 3.2$
10. $(1/7)X + 2/3 = (3/8)X - 4/9$
11. $2.4 - 3.5X = 7.8 - 1.2X$
12. $(\mathbf{LOG}54)X + 45^2 = \mathbf{SIN}(45^{o}) - (4.5)^2X$
13. $X - \mathbf{LN}(60) = 3 - 2X$
14. $45 - 17X = 8X + 76$

A5EA

AX + B = CX + D This is an easy Linear Equation   Answers: [ ]

Solve for **X**, the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30Xa**.

1. 3X + 7 = 5 - 7X [**-0.2**]
2. 3.2X - 9 = 4.1X +7.8 [**-18.7**]
3. -12X - 98 = 23X + 76 [**-4.97**]
4. 0.002X - 0.015 = 0.0087 - 0.005X [**3.39**]
5. (3/4)X - 2/7 = (4/5)X + 3/8 [**-13 3/14 = -185/14 = -13.21**]
6. **SIN**(28°)X - 1.4 = **COS**(28°)X +2.3 [**-8.95**]
7. -18.3X + 4.6X - 22.4 = 13.9X - 45.4 + 3.9X [**0.73**]
8. $(2.13)^2$X - **LOG**(345) = 1/**COS**(12.5°) + $\sqrt{(5 + 1/.15)}$X [**3.18**]
9. 2 5/6X - 7.1 = 7 2/3X + 3.2 [**-2.13**]
10. (1/7)X + 2/3 = (3/8)X - 4/9 [**4 92/117 = 560/117 =4.79**]
11. 2.4 - 3.5X = 7.8 - 1.2X [**-2.35**]
12. (**LOG**54)X + 45^2 = **SIN**(45°) - $(4.5)^2$X [**-92.09**]
13. X - **LN**(60) = 3 - 2X [**2.37**]
14. 45 - 17X = 8X + 76 [**-1.24**]

A5ES

AX + B = CX + D This is an easy Linear Equation   Answers: [ ]

1. 4x - 17 = -35 - 5X [**X = -2**]
2. 25 + 3.5X = -25 + 7.5X [**X = 12.5**]
3. 6^2X - 24 = 36 + 18X [**X = 3.333**]
4. 0.375 + 4.25X = 1.525 - 8.125X [**X = 0.0929**]
5. SIN(45°)X - 4 = 12 - COS(45°)X [**X = 11.31**]
6. (√144)X - 2^4 = 3^3 + (√36)X [**X = 7.167**]
7. LOG(15)X + 1 = LN(25) + 2X [**X = -2.693**]
8. 1/COS(0°) -4X = -1/SIN(90°) + (3/4)X [**X = 0.421**]
9. πX - 2/3π = 3πX - 8/3π **HINT: What can be removed from both sides of the equation?**

   [**Since Pi is on either side of the equation, it can be removed.**]
   [**X = 1**]

10. 2TAN(45°)X + 2X - 0.375 = SIN(12.5°)X - √0.025

    [**X = 0.0573**]
11. $(1/4)^2$X - 25.67 = 27X + 6.022 [**X = -1.176**]
12. $[LN(25-7.4)]^2$X - 17 = 1/LOG(2) - 3COS(37°)X

    [**X = 1.19**]

A6 LESSON: A/X = C/D THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

Flip both sides: X/A = D/C **then** X = Ax(D/C)

Example: 3/X = 12/5

Solution: X/3 = 5/12 **then** X = 3x(5/12) = 1.25

Example: 2.16/X = -56.3 **then** X/2.16 = 1/-56.3

Solution: X = 2.16/-56.3 = -.038 (check: 2.16/-.038 = -56.8)

Example: -37.8 = -6.78/X

Solution: -6.78/X = -37.8 (switch sides)

Then: X = (-6.78)/(-37.8) = .18 (flip and multiply by -6.78)

Example: $(3.85)^2$/X =√349/ **SIN**(79^0)

$(3.85)^2$ = 14.8 √349 = 18.7 **SIN** (79^0) = .982

So: 14.8/X = 18.7/.982 = 19.0 **or** X = 14.8/19.0 = .78

Always simplify the numbers first, and then solve the equation. Also, always **CHECK** your answer by plugging it back into the equation and being sure both sides are equal.

$(3.85)^2$/.78 = 19.0 √349/ **SIN**(79º) = 19.0

A6E

A/X = C/D THIS IS AN EASY LINEAR EQUATION.

Flip both sides: $X/A = D/C$ **then** $X = A\times(D/C)$

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30Xa**.

1. $3/X = 12/5$
2. $2.16/X = -56.3$
3. $-37.8 = -6.78/X$
4. $(3.85)^2/X = \sqrt{349}/\ \mathbf{SIN}(79^0)$

Always simplify the numbers first and then, solve the equation. Also, always **CHECK** your answer by plugging it back into the equation and being sure both sides are equal.

5. $\mathbf{SIN}(23^o)/X = \mathbf{COS}(54^o)$
6. $23^2 = (12.5)^2/X$
7. $(3/4)/X = 9/16$
8. $\mathbf{LOG}(4235)/X = \text{LN } 435$
9. $10.5/X = 9.8/4.1$
10. $(5^2 + 7^2)/X = 1/(0.05)^2$
11. $\mathbf{COS}(37^o)/\mathbf{SIN}(37^o) = 1/X$

A6EA

A/X = C/D This is an easy Linear Equation Answers: [ ]

Flip both sides: **X**/A = D/C **then** **X** = Ax(D/C)

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30Xa**.

1. 3/**X** = 12/5 [**1.25**]
2. 2.16/**X** = -56.3 [**-0.038**]
3. -37.8 = -6.78/**X** [**0.179**]
4. $(3.85)^2$/**X** =√349/ **SIN**(79^0) [**0.779**]

Always simplify the numbers first, and then solve the equation. Also, always **CHECK** your answer by plugging it back into the equation and being sure both sides are equal.

5. **SIN**(23^o)/**X** = **COS**(54^o) [**0.665**]
6. $23^2 = (12.5)^2$/**X** [**0.295**]
7. (3/4)/**X** = 9/16 [**1 1/3 = 4/3 = 1.33**]
8. **LOG**(4235)/**X** = **LN** 435 [**0.597**]
9. 10.5/**X** = 9.8/4.1 [**4.39**]
10. $(5^2 + 7^2)$/**X** = $1/(0.05)^2$ [**0.185**]
11. **COS**(37^o)/**SIN**(37^o) = 1/**X** [**0.754**]

A6ES

A/X = C/D   This is an easy Linear Equation   Answers: [ ]

1. $4/X = 1$ [X = 4]
2. $10/X = 2/4$ [X = 20]
3. $17/X = 1/17$ [X = 289]
4. $\sin(30°)/X = 1/\cos(60°)$ [X = 0.25]
5. $25.3/X = -98.1/27.6$ [X = -7.12]
6. $(\sqrt{225})/X = 12/19$ [X = 23.75]
7. $23.6/-0.025 = 1112/X$ [X = -1.178]
8. $\sin(56°)/X = \cos(27°)$ [X = 0.93]
9. $\tan(75°)/\cos(23.5°) = \sin(14°)/X$ [X = 0.0594]
10. $\log(92)/X = 15/\ln(25)$ [X = 0.4214]
11. $\pi/X = 1/2$ [X = 2π]
12. $-\cos(180°)/2X = 43\sin(25°)/3.643$ [X = 0.1002]

A7 LESSON: $AX^2 = B$ THIS IS AN EASY NON-LINEAR EQUATION

What can you do to both sides to get closer to a solution?

$X^2 = B/A$ (divide by A) **now** take the square root both sides

$X = \sqrt{(B/A)}$ [**Note:** Answer could be **+** or **-**]

Example: $X^2 = 387$ $X = 19.7$ **or** -19.7 [$\sqrt{387} = 19.7$]

Example: $\text{SIN}(125^o)X^2 = (5.4 + 3.4)^2$ (simplify numbers first)

$\text{SIN}(125^o) = .819$ $(5.4 + 3.4)^2 = (8.8)^2 = 77.4$

So: $.819X^2 = 77.4$ **or** $X^2 = 77.4/.819$ **or** $X^2 = 94.55$

So: $X = 9.7$

Check: $\text{SIN}(125^o)x(9.7)^2 = 77.07$ [close enough due to **r/o**]

Note: $X = \sqrt{94.55} = 9.724$ to more digits

Then: $\text{SIN}(125^o)x(9.724)^2 = 77.5$

Always be aware of how many digits are really significant and the unavoidable round off (**r/o**) error. Ask yourself: How accurate or precise can I measure, or do I need to measure?

A7E

$AX^2 = B$ THIS IS AN EASY NON-LINEAR EQUATION

$X^2 = B/A$ (divide by A) **now** take the square root both sides

$X = \sqrt{(B/A)}$ [**Note:** Answer could be **+** or **-**]

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30Xa**.

1. $X^2 = 387$
2. $\mathbf{SIN}(125^o)X^2 = (5.4 + 3.4)^2$
3. $X^2 = 23^2$
4. $X^2 = (\sqrt{78})^2$
5. $X^2 = \mathbf{LOG}(98)$
6. $\mathbf{SIN}(34^o) = \mathbf{COS}(23^o)X^2$
7. $(3/4)X^2 = 9/16$
8. $X^2 = 16A^2$
9. $X^2 = (\mathbf{SIN}(78^o))^2 + (\mathbf{COS}(78^o))^2$
10. $X^2 = \mathbf{COS}^{-1}[(3^2 + 4^2 - 6^2)/2x3x4]$
11. $X^2 = \sqrt{81}$

A7EA

AX^2 = B THIS IS AN EASY NON-LINEAR EQUATION Answers:[ ]

X^2 = B/A (divide by A) **now** take the square root both sides

$X = \sqrt{(B/A)}$ [**Note:** Answer could be **+** or **-**]

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30Xa**.

1. $X^2 = 387$ [**19.7**]
2. $SIN(125^o)X^2 = (5.4 + 3.4)^2$ [**9.7**]
3. $X^2 = 23^2$ [**23**]
4. $X^2 = (\sqrt{78})^2$ [**√78**]
5. $X^2 = LOG(98)$ [**1.41**]
6. $SIN(34^o) = COS(23^o)X^2$ [**0.779**]
7. $(3/4)X^2 = 9/16$ [**0.866**]
8. $X^2 = 16A^2$ [**4A**]
9. $X^2 = (SIN(78^o))^2 + (COS(78^o))^2$ [**1**]
10. $X^2 = COS^{-1}[(3^2 + 4^2 - 6^2)/2x3x4]$ [**10.8**]
11. $X^2 = \sqrt{81}$ [**3**]

A7ES

$AX^2 = B$ THIS IS AN EASY NON-LINEAR EQUATION Answers:[ ]

1. $X^2 = 81$ [X = ±9]
2. $X^2 = 169$ [X = ±13]
3. $3X^2 = 45$ [X = ±3.87]
4. $X^2 = 275^2$ [X = ±275]
5. $SIN(35°)X^2 = 65$ [X = ±10.645]
6. $(3/7)X^2 = (19/8)$ [X = ±2.354]
7. $LOG(8.756)X^2 = LN(253)$ [X = ±2.423]
8. $X^2 = \pi^2$ [X = ±π]
9. $3X^2 = \sqrt{121}$ [X = ±1.915]
10. $X^2 = SIN(65°) - COS(45°)$ [X = ±0.4463]
11. $4X^2 = (2^4 + 3^3 + 4^2)^2$ [X = ±29.5]
12. $X^2 = (3\pi^2)^2$ [$X = \pm 3\pi^2$]

A8 LESSON: $A\sqrt{X} = B$ THIS IS AN EASY NON-LINEAR EQUATION

What can you do to both sides to get closer to a solution?

$\sqrt{X} = B/A$ (divide by A) **now** take the square both sides

$X = (B/A)^2$ [**Note:** Answer will be positive]

Example: $\sqrt{X} = 387$ $X = 149{,}769$ which is $(387)^2$

How many digits are significant…**probably 3.**
150,000 is good enough.

Example: $\mathbf{SIN}(125^o)\sqrt{X} = (5.4 + 3.4)^2$ (simplify numbers first)

$\mathbf{SIN}(125^o) = .819$ $(5.4 + 3.4)^2 = (8.8)^2 = 77.4$

So: $.819\sqrt{X} = 77.4$ **or** $\sqrt{X} = 77.4/.819$ **or** $\sqrt{X} = 94.55$

or $X = 8940$

Check: $\mathbf{SIN}(125^o) \times \sqrt{8940} = 77.4$

Always be aware of how many digits are really significant and the unavoidable round off (**r/o**) error. Ask yourself: How accurate or precise can I measure, or do I need to measure?

A8E

$A\sqrt{X} = B$ This is an easy non-Linear Equation

$\sqrt{X} = B/A$ (divide by A) **now** take the square both sides

$X = (B/A)^2$ [**Note:** Answer will be positive]

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but is easy with the **TI-30Xa**.

1. $\sqrt{X} = 387$
2. $\sqrt{X} = -23.5$
3. $\sqrt{X} = 7/8$
4. $3.5\sqrt{X} = 98.2$
5. $78 = 4.2\sqrt{X}$
6. $\sqrt{X} = 6^2$
7. $\sqrt{X} = \sqrt{17}$
8. $\mathbf{SIN}(125^o)\sqrt{X} = (5.4 + 3.4)^2$ (simplify numbers first)
9. $\sqrt{X} = \mathbf{LOG}(6754)$
10. $\sqrt{X} = \mathbf{SIN}^2(65^o) + \mathbf{COS}^2(65^o)$

A8EA

$A\sqrt{X} = B$ This is an easy non-Linear Equation

Answers:[ ]

$\sqrt{X} = B/A$ (divide by A) **now** take the square both sides

$X = (B/A)^2$ [**Note:** Answer will be positive]

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30Xa**.

1. $\sqrt{X} = 387$ [**149,769**]
2. $\sqrt{X} = -23.5$ [**552.25**]
3. $\sqrt{X} = 7/8$ [**0.766 or 49/64**]
4. $3.5\sqrt{X} = 98.2$ [**787**]
5. $78 = 4.2\sqrt{X}$ [**345**]
6. $\sqrt{X} = 6^2$ [**1296**]
7. $\sqrt{X} = \sqrt{17}$ [**17**]
8. $\text{SIN}(125^\circ)\sqrt{X} = (5.4 + 3.4)^2$ [**8937**]
9. $\sqrt{X} = \text{LOG}(6754)$ [**14.67**]
10. $\sqrt{X} = \text{SIN}^2(65^\circ) + \text{COS}^2(65^\circ)$ [**1**]

A8ES

$A\sqrt{X} = B$ This is an easy non-Linear Equation

Answers:[ ]

1. $\sqrt{X} = 9$ [**X = 81**]
2. $\sqrt{X} = 3/4$ [**X = 9/16**]
3. $2.5\sqrt{X} = 10$ [**X = 16**]
4. $\sqrt{X} = COS(30°)$ [**X = 0.75**]
5. $\sqrt{X} = \sqrt{225}$ [**X = 225**]
6. $\sqrt{X} = COS(75°)/LOG(25)$ [**X = 0.0343**]
7. $\sqrt{X} = COS(45°) + SIN(45°)$ [**X = 2**]
8. $(\sqrt{X})^2 = (30.25)^2$ [**X = 915.0625**]
9. $\sqrt{X} = [COS(12.5°) + TAN(12.5°)]/SIN(12.5°)$ [**X = 30.636**]
10. $\sqrt{25}\sqrt{X} = 2000$ [**X = 160000**]
11. $\sqrt{(16X)} = 24$ **HINT: $\sqrt{(16X)} = \sqrt{16}\sqrt{X}$** [**X = 36**]
12. $SIN(87°)\sqrt{25X} = LOG(63)$ [**X = 0.3604**]

INTRODUCTION TO GEOMETRY

The Foundation Course is dedicated to your learning how to solve practical math problems that arise in a wide variety of industrial and "real world" situations.

In addition to learning how to use the power tool called a scientific calculator, you need to learn material from three fields, Algebra, Geometry and Trigonometry.

Geometry is the "Centerpiece" of math that you will use in most problems. It is all about physical space in one, two, and three dimensions: Lines, Flat Surfaces and 3-D objects.

Algebra is a tool that is often used along with Geometry to solve problems.

You use Geometry to set up an equation which you then solve for the unknown. The unknown might be a length, or some dimension you need to know, or area, or volume.

Trigonometry is a special subject used for triangles. There are occasions where you cannot solve a problem with just algebra and geometry alone and where you need trigonometry. It deals with triangles.

Geometry is one of humankind's oldest mathematical subjects along with numbers and algebra.

Geometry is the foundation of modern science and technology and much modern mathematics.

Mathematics is like a "contact" sport, or a game.

You learn by practicing and "doing."

Each Lesson will include a video discussion of the topic just as we did in Algebra.

Then you will be given Homework Problems to work.

You are encouraged to make up your own problems.

The more you "play" and the more questions you ask, the better you will learn.

When you think you are ready, take the Online Quiz.

This will give you an indicator if you have mastered the material. If not, go back and "play" some more.

Learning math is like climbing a ladder. If you do it one small step at a time, it is pretty easy. But, it is difficult to go from rung 4 to rung 9 directly.

This Foundation course has been designed to let you climb the ladder of math understanding in small steps.

But, **YOU** must do the climbing. Watching someone else climb isn't enough. Play the game.

G1 LESSON: WHAT IS GEOMETRY?

Mathematics is based on two fundamental concepts:

Numbers and Geometry

Numbers are used to count and measure things.

Geometry is used to model physical things.

There are actually several different kinds of geometry.

We will study the oldest of all geometries, **Euclidean**.

Euclidean Geometry is used in most practical situations.

We will study:

Points:	0 dimensional
Lines:	1 dimensional
Surface Objects:	2 dimensions
And:	3-D objects

We will learn how to analyze many geometric situations and then set up **Equations** to find the value of various unknowns. This could be how long something is, or how much area something is, or the volume of something.

Many of the practical problems one comes across in many walks of life involve some type of geometric object.

Historically, in our schools, emphasis has been placed on proving theorems (statements about geometric objects) with rigorous logic and step by step deductions.

This can be difficult and tedious, and sometimes seemingly meaningless. We will emphasis sound reasoning in the Foundation Course, but not formal "proofs."

G1E

WHAT IS GEOMETRY?

1. Math is based on what two fundamental concepts?
2. Numbers are used to?
3. **Geometry** is used to?
4. The oldest kind of **Geometry** is?
5. In **Geometry** we will study what four things?
6. What will we use to find unknowns in **Geometry**?
7. What kind of **Unknowns** might we wish to find?

G1EA

WHAT IS GEOMETRY? Answers: []

1. Math is based on what two fundamental concepts?

   [**Numbers and Geometry**]

2. Numbers are used to? [**Count and measure things**]

3. Geometry is used to? [**Model physical things**]

4. The oldest kind of geometry is? [**Euclidean**]

5. In Geometry we will study what four things?

   [**Points: 0 dimensional**]
   [**Lines: 1 dimension**]
   [**Surface Objects: 2 dimensions**]
   [**And: 3-D objects**]

6. What will we use to find Unknowns in Geometry?

   [**Equations and Algebra**]

7. What kind of Unknowns might we wish to find?

[**This could be how long something is, or how much area something is, or the volume of something.**]

Many of the practical problems one comes across in many walks of life involve some type of geometric object.

G2 LESSON: STRAIGHT LINES AND ANGLES

A **Point** is ideally a location in space with no length or width. It has zero area.

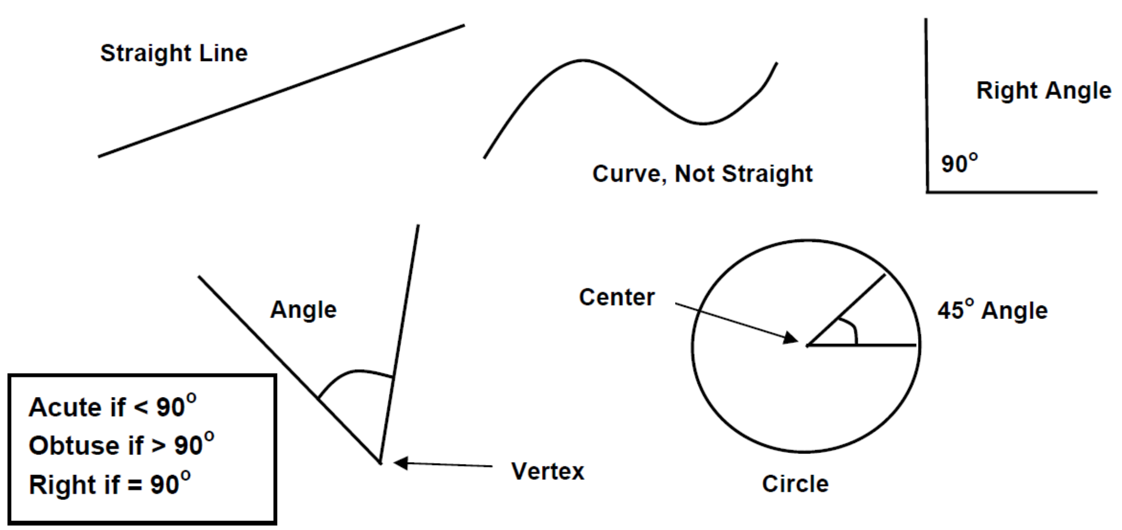
A **Plane** is a flat surface consisting of points. Think of a wall or blackboard as a plane. It is a surface with zero curvature.

A **Straight Line** (Segment) is the collection of points between two points that represents the shortest distance between them. It too has zero curvature. A **Straight Line** can be extended indefinitely.

The intersection of two lines (**straight**, unless I otherwise state), forms an **Angle** and their point of intersection is called the **Vertex.**

Angles are measured in **Degrees** (°) where there are 360° in a complete circle, a set of points equidistant from a point, center.

A **Right Angle** measures 90° and the two sides are **Perpendicular**.



G2E

STRAIGHT LINES AND ANGLES

1. What are: Point, Plane, and Straight Line?

2. What are an Angle and a Vertex?

3. How are Angles measured?

4. What is a Right Angle?

5. What are Acute and Obtuse Angles?

STRAIGHT LINES AND ANGLES Answers: []

1. What are: Point, Plane, and Straight Line?

   [A Point is ideally a location in space with no length or width. It has zero area.

   A Plane is a flat surface consisting of points. Think of a wall or blackboard as a plane. It is a surface with zero curvature.

   A Straight Line (Segment) is the collection of points between two points that represents the shortest distance between them.]

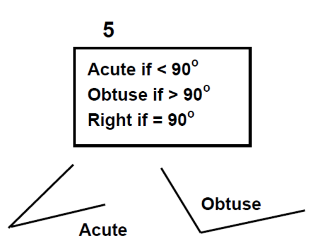
2. What are an Angle and a Vertex?

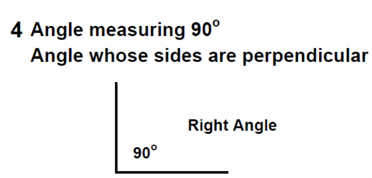
   [The intersection of two lines (straight, unless I otherwise state), forms an Angle and their point of intersection is called the Vertex.]

3. How are Angles measured?

   [Angles are measured in Degrees (^{o}) where there are 360^o in a complete circle, a set of points equidistant from a point, center.]

4. What is a Right Angle? [See Below Right]
5. What are Acute and Obtuse Angles? [See Below Left]





G3 LESSON: PARALLEL LINES

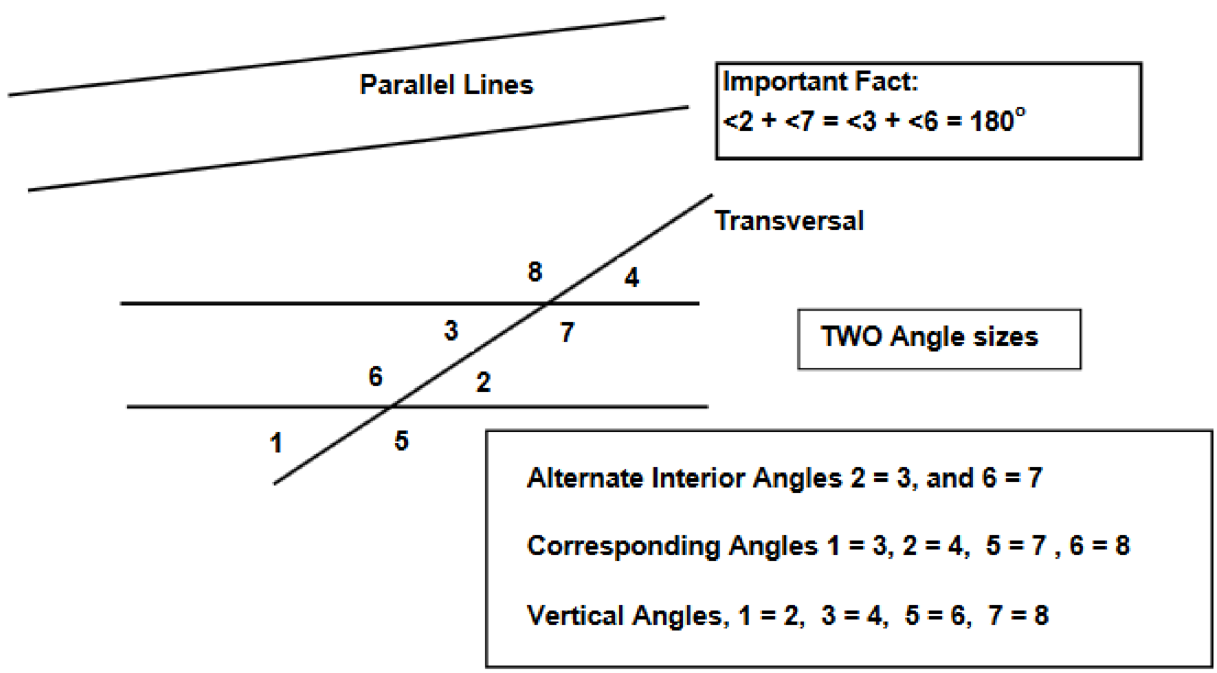
Two straight lines are **parallel** if they never intersect no matter how far they are extended in either direction.

The Fundamental Property in **Euclidean** Geometry is:

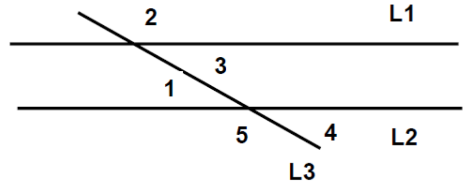
Given a straight line and an external point, there is exactly one straight line through this point parallel to the given line.

This is called the **Parallel Postulate** and it is not true for other **non-Euclidean** geometries.

When two parallel lines are crossed by another straight line, called a **transversal**, eight angles are created in two sets of four equal-sized angles. This is a critical property.

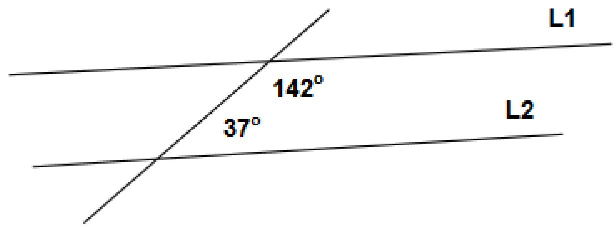


G3 Problems for Parallel Lines

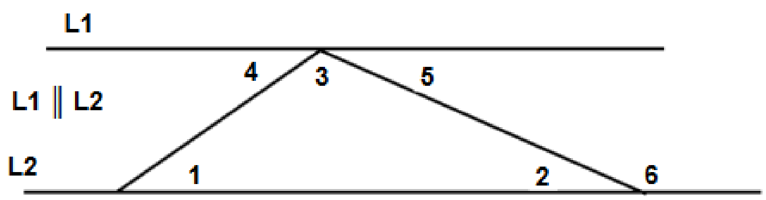


L1 ∥ L2

1. Given <1=42^{o}, what are the sizes of <2 and <3?
2. Given <3=50^{o}, what are the sizes of <4 and <5?
3. Given <2=132^{o}, what are the sizes of <1 and <4?



4. Are L1 and L2 parallel?
5. What is the sum of two interior angles if the lines are parallel?

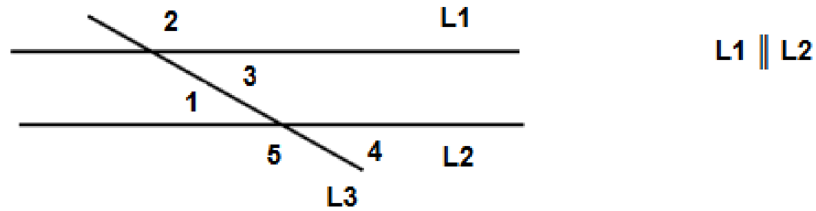


6. What is <4 + <3 +<5 =?
7. Which of the angles are equal <1, <2, <3, <4, <5=?
8. What is <1 + <2 + <3=?
9. If <1 = 43^{o} and <3 = 102^{o} then what does <6=?
10. In problem #9, what does <2=?

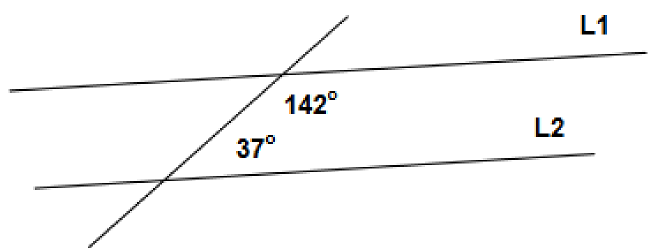
ANSWERS

1. <2 = 138^{o} and <3 = 42^{o}
2. <4 = 50^{o} and <5 = 130^{o}
3. <1 = <4 = 48^{o}
4. No
5. 180^{o}
6. 180^{o}
7. <1 = <4 and <2 = <5
8. 180^{o}
9. 43^{o} + 102^{o} = 145^{o}
10. 180^{o} -145^{o} = 35^{o}

PARALLEL LINES

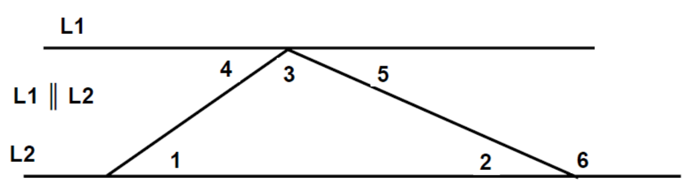


1. Given <1 = 38°, what are the sizes of <2 and <3?
2. Given <3 = 54°, what are sizes of <4 and <5?
3. Given <2 = 138°, what are sizes of <1 and <4?
4. Given:



Are L1 and L2 Parallel?

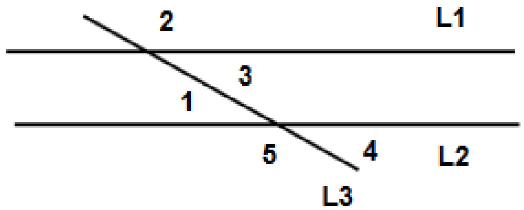
5. What is the sum of two opposite interior angles if the lines are parallel?



6. What is <4 + <3 + <5 =?
7. Which of these angles are equal <1, <2, <3, <4, <5?
8. What is <1 + <2 + <3 =?
9. If <1 = 42° and <3 = 105°, what does <6 =?
10. In problem #9, what does <2 =?
11. The sum of the three angles of a triangle equal?

G3EA

PARALLEL LINES Answers: []



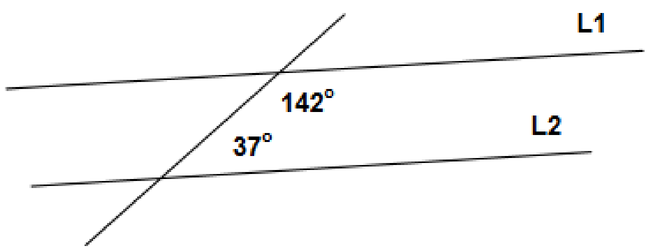
L1 ∥ L2

1. Given <1 = 38^{o}, what are the sizes of <2 and <3?
   [**<2 = 142^{o} and <3 = 38^{o}**]

2. Given <3 = 54^{o}, what are sizes of <4 and <5?
   [**<4 = 54^{o} and <5 = 126^{o}**]

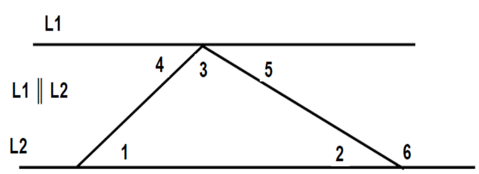
3. Given <2 = 138^{o}, what are sizes of <1 and <4?
   [**<1 = 42^{o} and <5 = 138^{o}**]

4. Given:



   Are L1 and L2 Parallel? [**NO because: 142 + 37 = 179 not 80**]

5. What is the sum of two interior angles if the lines are parallel?
   [**<5 + <6 = 180^{o}**]



6. What is <4 + <3 + <5 =? [**180^{o}**]

7. Which of these angles are equal <1, <2, <3, <4, <5?
   [**<1 = <4**]

G3 EA (cont'd)

PARALLEL LINES (cont'd) Answers: []

8. What is <1 + <2 + <3 =? [180°]

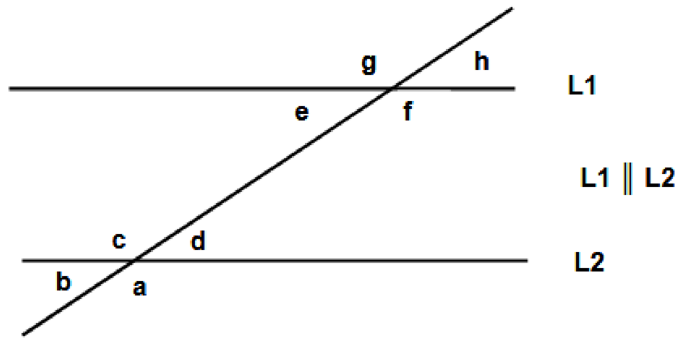
9. If <1 = 42° and <3 = 105°, what does <6 =? [**147°**]

10. In problem #9, what does <2 =? [**33°**]

11. The sum of the three angles of a triangle equal? [**180°**]

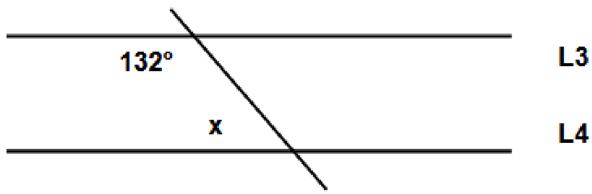
G3ES

PARALLEL LINES



1.) How many angles do you need to know in order to replace the letters in the diagram to the left?

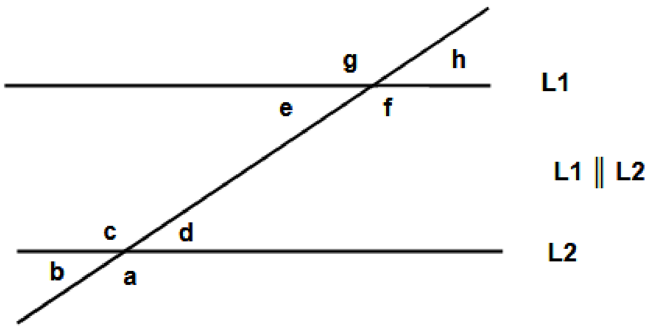
2.) If <a = 115, find the rest of the remaining angles.



3.) If L3 and L4 are parallel, what must <x equal?

4.) If two lines are truly parallel, what will they never do?
Hint: Think about intersecting lines

PARALLEL LINES

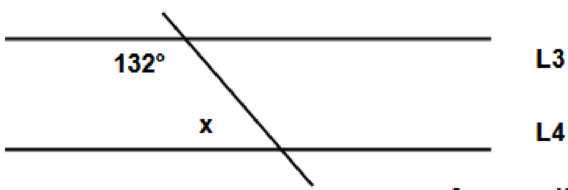


1.) How many angles do you need to know in order to replace the letters in the diagram to the left?

Answer: Only 1 angle.

2.) If <a = 115, find the rest of the remaining angles.

Answer: <b = <d = <e = <h = 65°, <a = <c = <f = <g = 115°



3.) If L3 and L4 are parallel, what must <x equal?

Answer: If L3 and L4 are parallel, <x and 132° must add up to 180°, therefore <x = 180 - 132 = 48°

4.) If two lines are truly parallel, what will they never do?
Hint: Think about intersecting lines

Answer: Two parallel lines will never touch.

G4 LESSON: TRIANGLES, DEFINITION, SUM OF ANGLES

A Triangle is a three-sided **polygon**, i.e., a geometric figure created by three intersecting straight lines. Thus, a triangle has three sides and three vertices.

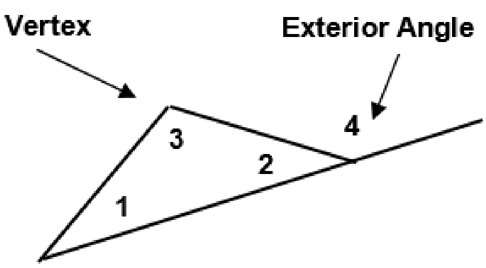
The sum of the three interior angles of a triangle is always 180°.
Exterior Angle = Sum of opposite Interiors

1 + 2 + 3 = 180° and   4 = 1 + 3

Triangles are often used to model a physical situation.

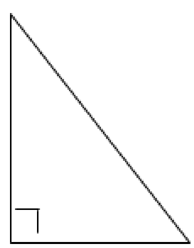
There are several types of triangles:

Right, Acute, Obtuse, Isosceles, and Equilateral. See below.

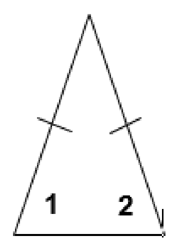


<1 + <2 + <3 = 180°

<4 = <1 + <3



Right Triangle

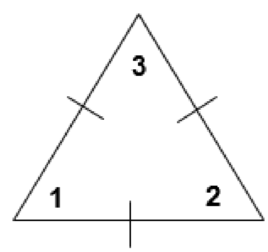


Isosceles Triangle
<1 = <2
Two Equal Sides

Acute **All angles < 90°**

Obtuse **One angle > 90°**

Right **One angle = 90°**



Equilateral Triangle
<1 = <2 = <3 = 60°

Three Equal Sides

G4 Triangle Problems

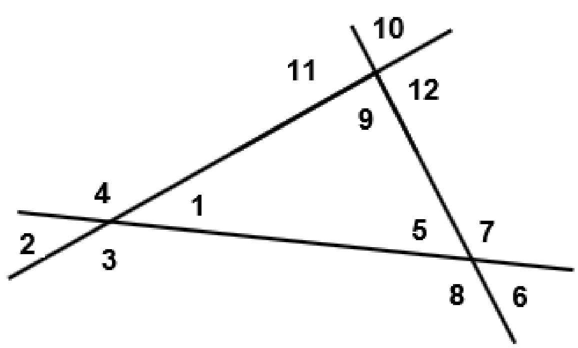
Finding unknown angles from known angles.

Each **vertex** of a triangle has four angles associated with it for a total of twelve angles for a triangle. There will be six values.

If you know any two angles from two different vertices, then you can calculate all the other angles.

This is demonstrated below.

Note 1: Angles do not have the < symbol



$1 + 5 + 9 = 180^\circ$

$4 = 3 = 180^\circ - 1$ $\quad 2 = 1$

$7 = 8 = 180^\circ - 5$ $\quad 6 = 5$

$11 = 12 = 180^\circ - 9$ $\quad 10 = 9$

Given any two angles from two vertices, we can calculate all the other angles.

Example 1 $\quad 1 = 40^\circ$ and $7 = 120^\circ$ $\quad$ **Find the other angles**

Answers $\quad 5 = 6 = 180^\circ - 120^\circ = 60^\circ$ $\quad 8 = 120^\circ$

$4 = 3 = 180^\circ - 40^\circ$ $\quad 2 = 40^\circ$

** $\quad 9 = 10 = 180^\circ - 1 - 5 = 180^\circ - 40^\circ - 60^\circ = 80^\circ$

$11 = 12 = 180^\circ - 80^\circ = 100^\circ$

Example 2 $\quad 9 = 75^\circ$ and $8 = 110^\circ$ **Find other angles**

Answers $\quad 5 = 6 = 180^\circ - 110^\circ = 70^\circ$ and $7 = 110^\circ$

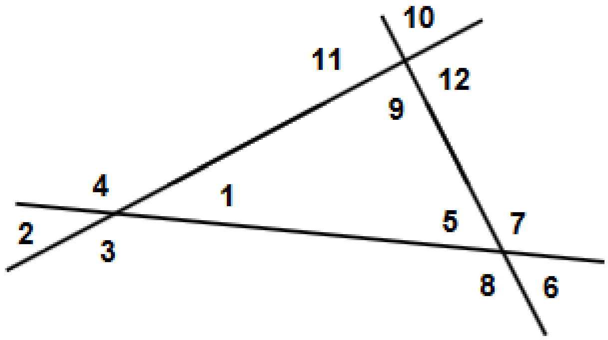
$11 = 12 = 180^\circ - 75^\circ = 105^\circ$ and $10 = 75^\circ$

** $\quad 2 = 1 = 180^\circ - 75^\circ - 70^\circ = 35^\circ$ and $4 = 3 = 180^\circ - 35^\circ = 145^\circ$

G4E

TRIANGLES

Find the unknown angles from known angles below.



$1 + 5 + 9 = 180^\circ$

$4 = 3 = 180^\circ - 1$ $2 = 1$

$7 = 8 = 180^\circ - 5$ $6 = 5$

$11 = 12 = 180^\circ - 9$ $10 = 9$

Given any two angles from two vertices, we can calculate all the other angles.

Exercise 1: $1 = 40^\circ$ and $7 = 120^\circ$ **Find the other angles.**

Exercise 2: $9 = 75^\circ$ and $8 = 110^\circ$ **Find the other angles.**

Exercise 3: $2 = 38^\circ$ and $10 = 70^\circ$ **Find the other angles.**

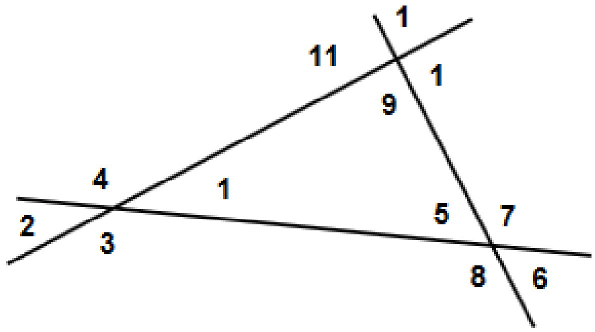
Exercise 4: $9 = 72^\circ$ and $6 = 68^\circ$ **Find the other angles.**

Exercise 5: $4 = 135^\circ$ and $7 = 118^\circ$ **Find the other angles.**

Exercise 6: $10 = 85^\circ$ and $12 = 95^\circ$ **Find the other angles.**

TRIANGLES

Find the unknown angles from known angles below.



$1 + 5 + 9 = 180^\circ$

$4 = 3 = 180^\circ - 1$ $2 = 1$

$7 = 8 = 180^\circ - 5$ $6 = 5$

$11 = 12 = 180^\circ - 9$ $10 = 9$

Given any two angles from two vertices, we can calculate all the other angles.

Exercise 1 $1 = 40^\circ$ and $7 = 120^\circ$ Find the other angles

Answers
$5 = 6 = 180^\circ - 120^\circ = 60^\circ$ $8 = 120^\circ$
$4 = 3 = 180^\circ - 40^\circ = 140^\circ$ $2 = 40^\circ$
$11 = 12 = 180^\circ - 80^\circ = 100^\circ$ $9 = 10 = 80^\circ$

Exercise 2 $9 = 75^\circ$ and $8 = 110^\circ$ Find other angles

Answers
$5 = 6 = 180^\circ - 110^\circ = 70^\circ$ and $7 = 110^\circ$
$11 = 12 = 180^\circ - 75^\circ = 105^\circ$ and $10 = 75^\circ$
$2 = 1 = 180^\circ - 75^\circ - 70^\circ = 35^\circ$ and $4 = 3 = 180^\circ - 35^\circ = 145^\circ$

Exercise 3 $2 = 38^\circ$ and $10 = 70^\circ$ Find the other angles

Answers
$1 = 38^\circ$ and $3 = 4 = 142^\circ$
$9 = 70^\circ$ and $11 = 12 = 110^\circ$
$5 = 6 = 72^\circ$ and $7 = 8 = 108^\circ$

Exercise 4 $9 = 72^\circ$ and $6 = 68^\circ$ Find the other angles

Answers
$5 = 6 = 68^\circ$ and $7 = 8 = 112^\circ$
$11 = 12 = 108^\circ$ and $10 = 72^\circ$
$2 = 1 = 40^\circ$ and $4 = 3 = 140^\circ$

Exercise 5 $4 = 135^\circ$ and $7 = 118^\circ$ Find the other angles

Answers
$5 = 6 = 62^\circ$ and $7 = 8 = 118^\circ$
$11 = 12 = 107^\circ$ and $9 = 10 = 73^\circ$
$2 = 1 = 45^\circ$ and $4 = 3 = 135^\circ$

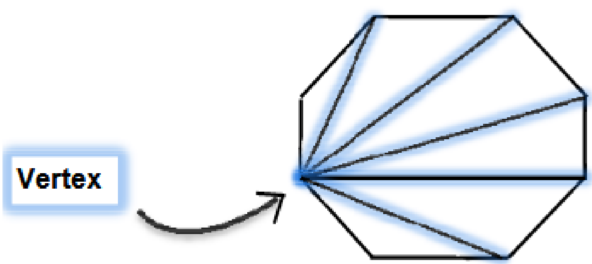
Exercise 6 $10 = 85^\circ$ and $12 = 95^\circ$ Find the other angles

Answers
$9 = 85^\circ$ and $11 = 95^\circ$
Not enough information for the other angles.

TRIANGLES

Note: The interior angles of any polygon add up to the number of sides the shape has - 2 and then multiplied by 180.

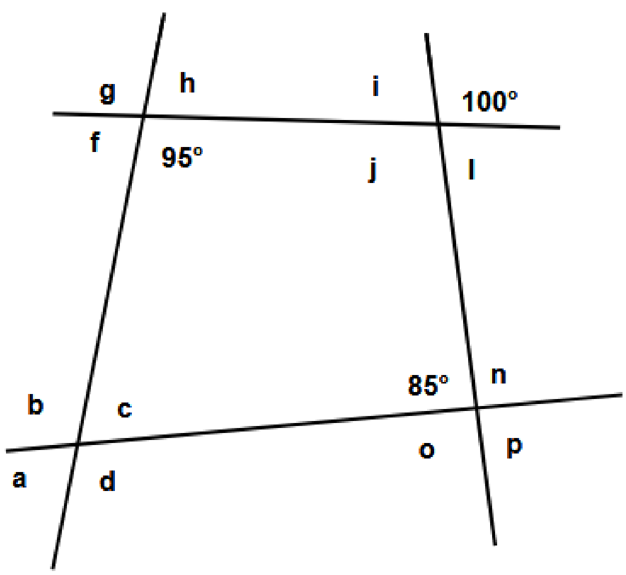
Ex. Triangles have 3 sides --> (3 - 2) times 180 = 180°
Ex. Rectangles have 4 sides --> (4 - 2) times 180 = 360°



1.) The reasoning behind this trick all comes back to triangles. How many degrees does a triangle's interior angles add up to?

2.) Now how many triangles can we break up this octagon into from a single vertex?

****Note: A vertex is just a corner made by two lines!****



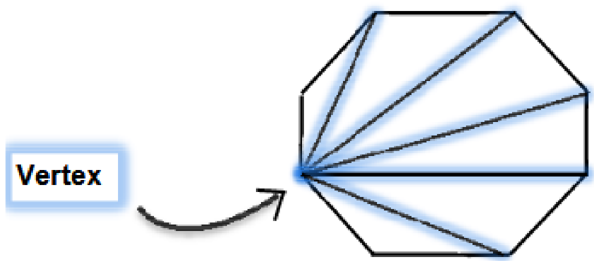
3.) With the help of this trick, find the remaining angles in the diagram to the left.

G4ESA

TRIANGLES

Note: The interior angles of any polygon add up to the number of sides the shape has - 2 and then multiplied by 180.

Ex. Triangles have 3 sides --> (3 - 2) times 180 = 180°
Ex. Rectangles have 4 sides --> (4 - 2) times 180 = 360°



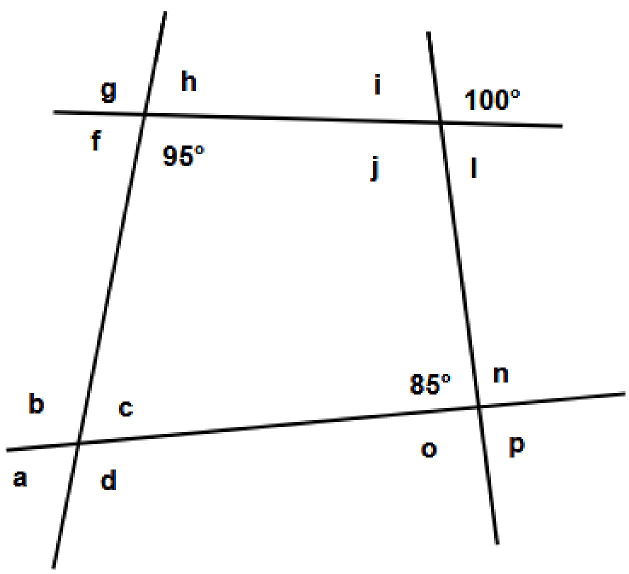
1.) The reasoning behind this trick all comes back to triangles. How many degrees does a triangle's interior angles add up to?

Answer: 180°

2.) Now how many triangles can we break up this octagon into from a single vertex?

****Note: A vertex is just a corner made by two lines!****

Answer: 6 triangles



3.) With the help of this trick, find the remaining angles in the diagram to the left.

Answer:
a = c = 80°
b = d = 100°
f = h = 85°
g = 95°
i = l = 80°
j = 100°
n = o = 95°
p = 85°

G5 LESSON: RIGHT TRIANGLES - PYTHAGOREAN THEOREM

A **Right Triangle** has one of its angles = 90^o

The side opposite the **right angle** is called the **Hypotenuse.**

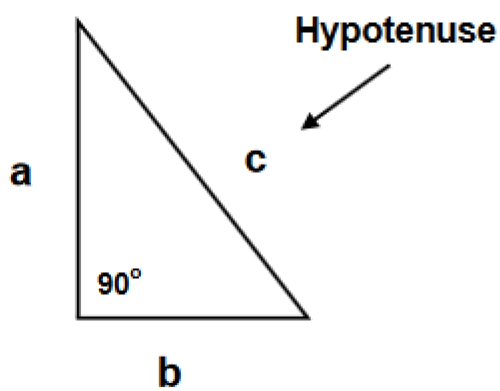
The sum of the **other two angles** will sum to 90^o

The Lengths of the three sides of a **Right Triangle** are related by the **Pythagorean Theorem.**

If, they are **a**, **b**, and **C** where "**C**" is the **hypotenuse**, then:

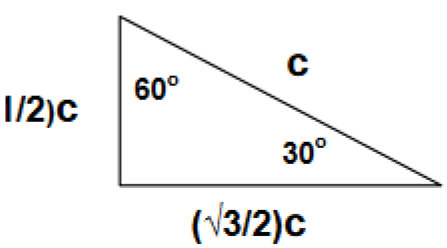
$$a^2 + b^2 = c^2$$

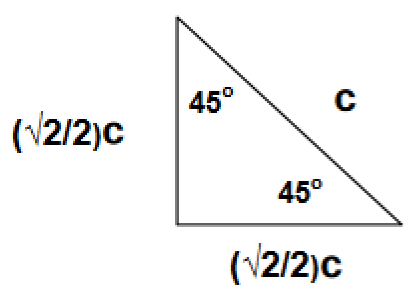
So, $c = \sqrt{a^2 + b^2}$; $b = \sqrt{c^2 - a^2}$; $a = \sqrt{c^2 - b^2}$



Pythogorean Theorem

$$a^2 + b^2 = c^2$$

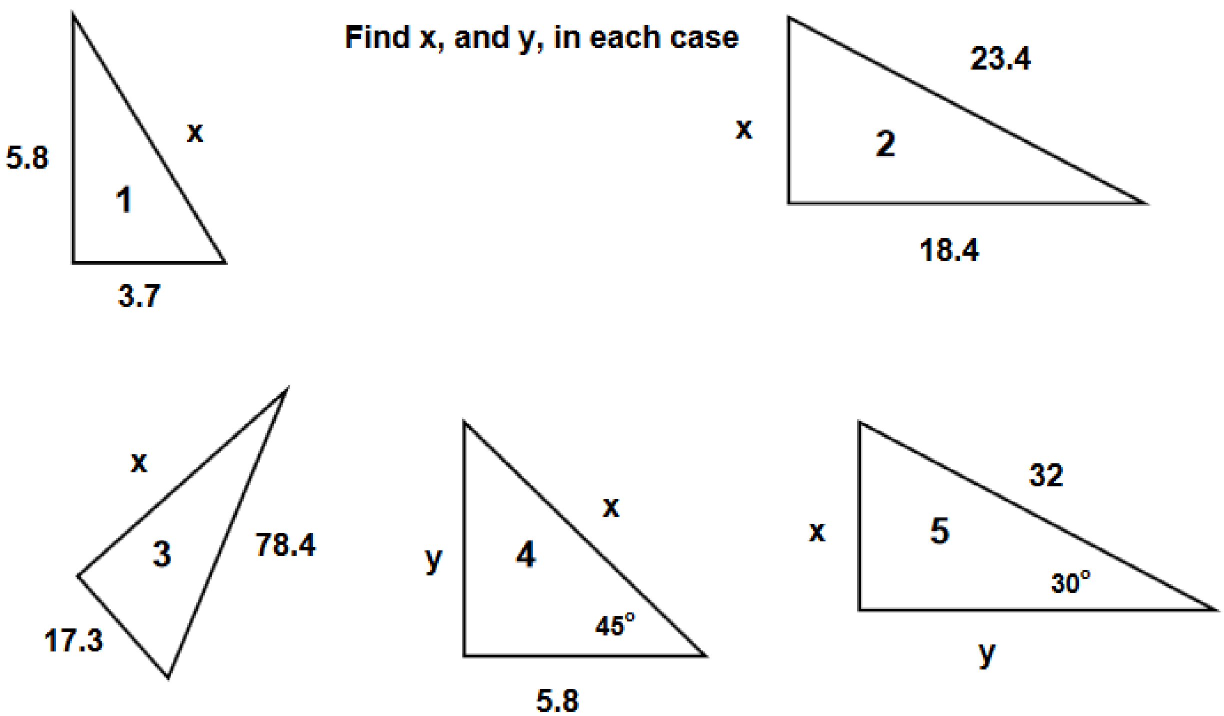




G5 Right Triangle Problems

Typically, you are given one or two sides or angles and want to figure out the other sides or angles.

Here are a few examples (You will typically use the **Pythagorean Theorem** and a calculator):



Answers 1. x = 6.9 2. x = 14.5 3. 76.5 4. y = 5.8, x = 8.2 5. x = 16, y = 27.7

G5E

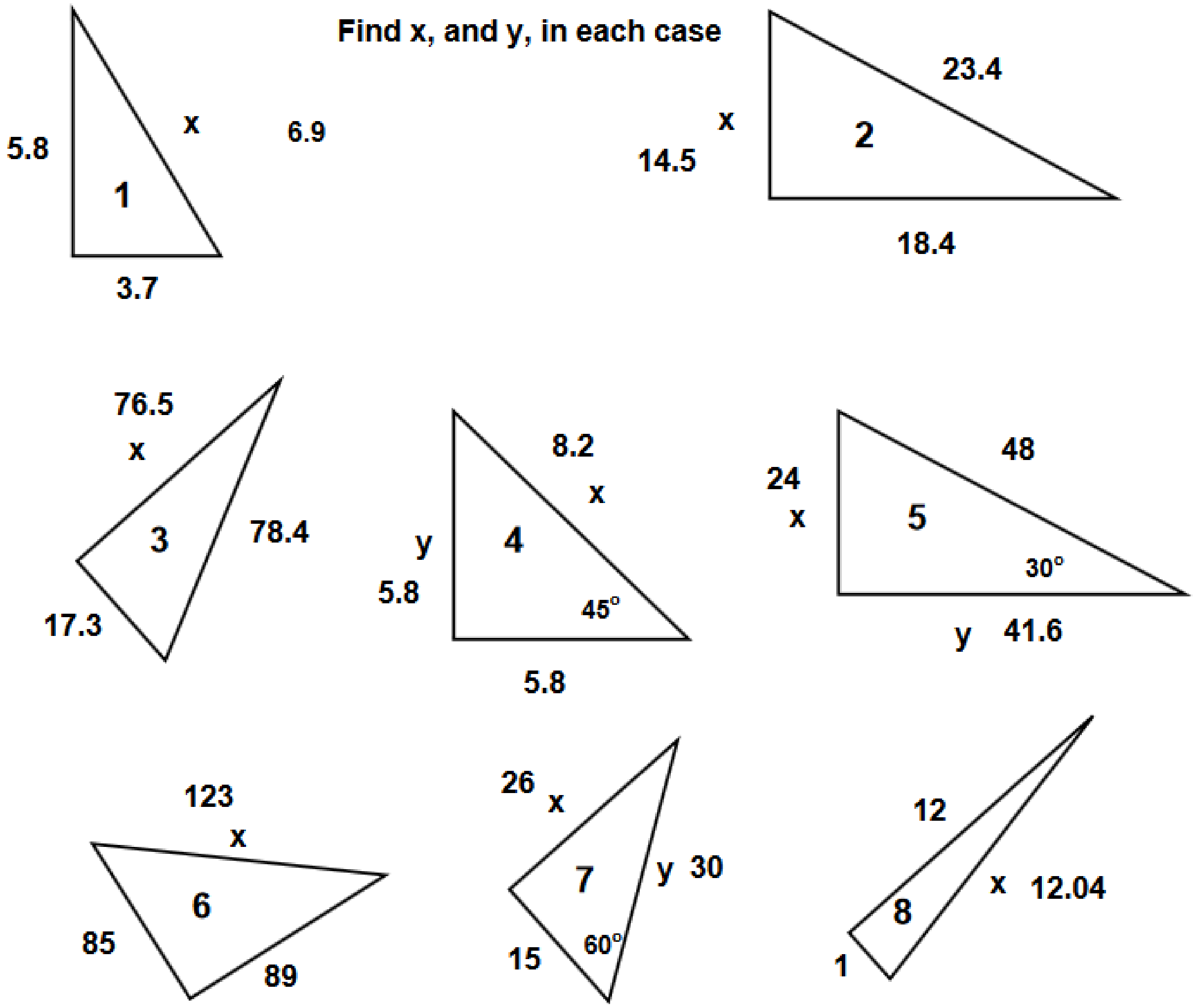
RIGHT TRIANGLES

Find **x** and **y** in each of the Exercises below.

You will typically use the **Pythagorean Theorem** and a calculator.

All triangles below are **right triangles**.

Find x, and y, in each case



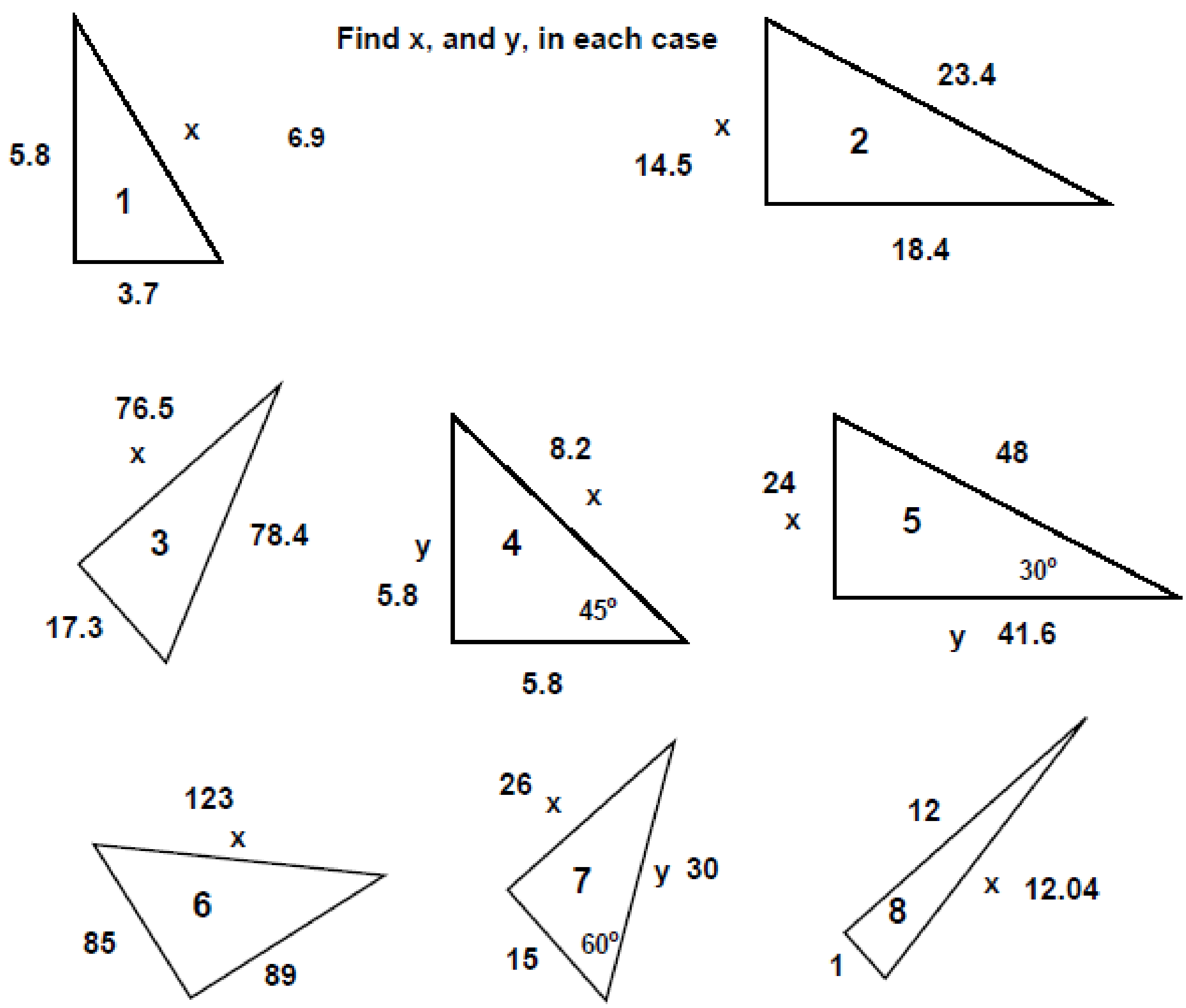
G5EA

RIGHT TRIANGLES

Find **x** and **y** in each of the Exercises below.

You will typically use the **Pythagorean Theorem** and a calculator.

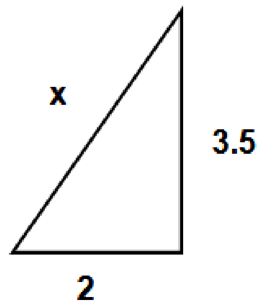
All triangles below are **right triangles**.

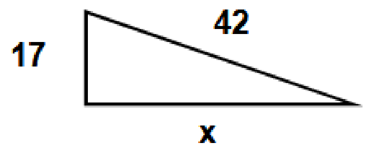


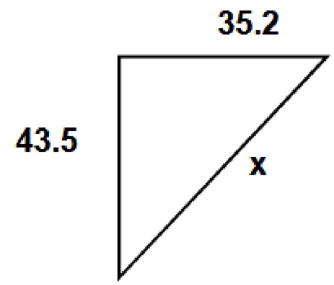
G5ES

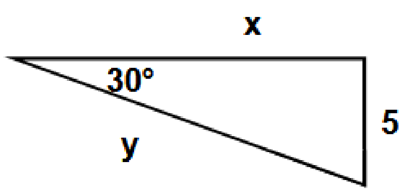
RIGHT TRIANGLES

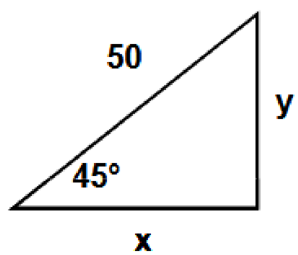
Find the unknowns, x and y.

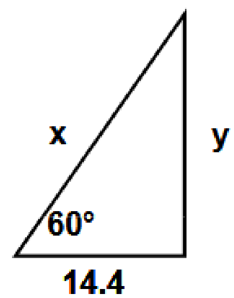






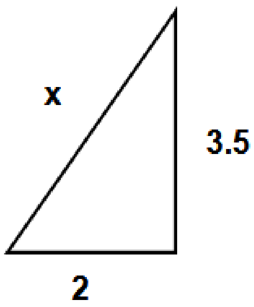




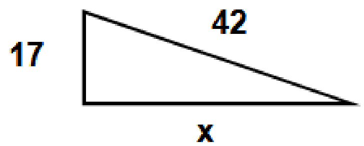


G5ESA

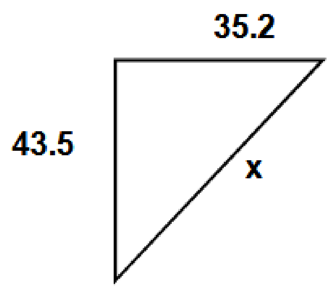
RIGHT TRIANGLES



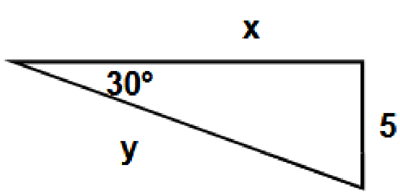
x = 4.03



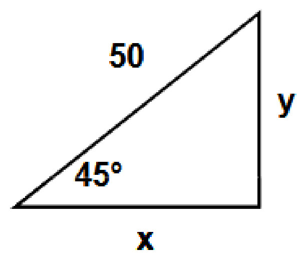
x = 38.41



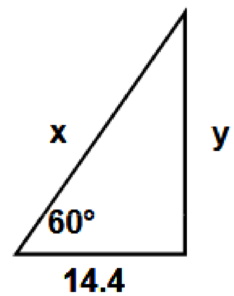
x = 55.95



x = 8.66, y = 10



x = 35.36, y = 35.36



x = 28.8, y = 24.94

G6 LESSON: SIMILAR TRIANGLES

Two Triangles are similar if they have equal angles.

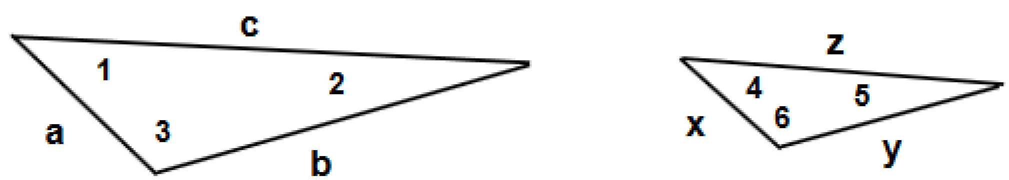
This means they have the same "**shape**" but may be of different sizes. If they also are the same size they are **congruent**.

Similar triangles appear frequently in practical problems.

Their corresponding ratios are equal, and that is what makes them so important and useful.

This is often the way you set up an **Equation** to find an **Unknown**.

Note: If **two** sets of angles are equal, the **third** must be equal also, and the triangles are similar.



Given: 1 = 4 ; 2 = 5 ; 3 = 6, Called corresponding angles

Corresponding sides are: a ↔ x ; b ↔ y ; c ↔ z

The Following Ratios are Equal

a/x = b/y = c/z	and	**x/a = y/b = z/c**
a/b = x/y	**a/c = x/z**	**b/c = y/z**
b/a = y/x	**c/a = z/x**	**c/b = z/y**

G6 Similar Triangles Problems

When you have **two equal ratios** with one **unknown** it is a simple algebra problem to solve for the unknown **X**.

X/a = b/c **and X** = a(b/c) **X**/3 = 7/12 **and X** = 3x(7/12) = 1.75

a/**X** = b/c **and X** = a(c/b) 3/**X** = 7/12 **and X** = 3x(12/7) = 5.15

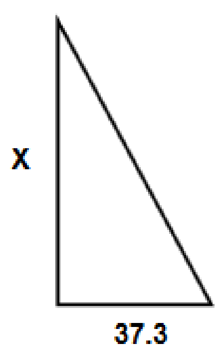
Find two similar triangles where the **unknown** is one side and you know three more sides, one of which is opposite the corresponding angle of the unknown.

Given: 1 = 4 ; 2 = 5 ; 3 = 6 Find x

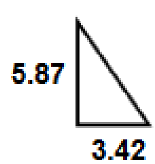
x/ 12.3 = 3.7/5.4 so x = 12.3(3.7/5.4) = 8.4

or x/3.7 = 12.3/5.4 so x = 3.7(12.3/5.4) = 8.4

Wrong: x/3.7 = 5.4/12.3 See why?



How tall is the Pole? The horizontal lines are shadows



x/5.87 = 37.3/3.42

x = 5.87(37.3.3/3.42)

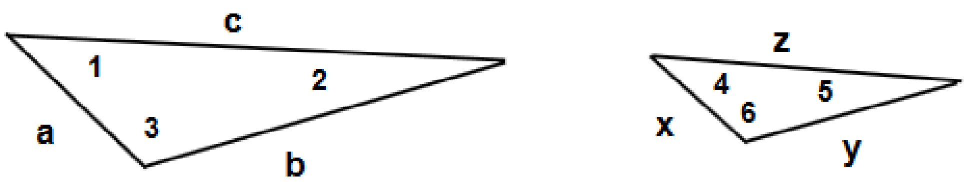
= 64.02 = 64.0

G6E

SIMILAR TRIANGLES

In each Exercise assume the triangles are similar.

Find lengths that you can.

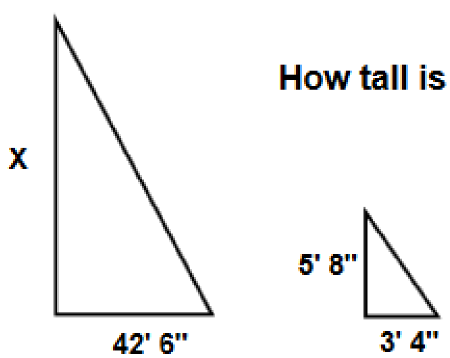


Given: <1 = <4 and <2 = <5

1. **What can you conclude about <3 and <6 and why?**
2. **What are the corresponding sides in pairs?**
3. **a = 12.3, b = 18.7, x = 5.4, y = ?, z = ?**
4. **c = 1435, z = 765, y = 453, What can you figure?**
5. **a = .05, x = .02, y = .04, What can you figure?**
6. **c = 4, b = 3, x = 1.5, What can you figure?**
7. **b = 23/8, x = 3/4, y = 4/5, What can you figure?**
8. **In Drawing below, how tall is the pole?**

How tall is the Pole? The horizontal lines are shadows

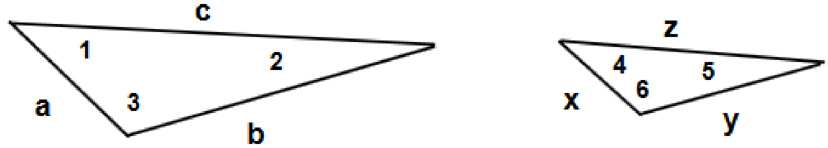
Hint: 1" = 1/12', So, 5'8" = (58/12)'



G6EA

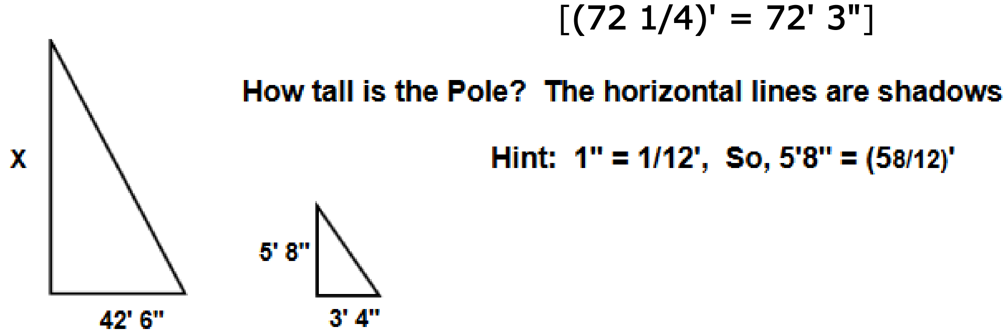
SIMILAR TRIANGLES   Answers: []

In each Exercise assume the triangles are similar. Find lengths that you can.



Given: <1 = <4 and <2 = <5

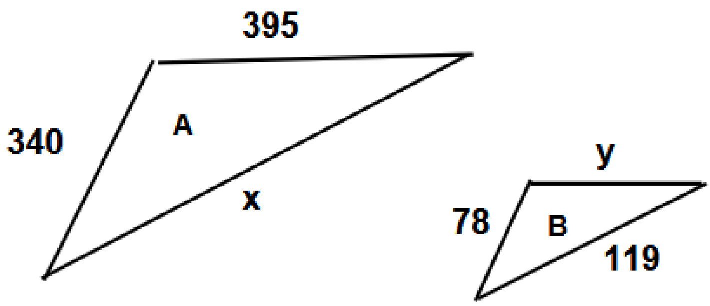
1. What can you conclude about <3 and <6 and why?
   [They are equal due to sum of angles of triangle equals 180°]
2. What are the corresponding sides in pairs?
   [a ↔ x, b ↔ y, c ↔ z]
3. a = 12.3, b = 18.7, x = 5.4, y = ?, z = ?
   [y = 8.2 Have not yet learned how to calculate z]
4. c = 1435, z = 765, y = 453 What can you figure?
   [b = 850]
5. a = 0.05, x = 0.02, y = 0.04 What can you figure?
   [b = 0.1]
6. c = 4, b = 3, x = 1.5 What can you figure?
   [Nothing with just similar triangles]
7. b = 2 3/8, x = 3/4 y = 4/5, What can you figure?
   [a = 2 29/128 = 2.23]
8. In drawing below, how tall is the pole?
   [(72 1/4)' = 72' 3"]



G6ES

SIMILAR TRIANGLES

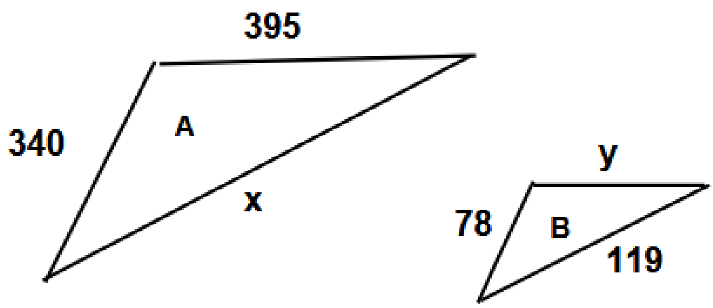
Find the unknowns, x and y.



Assume that the two triangles to the left are similar. Using this knowledge, find the unknown lengths.

G6ESA

SIMILAR TRIANGLES



Assume that the two triangles to the left are similar. Using this knowledge, find the unknown lengths.

x = 518.72, y = 90.62

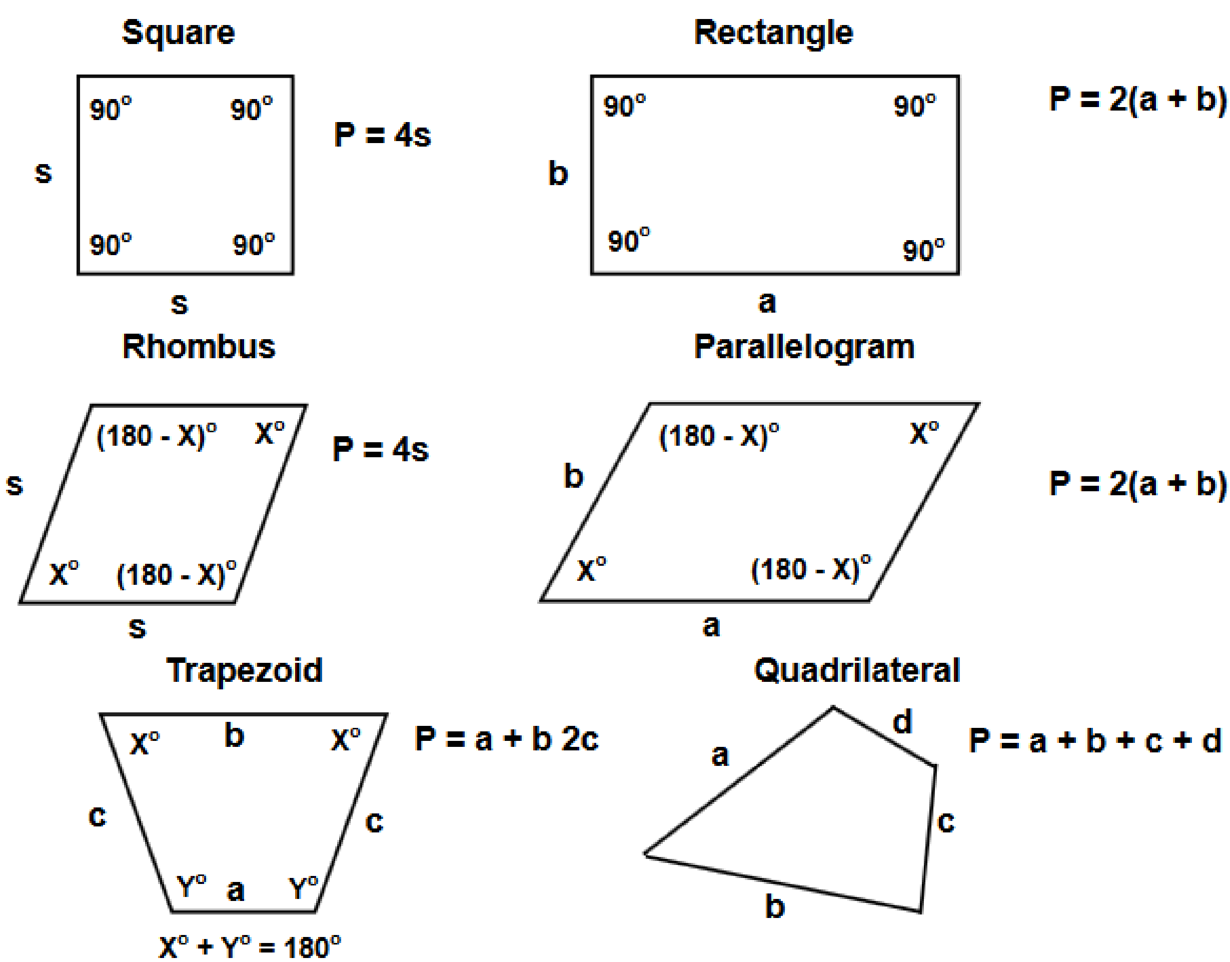
G7 LESSON: QUADRILATERALS, POLYGONS, PERIMETERS (P)

A **Polygon** is a closed geometric figure whose boundary is straight line segments. The **Perimeter** (**P**) is the distance around the polygon.

A **Quadrilateral** is a **polygon** with four sides.

Common **Quadrilaterals** are **Square**, **Rectangle**, **Rhombus**, **Parallelogram**, and **Trapezoid**.

There are three things one is usually interested in for any **quadrilateral**: **Dimensions**, **Perimeter** and **Area**.



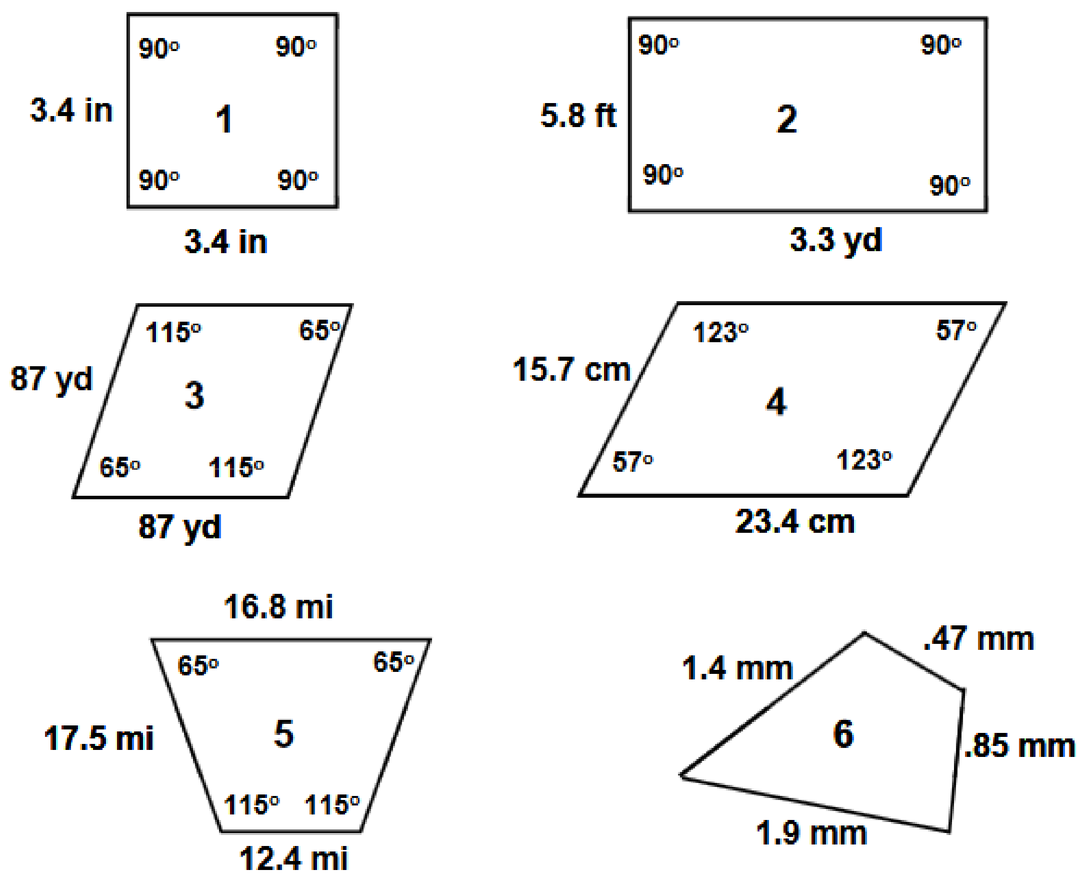
G7 Quadrilaterals, Polygons, Perimeters (P) Problems

Identify the figures below and compute their **Perimeters.**

Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.

Suppose a **rectangle** has one side 11/2 feet, and the other side 8 inches. Then, convert feet to inches or inches to feet.

Answers are at bottom of page - Number, name, Perimeter.



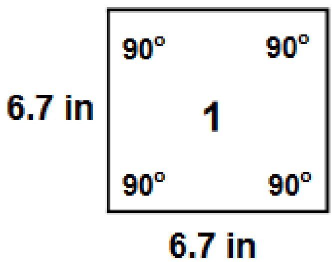
Answer: 1. Square 13.6 in 2. Rectangle 31.4 ft or 10.5 yd 3. Rhombus 348 yd
4. Parallelogram 78.2 cm 5. Trapezoid 64.2 mi 6. Quadrilateral 4.62 mm

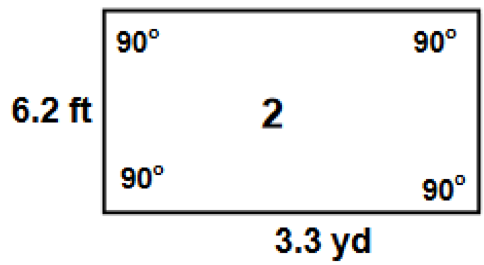
G7E

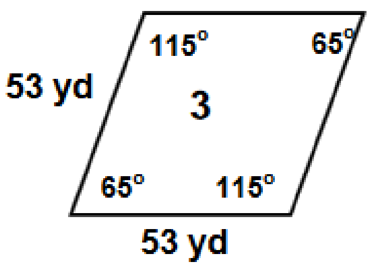
QUADRILATERALS, POLYGONS, PERIMETERS (P)

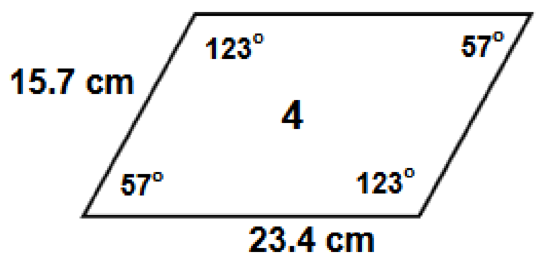
Identify the figures below and compute their Perimeters

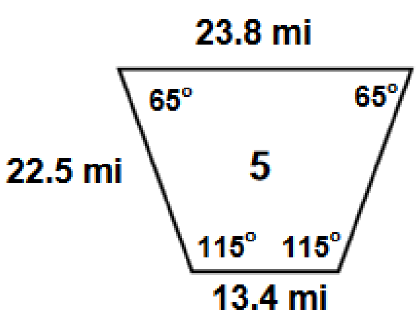
Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.

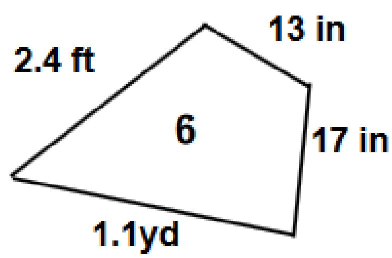


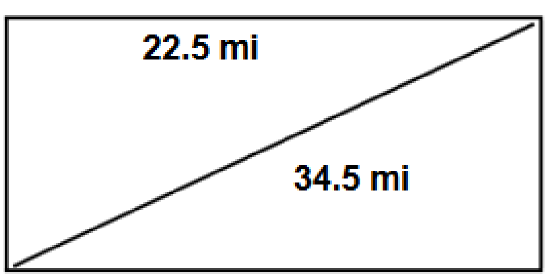












Hint: Use Pythogorean Theorem first

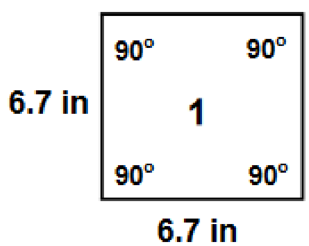
G7EA

QUADRILATERALS, POLYGONS, PERIMETERS (P)

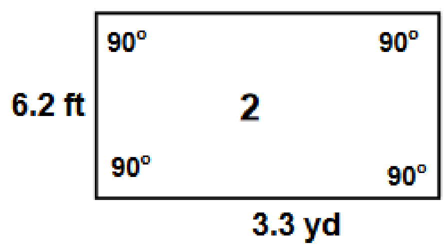
Identify the figures below and compute their Perimeters

Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.

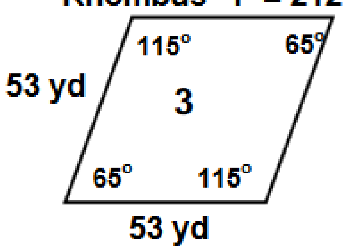
Square P = 26.8



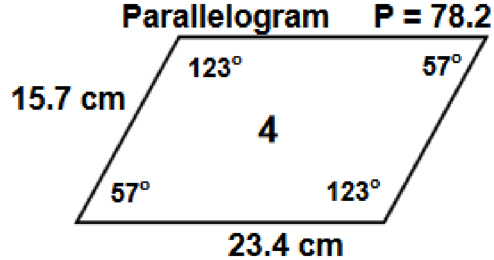
Rectangle P = 32.2 ft = 10.7 yd



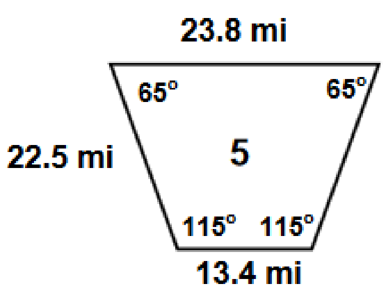
Rhombus P = 212 yd



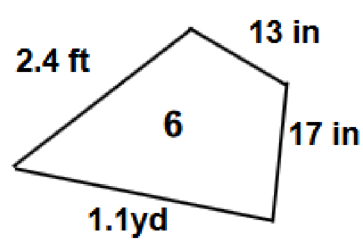
Parallelogram P = 78.2

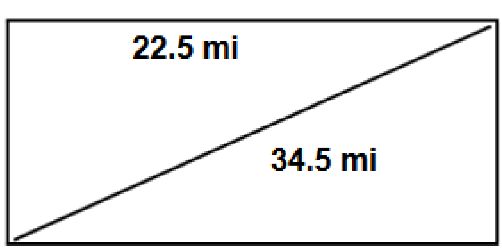


Trapezoid P = 82.2 mi



Polygon P = 98.4 in = 8.2 ft = 2.7 yd



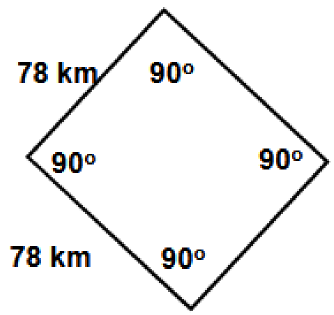


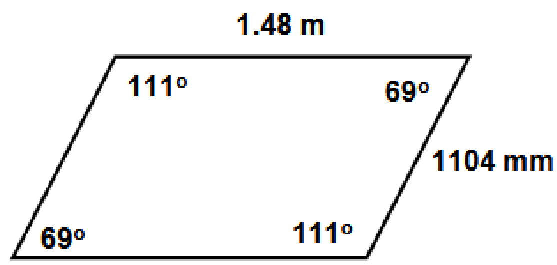
Rectangle P = 97.3 mi

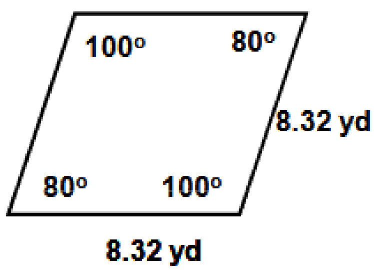
G7ES

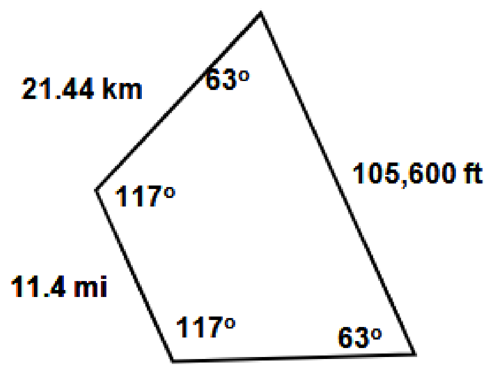
QUADRILATERALS, POLYGONS, PERIMETERS (P)

Identify the figures and calculate their perimeters. Be sure to check units and convert all numbers to the same unit where necessary.





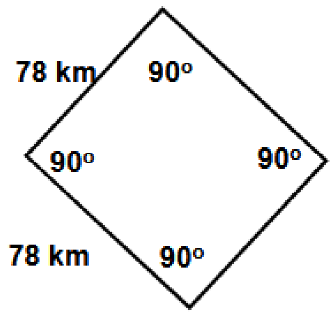




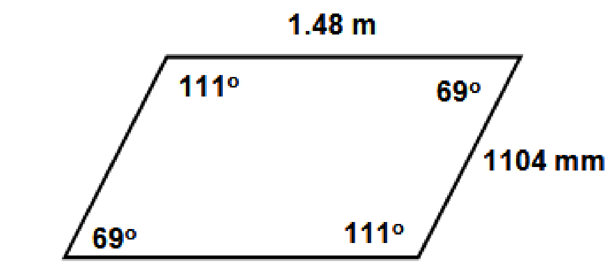
G7ESA

QUADRILATERALS, POLYGONS, PERIMETERS (P)

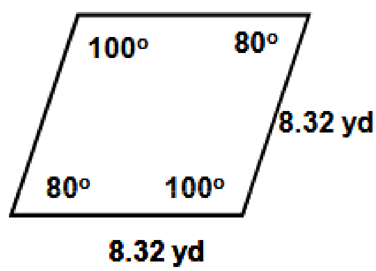
Identify the figures and calculate their perimeters. Be sure to check units and convert all numbers to the same unit where necessary.



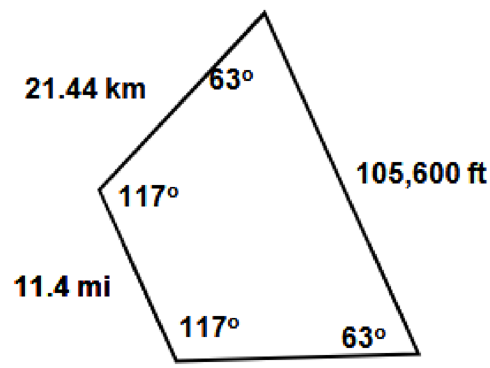
Square, P = 312 km



Parallelogram, P = 5.168 m = 5168mm



Rhombus, P = 33.28 yd



Trapezoid, P = 93.12 km = 58.2 mi = 307,296 ft

G8 LESSON: AREA OF TRIANGLES AND RECTANGLES

The **Area** of any **polygon** is a measure of its size.

The **Rectangle** is the simplest **polygon** and its **Area** is defined to be:

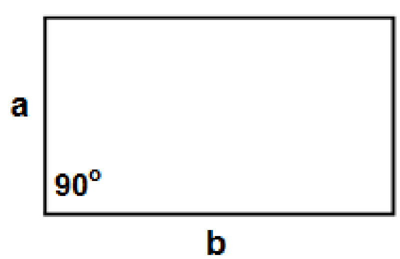
Area = ab where a and b are the lengths of its two sides.

A **Parallelogram** is a "**lopsided**" rectangle whose two adjacent sides have an angle **X^o** instead of 90^o.

Its **Area** can be calculated with a "**Correction Factor**" which is **SIN**(X^o)

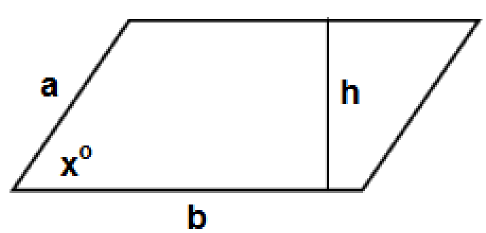
A **Triangle** is one-half of a **parallelogram**. So, its **Area** can be expressed with this same correction factor. **See Below.**

Of course, if one does know the "**height**" then one can use an alternative formula for the **Area**, which is usually given.

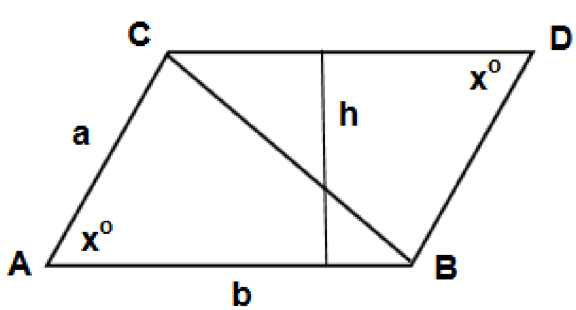


Area = ab

h = height



Area = abSIN(x^o)
Area = hb



Triangle ABC = Triangle BDC

Area = .5abSIN(x^o)

Area = (1/2)hb

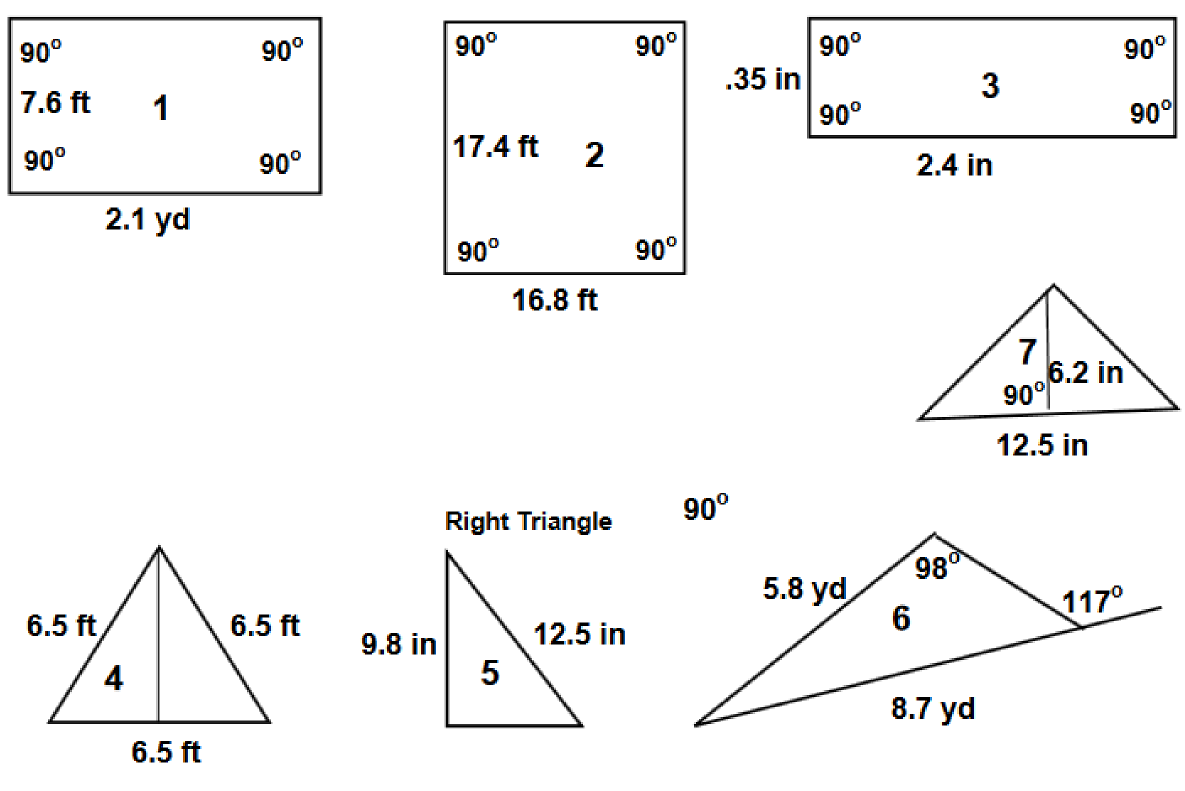
G8 Area of Triangles and Rectangles Problems

Calculate the areas of the triangles and rectangles.

Note: The lateral units of measurement must be the same.

DO NOT multiply **ft** times **yd** for example.

Answers: # Area.



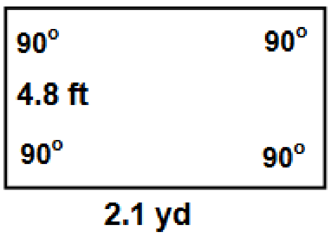
Answers: 1. 47.9 ft^2 or 5.3 yd^2 2. 292 ft^2 3. .84 in^2
4. 18.3 ft^2 5. 76 in^2 6. 8.2 yd^2 7. 38.8 in^2

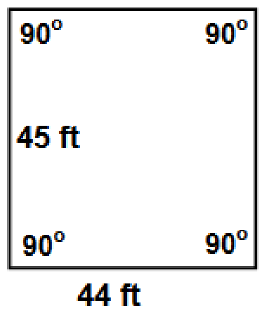
G8E

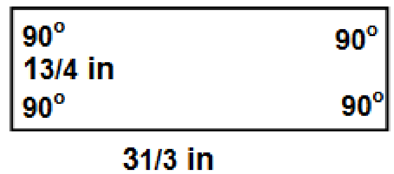
AREA OF TRIANGLES AND RECTANGLES

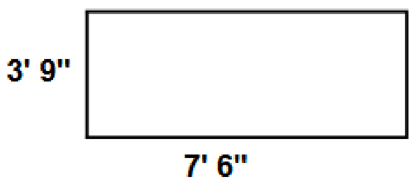
Calculate the areas of the triangles and rectangles.

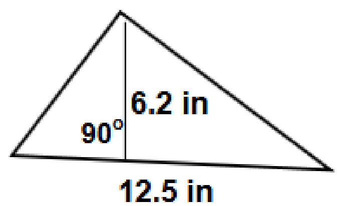
Note: The lateral units of measurement must be the same.

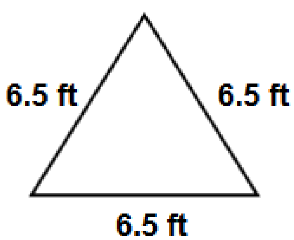


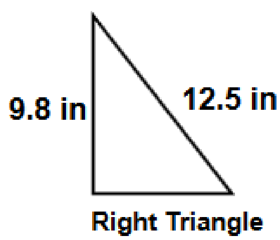


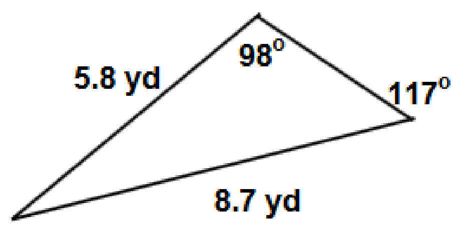












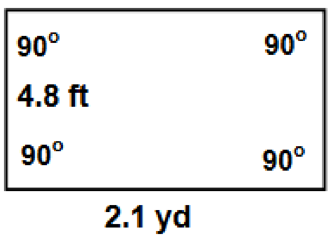
G8EA

AREA OF TRIANGLES AND RECTANGLES

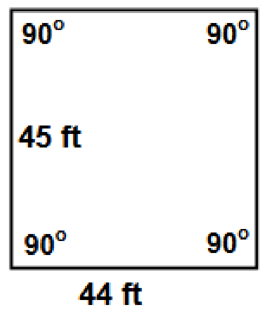
Calculate the areas of the triangles and rectangles.

Note: The lateral units of measurement must be the same.

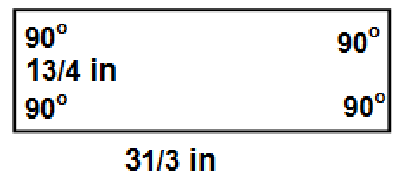
$A = 30.2\ ft^2 = 3.4\ yd^2$



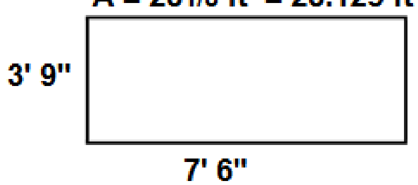
$A = 1980\ ft^2$



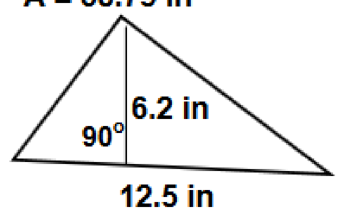
$A = 5\,5/6\ in^2$



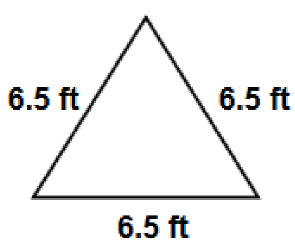
$A = 28\,1/8\ ft^2 = 28.125\ ft^2$



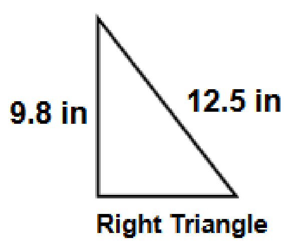
$A = 38.75\ in^2$



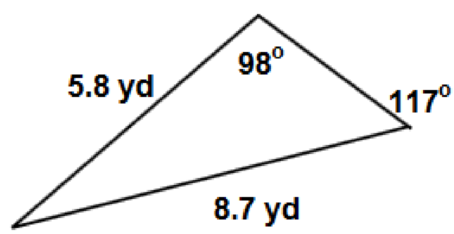
$A = 18.3\ ft^2$



$A = 38.0\ in^2$



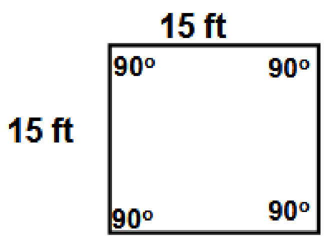
$A = 8.2\ yd^2$

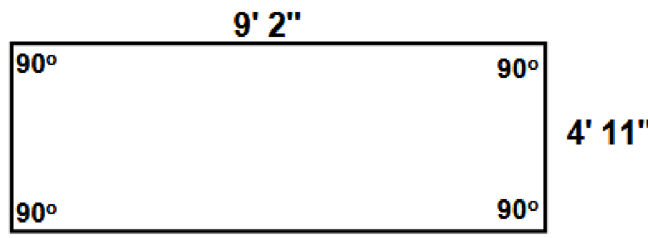


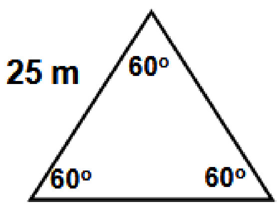
G8ES

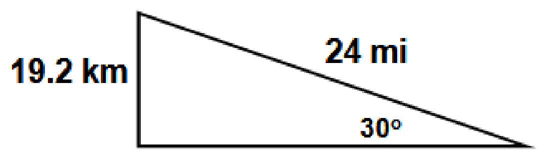
AREA OF TRIANGLES AND RECTANGLES

Identify the figures and calculate their areas. Be sure to check units and convert all numbers to the same unit where necessary.





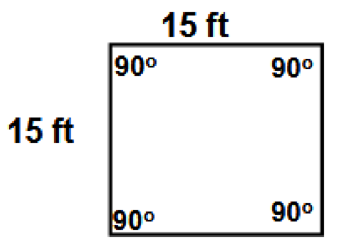




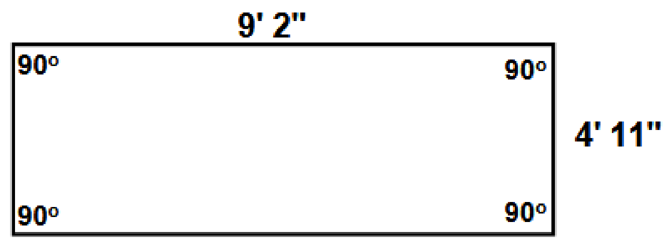
G8ESA

AREA OF TRIANGLES AND RECTANGLES

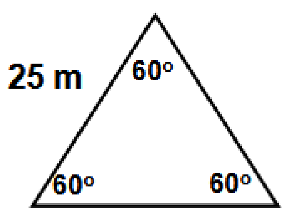
Identify the figures and calculate their areas. Be sure to check units and convert all numbers to the same unit where necessary.



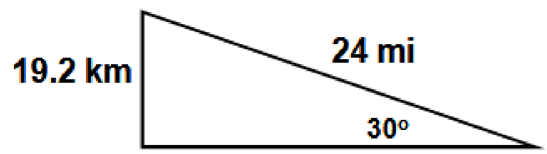
Square, A = 225 ft²



Rectangle, A = 45.07 ft²



Triangle, A = 270.6 m²



Right triangle, A = 124.7 mi² = 199.5 km²

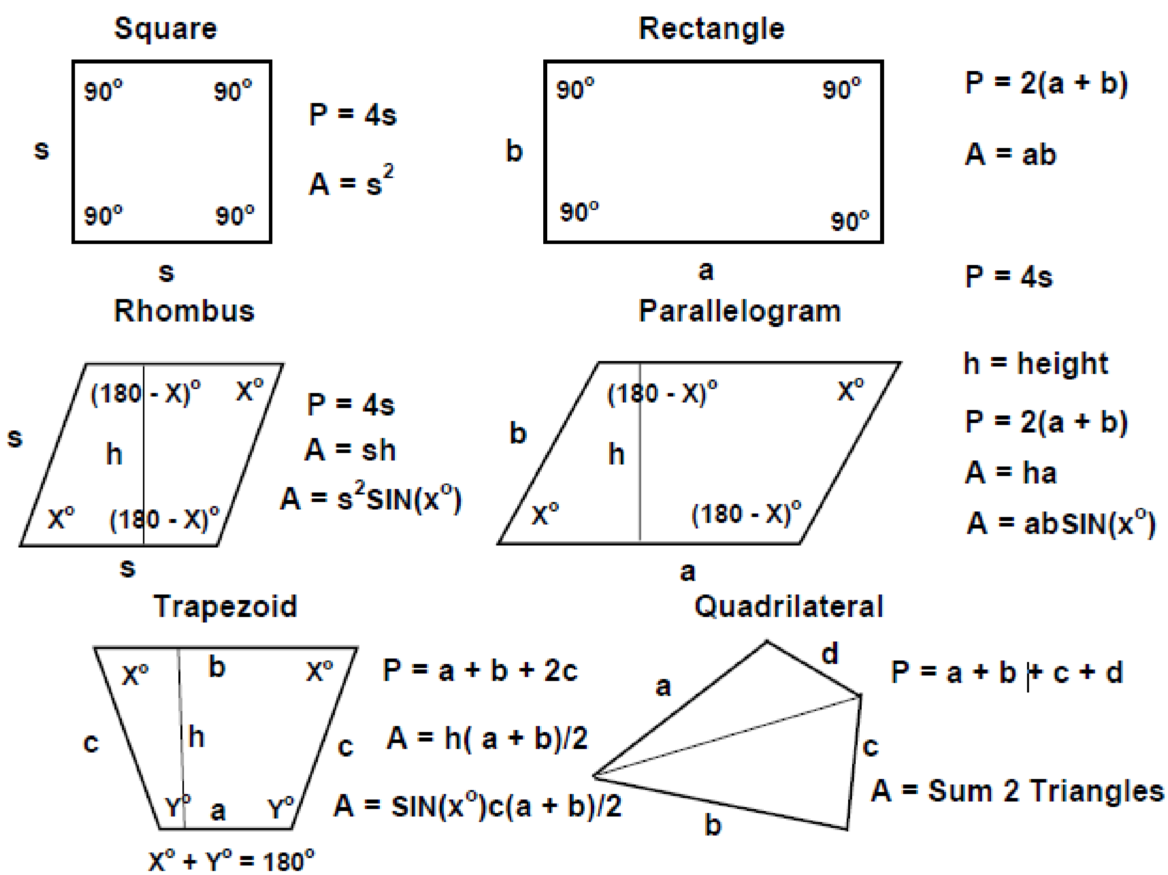
G9 LESSON: FORMULAS FOR POLYGONS

The **Area** of any geometric object is a measure of its size.

The basic unit of **Area** measure is a **square** which measures **one linear unit (U) per side**. Then, by definition, the **Area** of such a **square is 1 U^2** of **1 Square Unit**.

The **Area** of any other closed geometric figure is defined to be the sum of **areas** of inscribed, non-overlapping, squares which are so small they fully fill up the figure.

A rigorous definition is possible, but challenging. However; intuitively, the idea of **Area** is pretty easy.



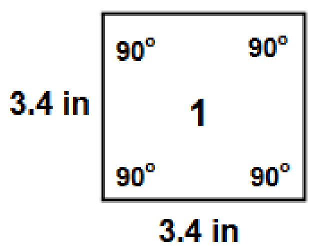
G9 Formulas for Polygons Problems

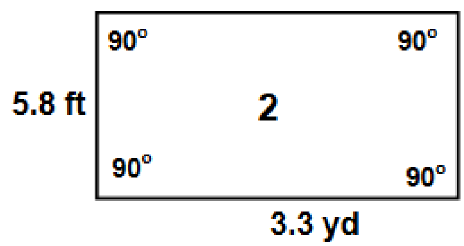
Identify the figures below and compute their Areas

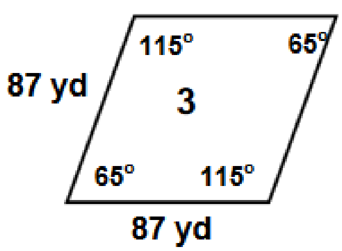
Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.

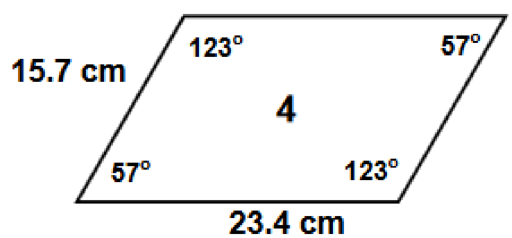
Suppose a rectangle has one side 11/2 feet, and the other side 8 inches. Then, convert feet to inches.

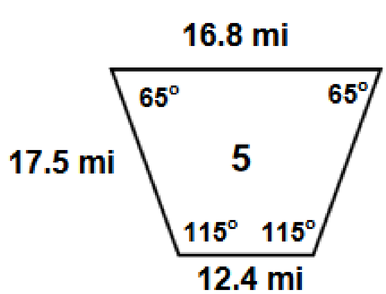
Answers are at bottom of page # Name, Area.

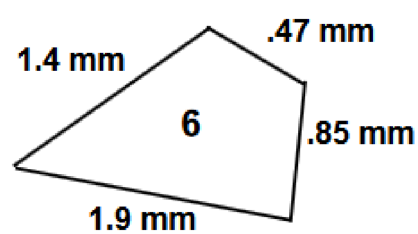










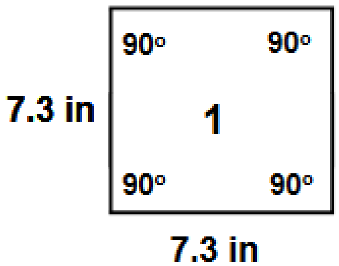


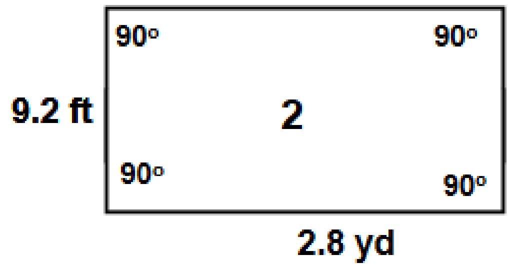
Answer: 1. Square 11.6 in^2 2. Rectangle 57.4 ft^2 or 6.4 yd^2 3. Rhombus 6860 yd^2
4. Parallelogram 308 cm^2 5. Trapezoid 231 mi^2 6. Quadrilateral Not enough Info

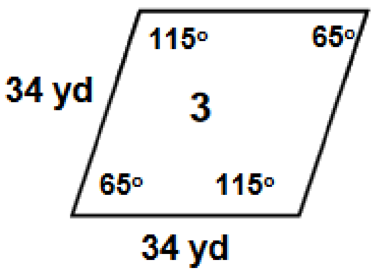
G9E

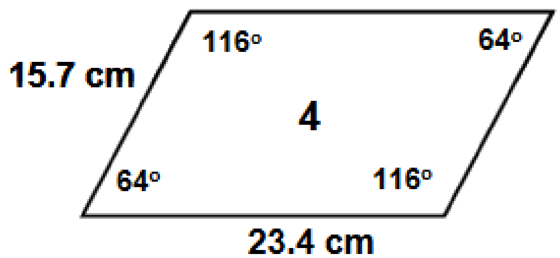
FORMULAS FOR POLYGONS

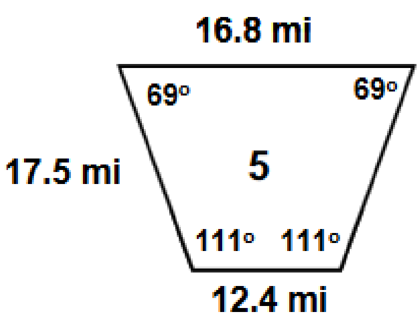
Identify the figures and calculate their areas.

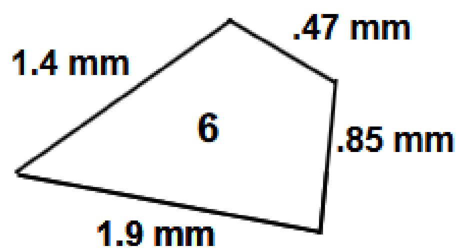


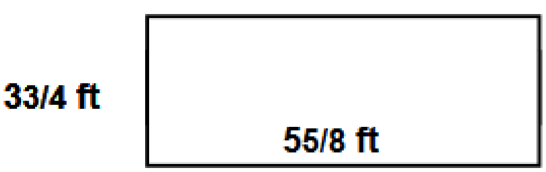


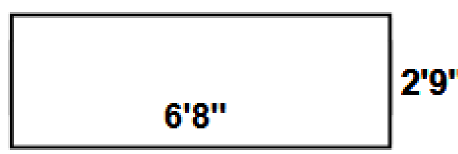










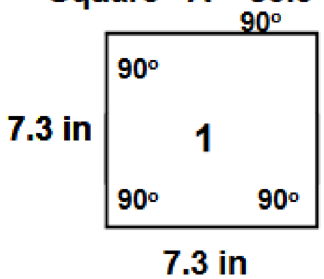


G9EA

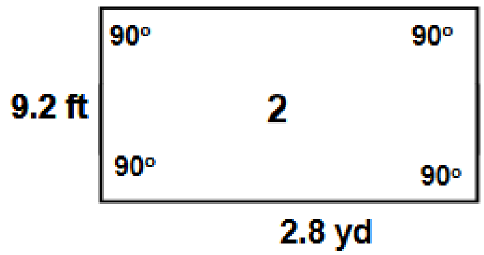
FORMULAS FOR POLYGONS

Identify the figures and calculate their areas.

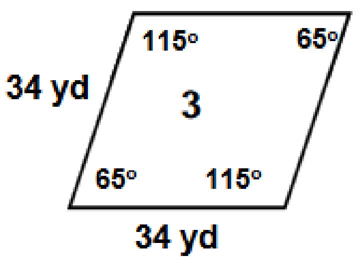
Square A = 53.3 in^2



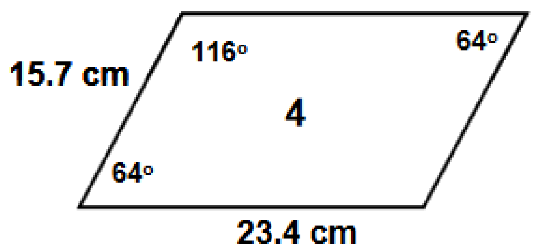
Rectangle A = 8.6 yd^2 = 77.3 ft^2



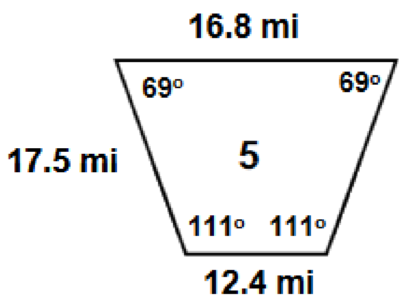
Rhombus A = 1048 yd^2



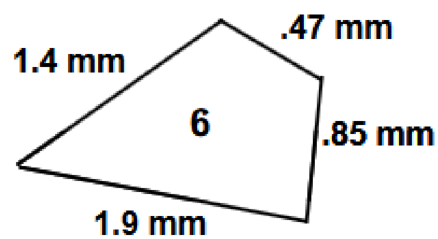
Parallelogram A = 330.2cm^2



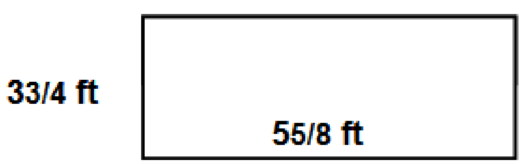
Trapezoid A = 238.5 mi^2



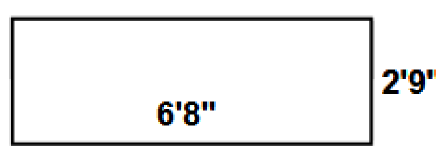
Polygon A = Insufficient Information



Rectangle A = 213/32 *ft*^{2} = *21.1ft*^{2}



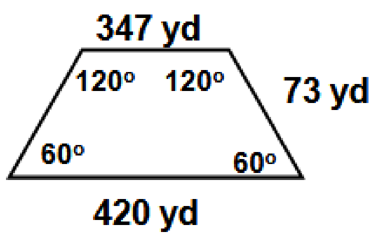
Rectangle A = 181/3 ft^2

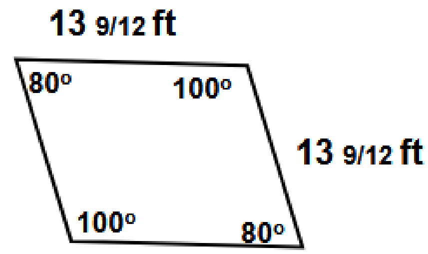


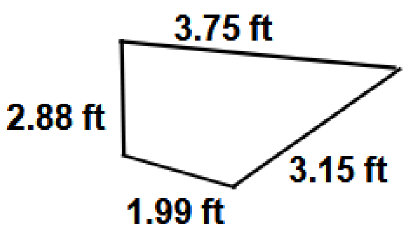
G9ES

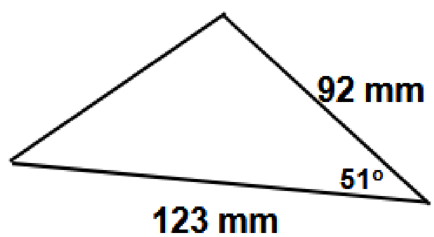
FORMULAS FOR POLYGONS

Identify the figures and calculate their areas.



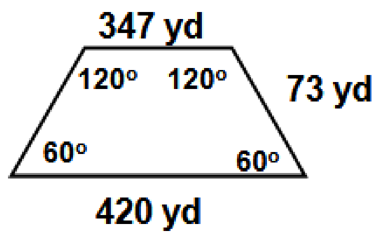




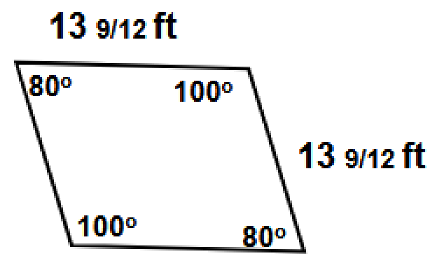


FORMULAS FOR POLYGONS

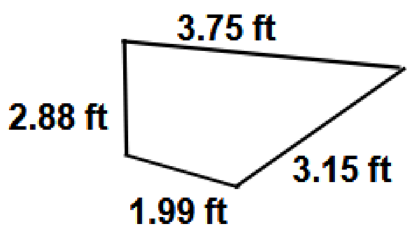
Identify the figures and calculate their areas.



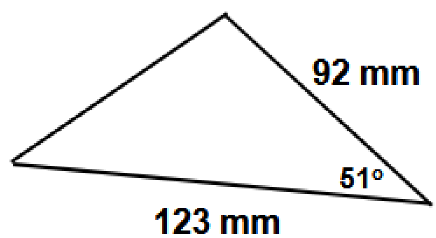
Trapezoid, A = 24,244.87 yd^2



Rhombus, A = 189.1 ft^2



Polygon, A = Insufficient Information



Triangle, A = 4,397.1 mm^2

G10 LESSON: CIRCLES π CIRCUMFERENCE

A **Circle** is a set of points equidistant from a point called the Center. This distance is called the **Radius** of the circle.

The distance across the **Circle** from one side to the other through the center is called the **Diameter** = 2x**Radius**

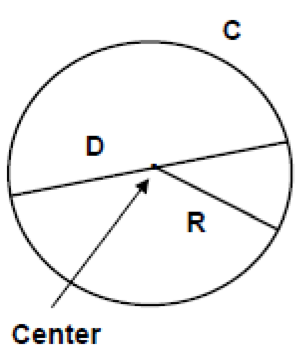
The **Circumference**, (**C**) of the **Circle** is the distance around the **Circle**, sort of its **perimeter**.

The ratio of the **Circumference** to the **Diameter** is always the same number for any circle. It is called **Pi** or π

Thus **$C = \pi D = 2\pi R$**

$\pi = 3.141592654\ldots$ 22/7 is an approximation.

I usually use 3.14 unless I need a lot of accuracy, then I use 3.1416. π is called a "**transcendental number**."



R = Radius = Distance from center to any point on the circle.

D = Diameter = Distance across circle

C = Circumference

$C = \pi D = 2\pi R$

G10 Circles π Circumference Problems

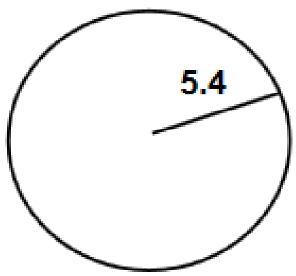
The TI-30Xa has a "π **Key**" we will use for π.

The three formulas we must remember are:

$$D = 2R \quad \textbf{and} \quad C = 2\pi R \quad \textbf{and} \quad A = \pi R^2 \text{ (next lesson)}$$

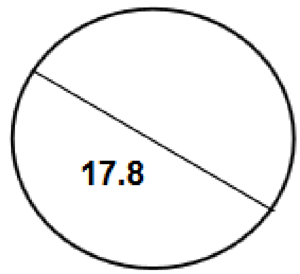
Find the unknown in the following problems.

Answers: **#, R, D, C**



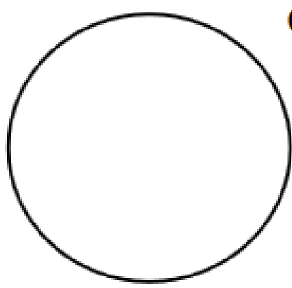
C =
D =

1



C =
R =

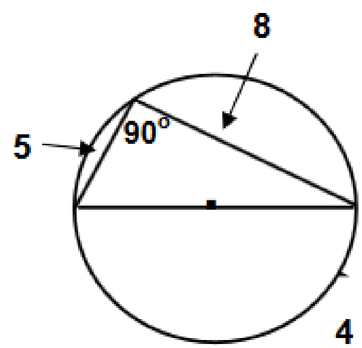
2



Circumference = 32

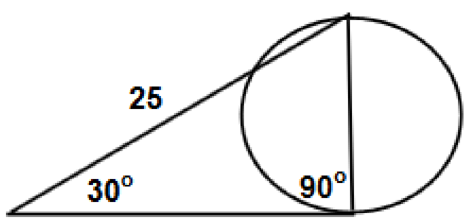
R =
D =

3



R =
D =
C =

4



R =
D =
C =

5

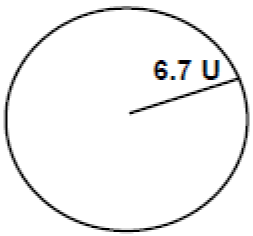
Answers 1. 5.4, 10.8, 33.9 2. 8.9, 17.8, 55.9 3. 5.1, 10.2, 32 4. 4.7, 9.4, 29.6 5. 6.25, 12.5, 39.3

G10E

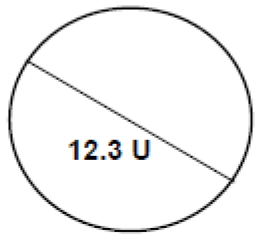
CIRCLES π CIRCUMFERENCE

R = Radius D = Diameter C = Circumference

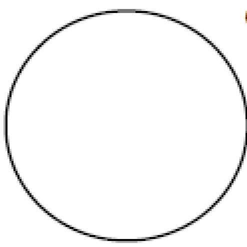
Find Unknowns



C = ? U
D = ? U

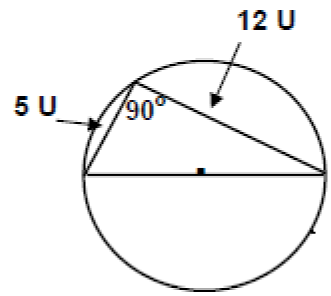


C = ? U
R = ? U

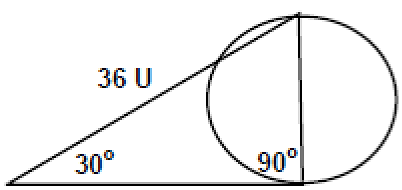


C = 53 U

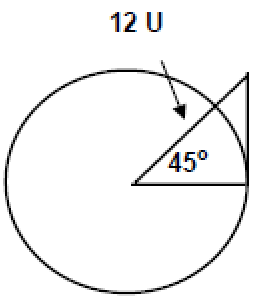
R = ? U
D = ? U



R = ? U
D = ? U
C = ? U

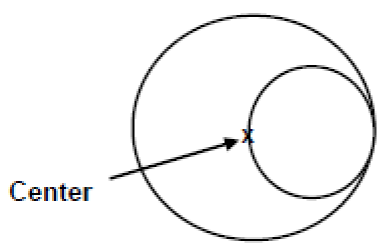


R = ? U
D = ? U
C = ? U



R = ? U
D = ? U
C = ? U

Inner Circle C = 15



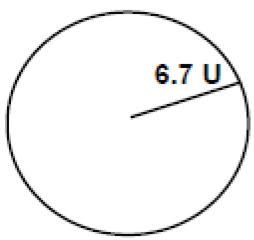
R = ? U
D = ? U
C = ? U

G10EA

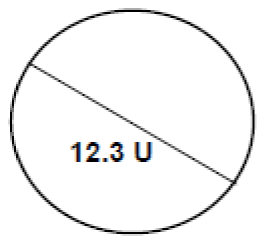
CIRCLES π CIRCUMFERENCE

R = Radius **D** = Diameter **C** = Circumference

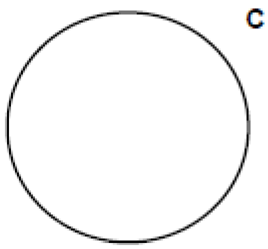
Find Unknowns



C = 42.1 U
D = 13.4 U

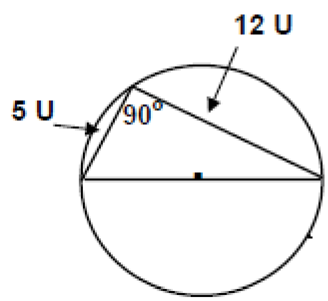


C = 38.6 U
R = 6.15 U

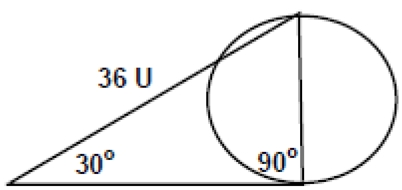


C = 53 U

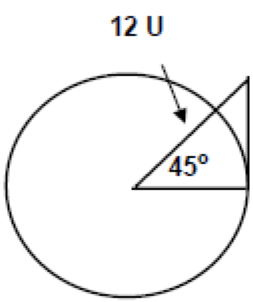
R = 8.4 U
D = 16.9 U



R = 6.5 U
D = 13 U
C = 40.8 U

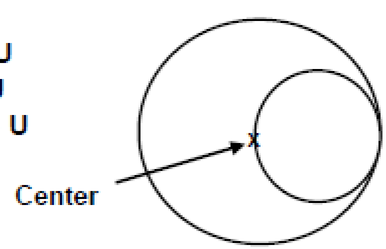


R = 9 U
D = 18 U
C = 56.6 U



R = 8.5 U
D = 17 U
C = 53.3 U

Inner Circle C = 15

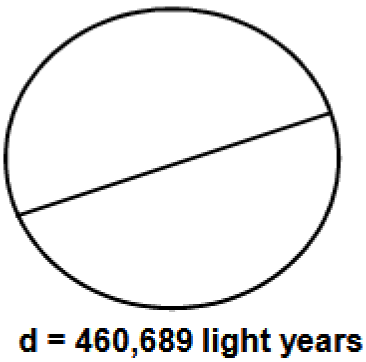


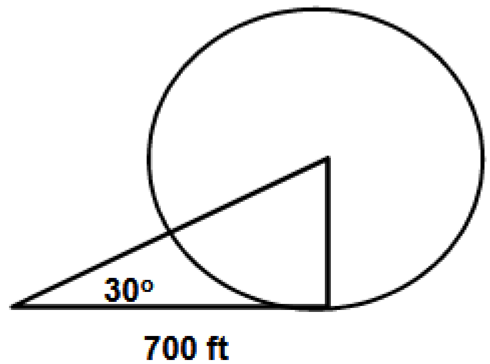
R = 4.8 U
D = 9.6 U
C = 30 U

G10ES

CIRCLES π CIRCUMFERENCE

Identify the figures and calculate their perimeters. Be sure to check units and convert all numbers to the same unit where necessary.

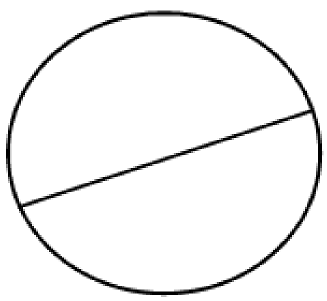




G10ESA

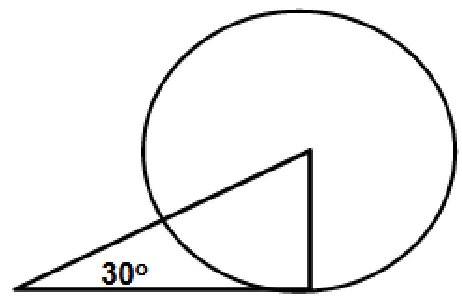
CIRCLES π CIRCUMFERENCE

Identify the figures and calculate their perimeters. Be sure to check units and convert all numbers to the same unit where necessary.



d = 460,689 light years

Circle, C = 460,689π ly = 1,447,297.2 ly



700 ft

Circle, C = 2539.32 ft

G11 LESSON: CIRCLES AREA A = π R2

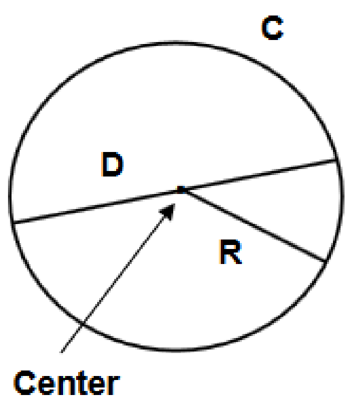
A **Circle** is a set of points equidistant from a point called the **Center**. This distance is called the **Radius** of the **circle**.

π is defined to be **C/D = Circumference/Diameter**

The **Area** (**A**) of the **Circle** turns out to be **A =** πR^2

This is a remarkable fact first discovered by the Greek genius mathematician **Archimedes**. It now is very easy to calculate the **Area** of any **Circle** using a calculator.

Remember: π is about 3.14



R = Radius = Distance from center to any point on the circle.

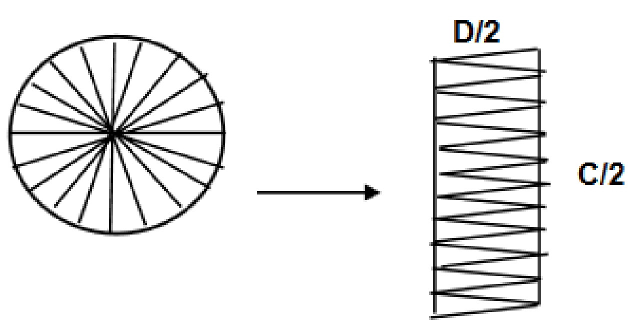
D = Diameter = Distance across circle

C = Circumference

C = πD = 2πR

A = πR^2

Archimedes "Proof" of Area. $A = (C/2)x(D/2) = (2\pi R/2)x(2R/2) = \pi R^2$



A = (D/2)X(C/2)

G11 Circles π Area Problems

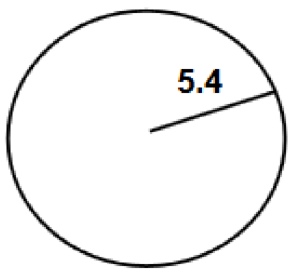
The TI-30Xa has a "π **Key**" we will use for π.

The three formulas we must remember are:

D = 2R **and** **C** = 2πR **and** **A** = πR2 (**next lesson**)

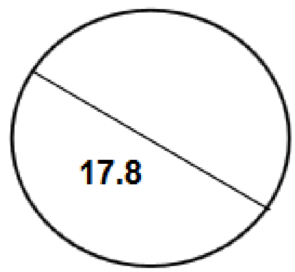
Find the Area in the following problems.

Answers: #, R, A



R =
A =

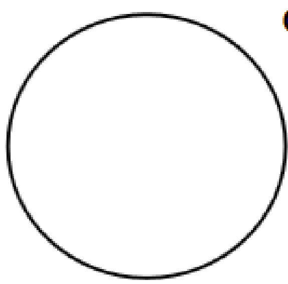
1



R =
A =

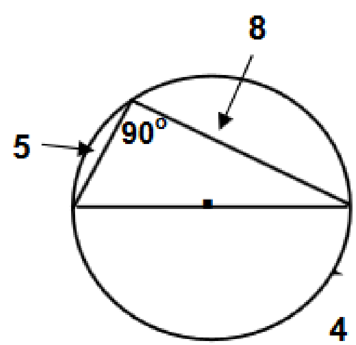
2

Circumference = 32



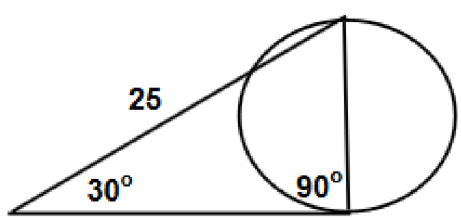
R =
A =

3



R =
A =

4



R =
A =

5

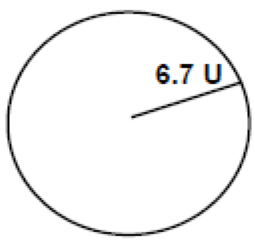
Answers **1. 5.4, 91.6** **2. 8.9, 249** **3. 5.1, 81.5** **4. 4.7, 70**
5. 6.25, 123

G11E

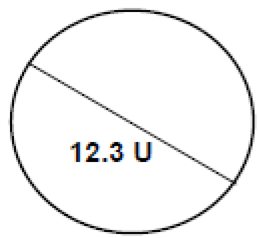
CIRCLES π AREA

R = Radius, **D** = Diameter, **C** = Circumference

Find Area

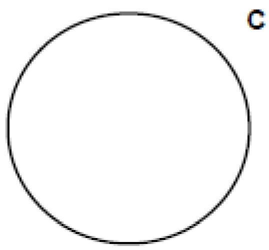


A = ? U^2

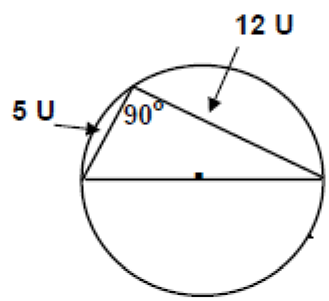


A = ? U^2

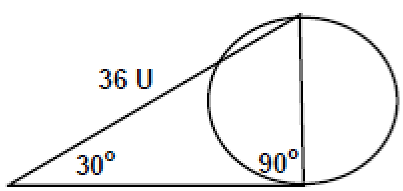
C = 53 U



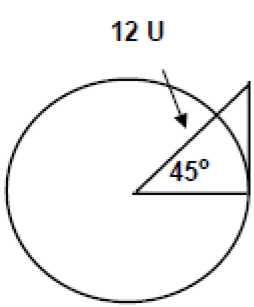
A = ? U^2



A = ? U^2

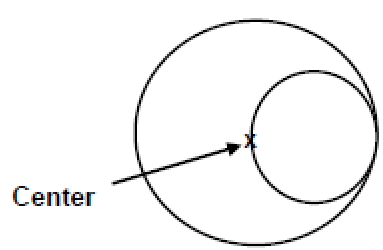


A = ? U^2



A = ? U^2

Inner Circle C = 15



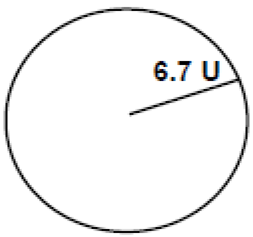
A = ? U^2

G11EA

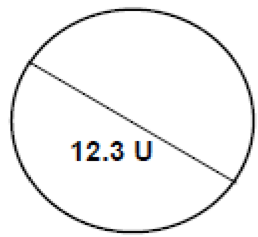
CIRCLES π AREA

R = Radius, **D** = Diameter, **C** = Circumference

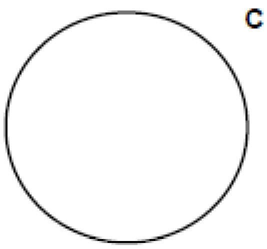
Find Area



A = 141 U^2

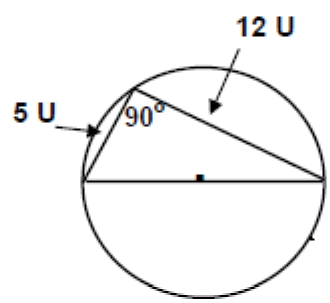


A = 118.9 U^2

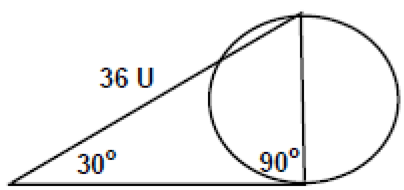


C = 53 U

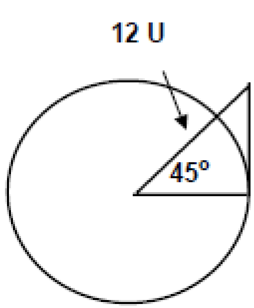
A = 223.5 U^2



A = 132.7 U^2

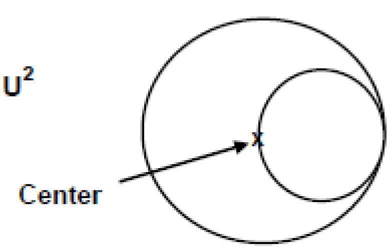


A = 254.5 U^2



A = 226 U^2

Inner Circle C = 15



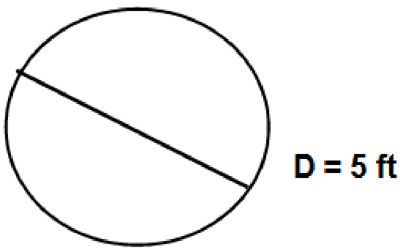
A = 71.6 U^2

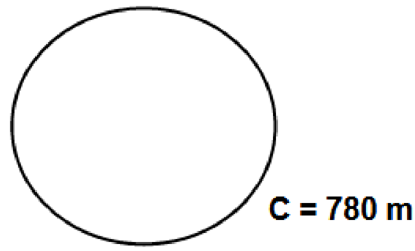
G11ES

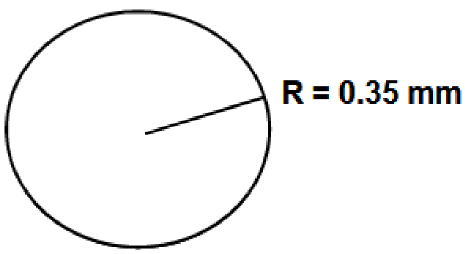
CIRCLES π AREA

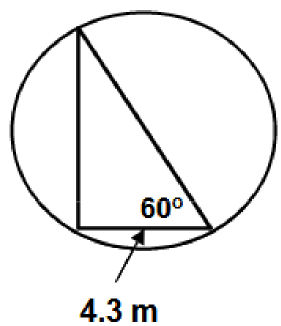
Calculate the areas of the figures below. Be sure to treat units appropriately!

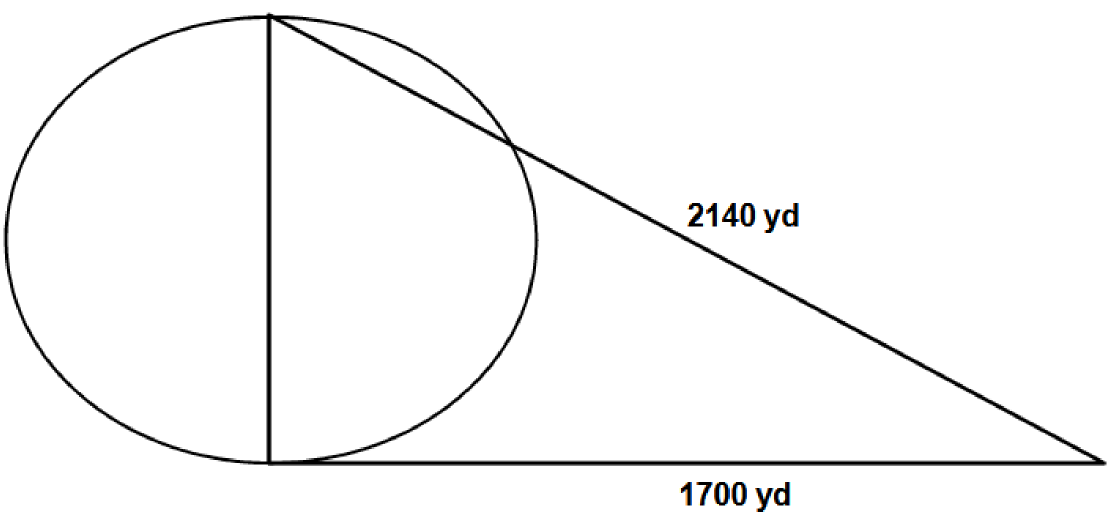
R = Radius, **D** = Diameter, **C** = Circumference









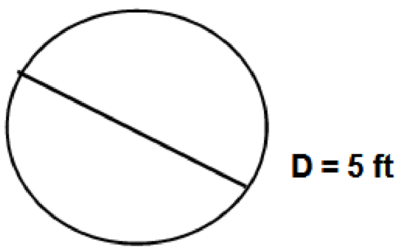


G11ESA

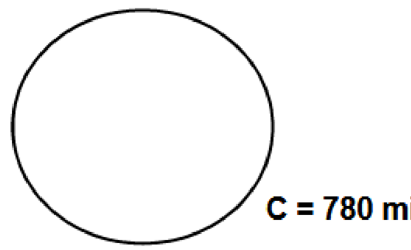
CIRCLES π AREA

Calculate the areas of the figures below. Be sure to treat units appropriately!

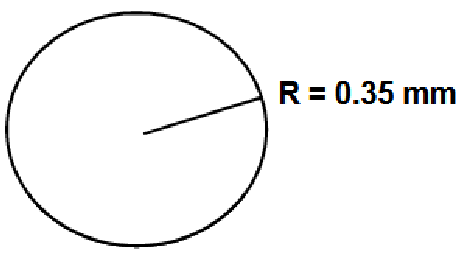
R = Radius, **D** = Diameter, **C** = Circumference



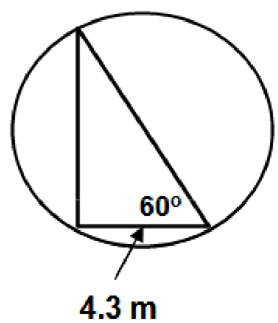
A = 19.6 ft²



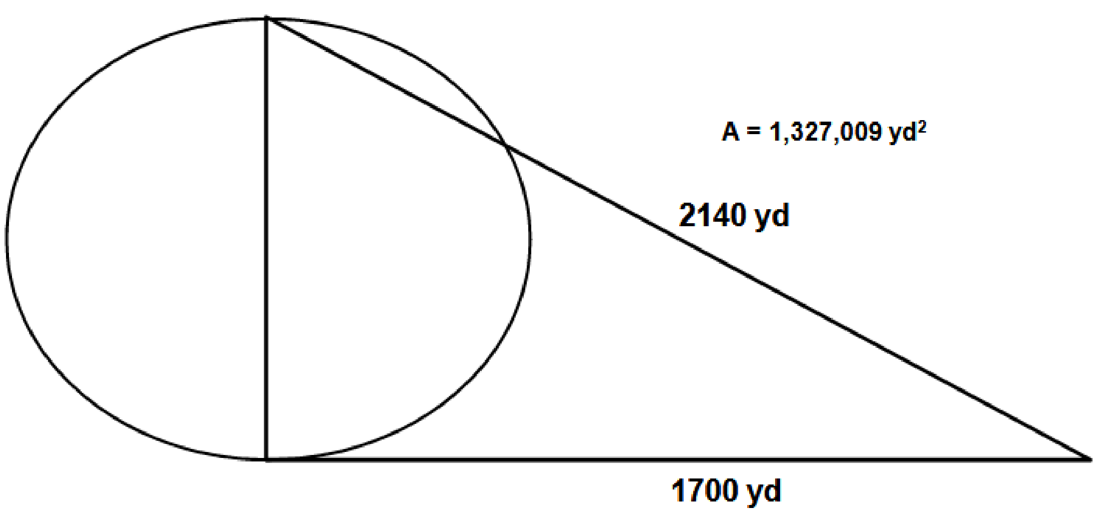
A = 48,415 mi²



A = 0.385 mm²



A = 43.57 m²



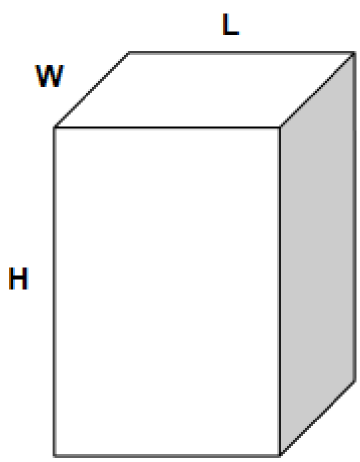
A = 1,327,009 yd²

G13 LESSON: SURFACE AREAS BLOCKS AND CYLINDERS

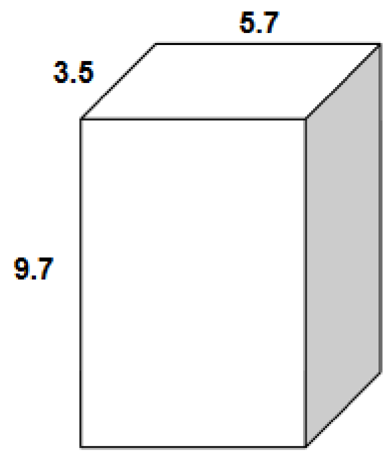
Calculate the Area of each "**face**" or "**side**" for a block.

The **Ends** and then the **Lateral Area** for the **Cylinder**

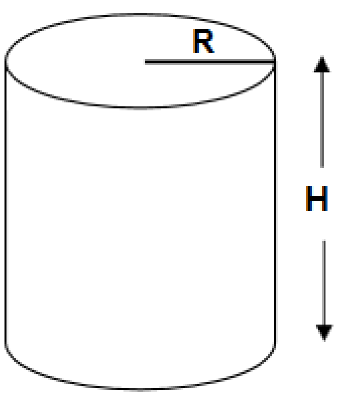
Area is measured in **Square Units, U^2**



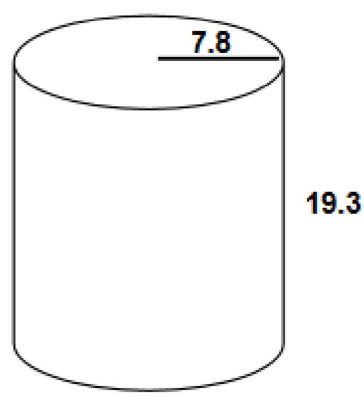
$$A = 2HL + 2HW + 2LW$$
$$= 2(HL + HW + LW)$$



$$A = 2(3.5x5.7 + 3.5x9.7 + 5.7x9.7) = 218\ U^2$$



$$A = 2\pi R^2 + 2\pi RH$$
$$= 2\pi R(R + H)$$

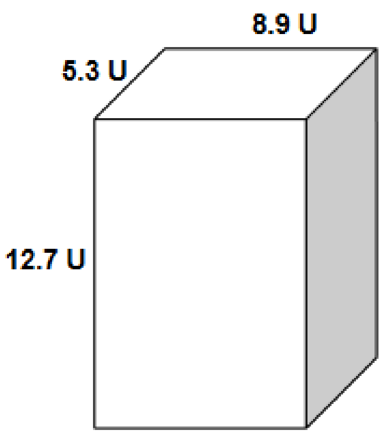


$$A = 2x\pi x7.8^2 + 2\pi x7.8x19.3 = 1328\ U^2$$

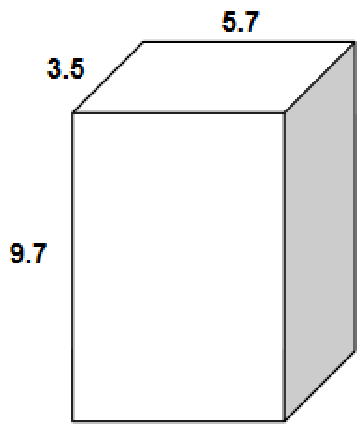
G13E

SURFACE AREAS BLOCKS AND CYLINDERS

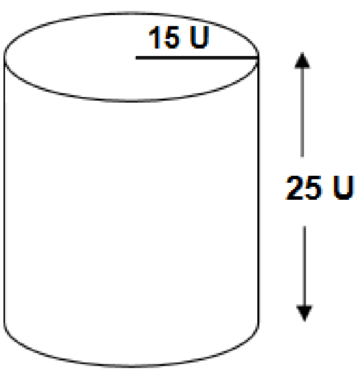
Calculate the Total Surface Area, **U^2**, in each case.

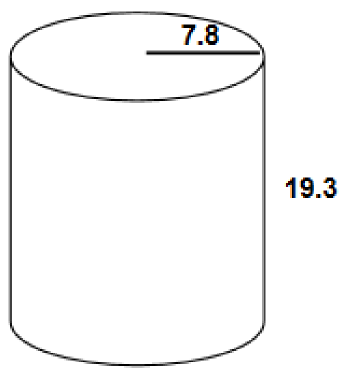


Area = ? U^2



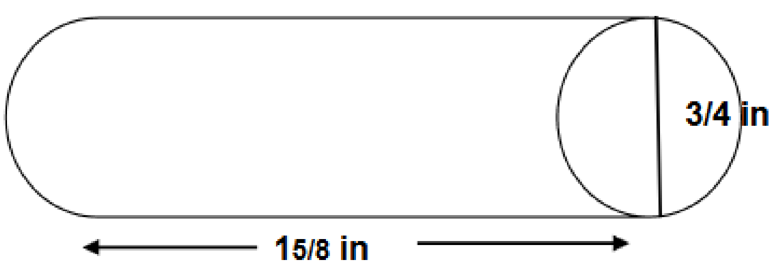
A = ? U^2





Area = ? U^2

A = ? U^2

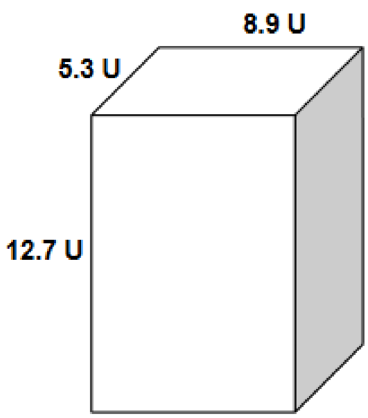


Total Surface Area = ? in^2

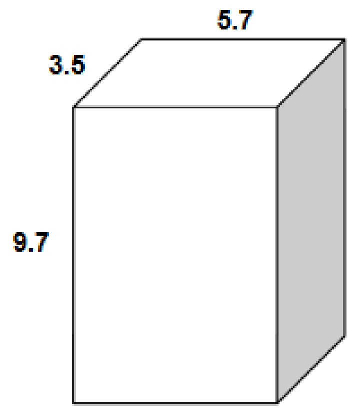
G13EA

SURFACE AREAS BLOCKS AND CYLINDERS

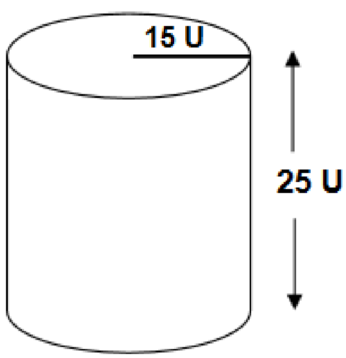
Calculate the Total Surface Area, **U^2**, in each case.



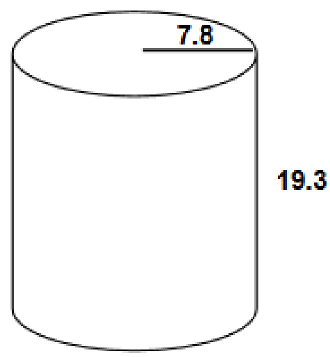
Area = 455 U^2



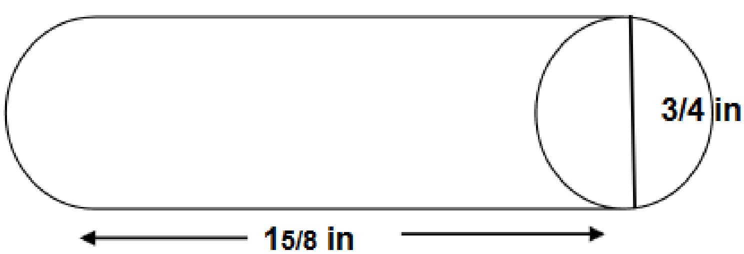
A = 2(3.5x5.7 + 3.5x9.7 + 5.7x9.7) = 218 U^2



Area = 3770 U^2



A = $2\pi x7.8^2$ + $2\pi x7.8x19.3$ = 1328 U^2

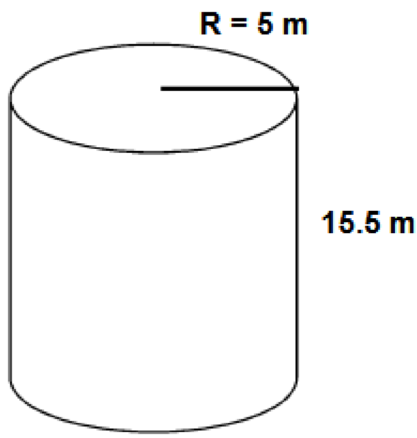


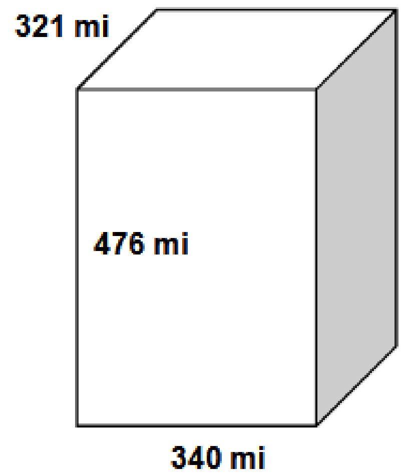
Total Surface Area = 4.71 in^2

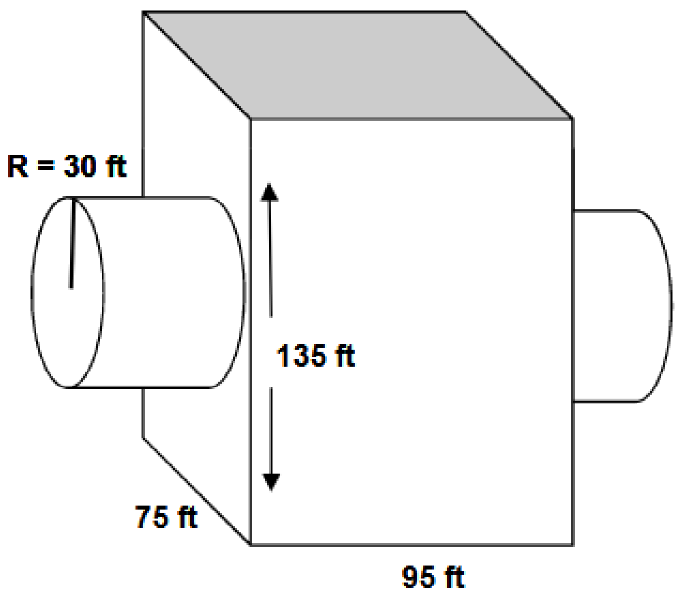
G13ES

SURFACE AREAS BLOCKS AND CYLINDERS

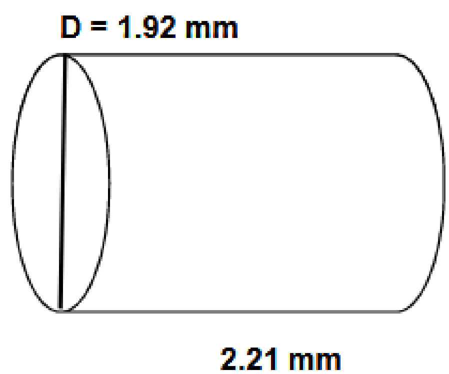
Calculate the surface area of the figures below. Be sure to treat units appropriately!







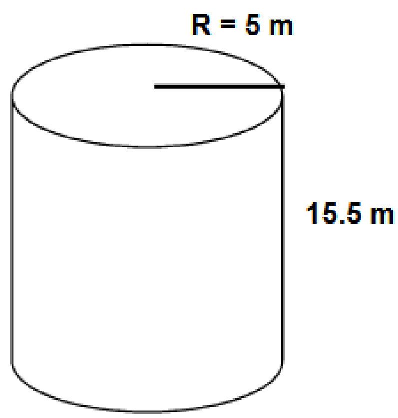
Note: the cylinder of length 140 ft is centered inside the block.



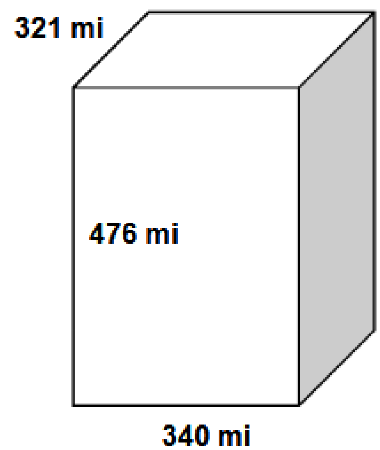
G13ESA

SURFACE AREAS BLOCKS AND CYLINDERS

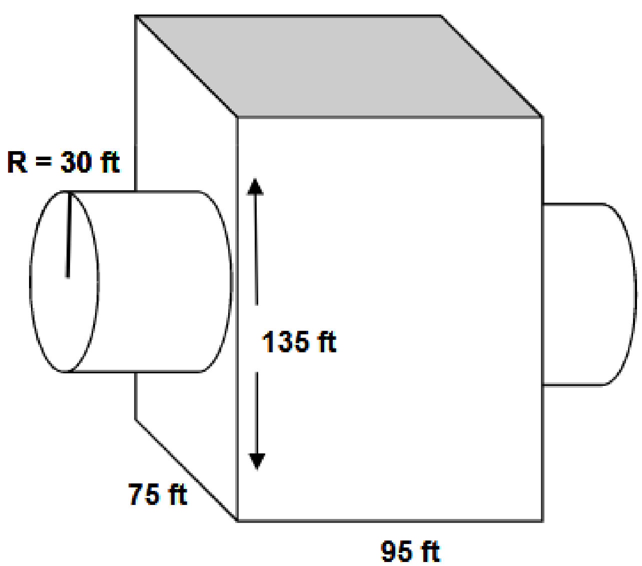
Calculate the surface area of the figures below. Be sure to treat units appropriately!



SA = 644 m²

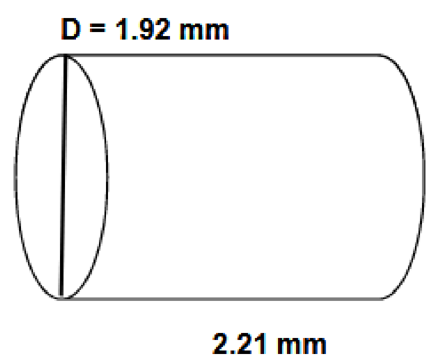


SA = 847,552 mi²



Note: the cylinder of length 140 ft is centered inside the block.

SA = 68,632.3 ft²

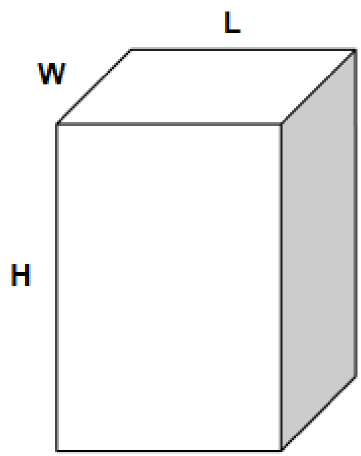


SA = 19.12 mm²

G15 LESSON: VOLUMES BLOCKS AND CYLINDERS

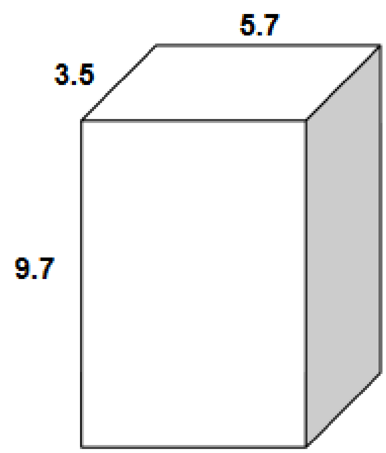
Volume = (**Area of Base**) x **Height**

Volume is measured in **Cubic Units, U^3**



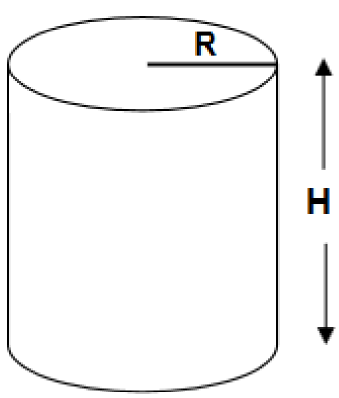
$A = 2HL + 2HW + 2LW$

$V = LWH$



$A = 2(3.5x5.7 + 3.5x9.7 + 5.7x9.7) = 218\ U^2$

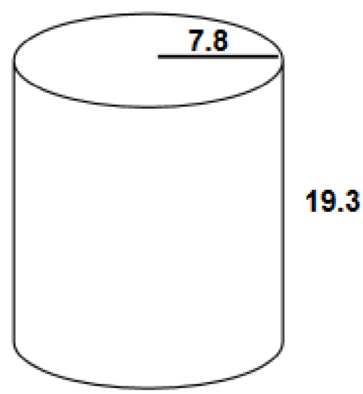
$V = 3.5x5.7x9.7 = 194\ U^3$



$A = 2\pi R^2 + 2\pi RH$

$= 2\pi R(R + H)$

$V = \pi R^2 H$



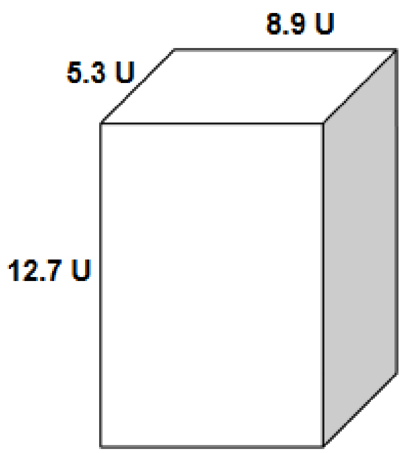
$A = 2x\pi x7.8^2 + 2\pi x7.8x19.3 = 1328\ U^2$

$V = \pi x7.8^2 x19.3 = 1174\ U^3$

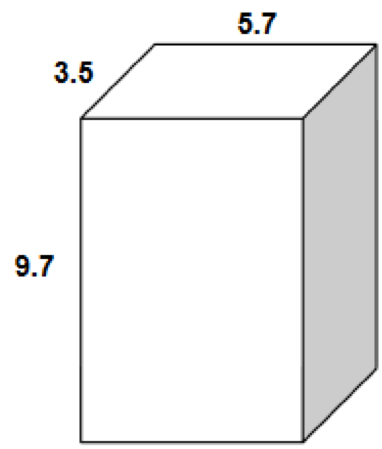
G15E

VOLUMES   BLOCKS AND CYLINDERS

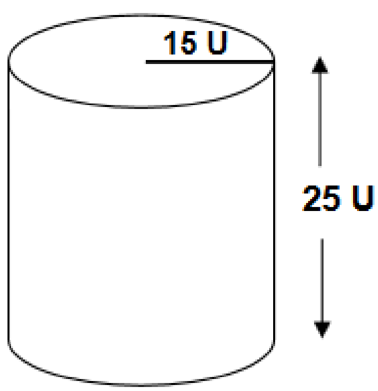
Calculate the **Volume, U^3**, in each case.



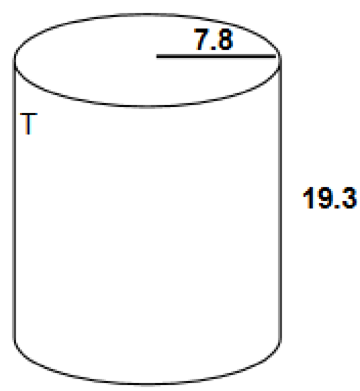
Volume = ? U^3



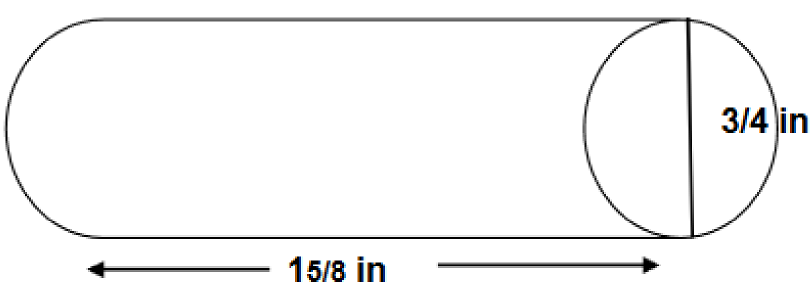
Volume = ? U^3



Volume = ? U^3



Volume = ? U^3

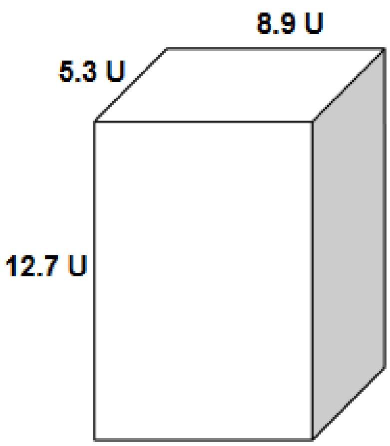


Volume = ? U^3

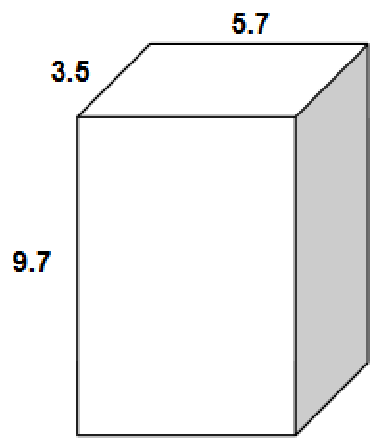
G15EA

VOLUMES BLOCKS AND CYLINDERS

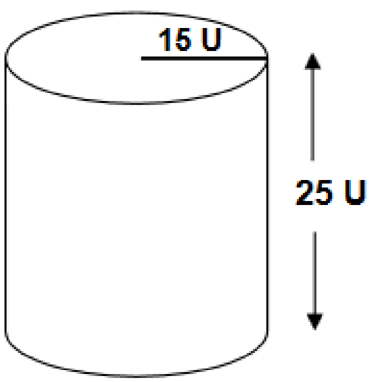
Calculate the **Volume, U^3**, in each case.



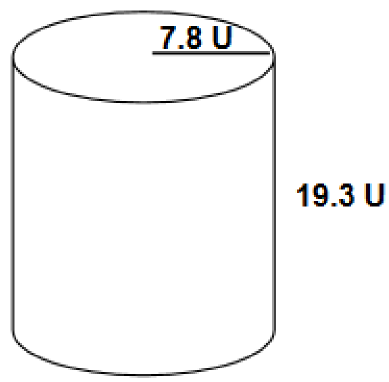
Volume = 599 U^3



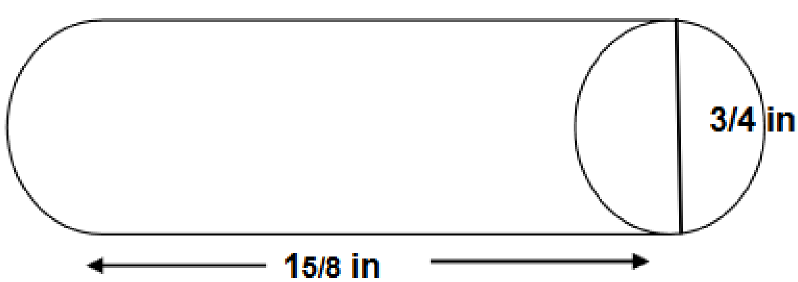
Volume = 193.5? U^3



Volume = 17,671 U^3



Volume = 3689 U^3

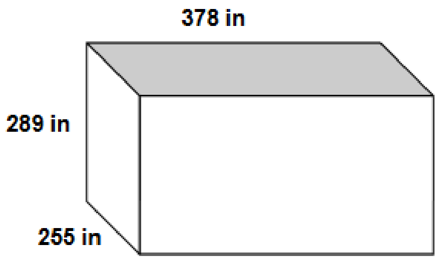


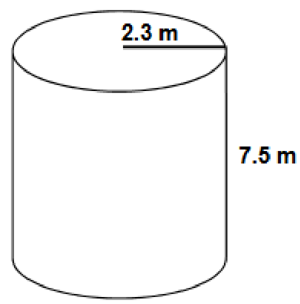
Volume = .718 in^3

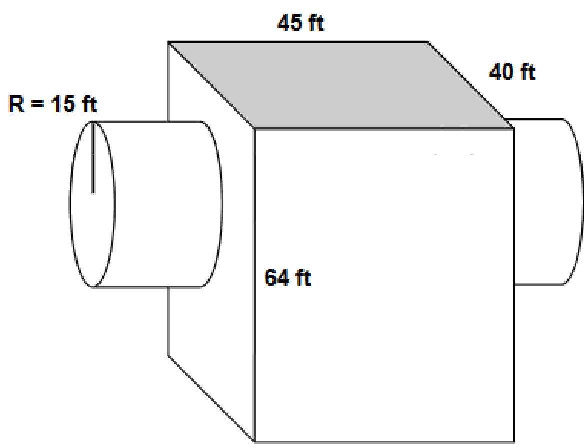
G15ES

VOLUMES BLOCKS AND CYLINDERS

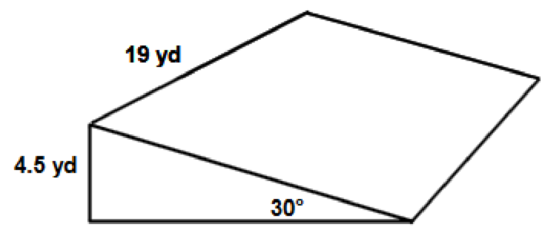
Find the volumes of the figures below. Be mindful of units!







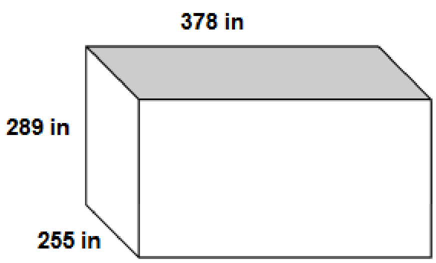
The cylinder of length 65 ft is centered inside the block.



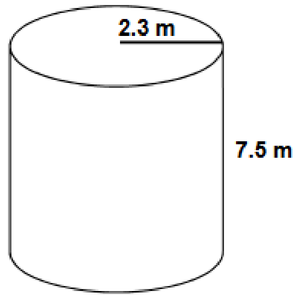
G15ESA

VOLUMES BLOCKS AND CYLINDERS

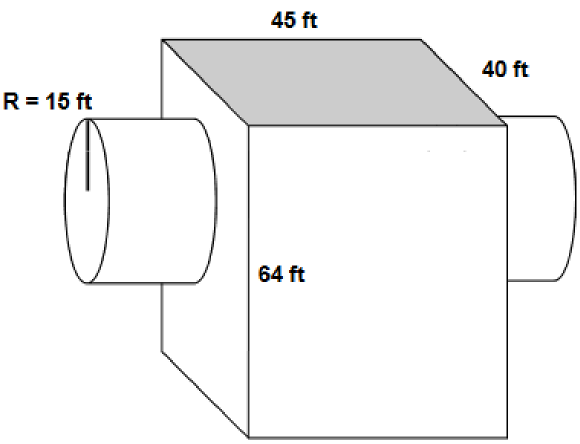
Find the volumes of the figures below. Be mindful of units!



V = 27,856,710 in^3

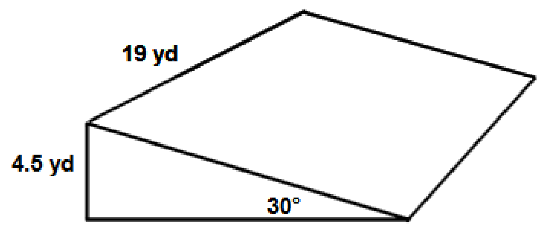


V = 124.6 m^3



The cylinder of length 65 ft is centered inside the block.

V = 129,337 ft^3



V = 333.2 yd^3

S1 LESSON: UNITS CONVERSION

Suppose you have two Units of Measurement

U_1 and U_2 and you wish to convert from one unit to the other, for example, cm and inches.

For example, you want to convert 23.4 cm to inches.

First, you must determine the conversion number.

You may look this up in some type of unit conversion table, or you can go to www.wolframalpha.com and get the answer or find the conversion number.

WA1 **Convert 1 cm to inches**

Answer: 1 cm = 0.3937 inches

Now, you have 23.4 cm = X inches and you want X.

Multiply both sides by 23.4 and get:

23.4 cm = 23.4x0.3937 inches = 9.2 inches

Of course, we could have gotten this directly from www.wolframalpha.com

WA2 **Convert 23.4 cm to inches**

Answer: 9.213

Suppose you wanted to convert 15.7 inches to cm?

1 cm = 0.3937 inches same as 1/0.3937 cm = 1 inch

Or, 1 inch = 2.54 cm since 1/0.3937 = 2.54

Then, 15.7 inches = 15.7x2.54 cm = 39.88 cm

Of course,

WA3 **convert 1 inch to cm**

Answer: 2.54

WA4 **convert 15.7 inches to cm**

Answer: 39.88

This type of process applies to any type of conversion of units. Of course, the units must be measuring the same thing like length or weight.

Example 1: convert 18.3 grams to ounces

First you must find a conversion factor for grams to ounces:

1 gm = .0353 oz you find somewhere.

Then, 18.3 gm = .0353x18.3 oz = .646 oz

WA5 **1 gram to ounce**

Answer: .03527

WA6 **18.3 gram to ounce**

Answer: .6455

The same process applies to any type of unit conversion.

For example, square feet to square meters:

1 sq meter = 10.76 square feet

Thus, 1 square foot = 1/10.76 sq m = .093m^2

Example 2: 4.7 sq m are how many sq ft?

Answer: $4.7 \times 10.76\ ft^2 = 50.57\ ft^2$

WA7 **4.7 sq m to sq ft**

Answer: 50.6

To get more accuracy:

WA8 **4.70 sq m to sq ft**

Answer: 50.59

WA9 **1 square meter to square feet**

Answer: 10.76

Example 3: 12.3 Kilograms is how many pounds?

WA10 **12.3 kilograms to pounds**

Answer: 27.12 lb = 27 lb 1.9 oz

Example 4: 3.4 cubic meters is how many cubic yards

1 m = 1.094 yd

$1\ m^3 = 1.0943^3\ yd^3 = 1.309$ yd

So 3.4 cu m = 3.4x1.309 cu yd = 4.45 cu yd

WA11 **3.4 cubic meter to cubic yard**

Answer: 3.45 cu yd

In general, if you have two units which measure the same quantity, U_1 and U_2, and you wish to convert from one unit to the other, then:

If you have access to www.WolframAlpha.com, you simply enter the command:

convert N U_1 to U_2

where N is the amount of the quantity you have expressed in U_1 and you will get the amount expressed in U_2.

If you don't have access to Wolfram Alpha, then you must find the conversion factor, C, where:

$1\ U_1 = CU_2$

Multiply both sides by N to obtain the answer:

$N\ U_1 = CxN\ U_2$

Example: you know 1 mile = 1.609 kilometers

60 miles = 1.609x60 km = 96.54 km

So, you can see for example that:

100 km/hr is about 60 m/hr.

S1E

Units Conversion

1. Given the conversion factor 1 ft = 12 in, how many inches are in 1.5 ft?

2. Given the conversion factor 1 ft = 12 in, how many feet are in 14 in?

3. Given the conversion factor 1 m = 39.37 in, how many inches are in 2.8 m?

4. Given the conversion factor 1 m = 39.37 in, how many meters are in 76 in?

5. Given the conversion factor 1 in^2 = 6.452 cm^2, how many cm^2 are on an 8 ½ in x 11 in sheet of paper?

6. Given the conversion factor 1 in^2 = 6.452 cm^2, how many in^2 are in 100 cm^2?

7. Given the conversion factor 1 gal = 3.785 L, how many liters are in 19 gal?

8. Given the conversion factor 1 km^2 = 0.3861 mi^2, how many mi2 are in 15 km2?

9. Given the conversion factor 1 gal = 3.785 L, how many gallons are in 2 L?

10. If I want to pour a concrete house slab that is 52 feet long by 28 feet wide by 4 inches deep, how would I determine how many cubic yards of concrete would be needed?

S1EA

UNITS CONVERSION

1. Given the conversion factor 1 ft = 12 in, how many inches are in 1.5 ft?

   1 ft = 12 in (You will also see this written as 12 in/ft.)

   1.5 ft = X in

   (12 in/ft)*(1.5 ft) = 18 in

   or

   WA **convert 1.5 ft to in**

   18 in

2. Given the conversion factor 1 ft = 12 in, how many feet are in 14 in?

   1 ft = 12 in

   1/12 ft = 12/12 in

   0.0833 ft = 1 in (You will also see this written as 0.0833 ft/in.)

   14 in = X feet

   (0.0833 ft/in)*(14 in) = 1.167 ft

   or

   WA **convert 14 in to ft**

   1.167 ft

3. Given the conversion factor 1 m = 39.37 in, how many inches are in 2.8 m?

   2.8 m = X in

   (39.37 in/m)*(2.8 m) = 110.24 in

4. Given the conversion factor 1 m = 39.37 in, how many meters are in 76 in?

   1 m = 39.37 in

   1/39.37 m = 39.37/39.37 in

   0.0254 m = 1 in (You will also see this written as 0.0254 m/in.)

   76 in = X m

   (0.0254 m/in)*(76 in) = 1.930 m

   or

   WA **convert 76 in to m**

   1.93 m

5. Given the conversion factor 1 in^2 = 6.452 cm^2, how many cm^2 are on an 8 ½ in x 11 in sheet of paper?

   (8 ½ in)*(11 in) = 93.5 in^2

   (6.452 cm^2/in^2)*(93.5 in^2) = 603.262 cm^2

   or

   WA **convert 93.5 inches^2 to cm^2**

   603.2 cm^2

   or

   WA **convert (8.5 inches)*(11 in) to cm^2**

   603 cm^2

   Note: The answers are actually the same. The slight differences occur during rounding.

6. Given the conversion factor 1 in^2 = 6.452 cm^2, how many in^2 are in 100 cm^2?

   1 in^2 = 6.452 cm

   1/6.452 in^2 = 6.452/6.452 cm^2

   0.155 in^2 = 1 cm^2 (You will also see this written as 0.155 in^2/ cm^2.)

   100 cm^2 = X in^2

   (0.155 in^2/ cm2)*(100 cm^2) = 15.5 in^2

   or

   WA **convert 100 cm^2 to in^2**

   15.5 in^2

7. Given the conversion factor 1 gal = 3.785 L, how many liters are in 19 gal?

   19 gal = X L

   (3.785 L/gal)*(19 gal) = 71.915 L

   or

   WA **convert 19 gal to L**

   71.92 L

8. Given the conversion factor 1 km^2 = 0.3861 mi^2, how many mi^2 are in 15 km^2?

   15 km^2 = X mi^2

   (0.3861 mi^2/km^2)(15 km^2) = 5.7915 mi^2

   or

   WA **convert 15 km^2 to mi^2**

   5.792 mi^2

9. Given the conversion factor 1 gal = 3.785 L, how many gallons are in 2 l?

1 gal = 3.785 L

1/3.785 gal = 3.785/3.785 L

0.2642 gal = 1 L (You will also see this written as 0.2642 gal/L.)

2 L = X gal

(0.2642 gal/L)*(2 L) = 0.5284 L

or

WA **convert 2 L to gal**

0.5283 L

10. If I want to pour a concrete house slab that is 52 feet long by 28 feet wide by 4 inches deep, how would I determine how many cubic yards of concrete would be needed?

27 ft^3 = 1 yd^3

27/27 ft^3 = 1/27 yd^3

1 ft^3 = 0.0370 yd^3

1 ft = 12 in

1 in = 0.0833 ft. (See A1 for math conversion.)

First, convert in to ft.

4 in = X ft

(0.0833 ft/in)(4 in) = 0.3332 ft

Next, calculate number of ft^3.

(52 ft)(28 ft)(0.3332 ft) = 485.1392 ft^3

Finally, convert ft^3 to yd^3

485.1392 ft^3 = X yd^3

(0.0370 yd^3/ft^3)(485.1392 ft^3) = 17.968 yd^3

S2 LESSON: DMS Degrees – Minutes - Seconds

There are 360°, or Degrees, in one revolution or circle.

In the DD (decimal degrees) system we express degrees with decimal notation. 37.45 degrees means 37 and 45/100 degrees.

In the DMS system, 1 degree = 60 minutes, or 1° = 60′

And 1 minute = 60 seconds, or 1′ = 60″

So, 1′ = (1/60)° and 1″ = (1/60)′ = (1/3600)°

We can express degrees in either DD or DMS format and convert degrees from DD to DMS and DMS to DD using the TI-30Xa calculator.

DMS → DD is 2^{nd} +

DD → DMS is 2^{nd} =

Example:

6.5° = 6°30′ 00″00

6.55° = 6°33′ 00″00

6.57° = 6°34′ 12″00

6.573° = 6°34′ 22″80 (this means 22.80″)

127.875° = 127°52′ 30″

57.382° = 57°22′ 55″2 (this means 55.2″)

To apply the DMS → DD conversion you must enter the angle in the following format:

6°34’ 22”80 is entered: 6.342280 2nd +

Answer: 6.573°

26°4’ 2”50 is entered: 26.040250 2nd +

Answer: 26.06736

Now enter 26.06736° and get 26°04’ 02”5

It is possible to do these conversions manually with formulas, but it is best to do it with a calculator.

S2E

DMS Degrees – Minutes - Seconds

Convert the following decimal degree (DD) numbers to degrees-minutes-seconds (DMS).

1. 87.625
2. 137.6489
3. 65.475698
4. 19.01325
5. 45.4557

Convert the following degrees-minutes-seconds (DMS) to decimal degree (DD) numbers.

6. 66°18’12”0
7. 78°45’06”4
8. 180°04’07”
9. 97°09’45”7
10. 54°57’27”4

S2EA

THE NUMBER LINE, NEGATIVE NUMBERS

Answers: [ ]'s

Convert the following decimal degree (DD) numbers to degrees-minutes-seconds (DMS).

1. 87.625

   [**87°37'30"00**]

2. 137.6489

   [**137°38'56"**]

3. 65.475698

   [**65°28'32"5**]

4. 19.01325

   [**19°00'47"7**]

5. 45.4557

   [**45°27'20"5**]

Convert the following degrees-minutes-seconds (DMS) to decimal degree (DD) numbers.

6. 66°18'12"0

   [**66.30333333**]

7. 78°45'06"4

   [**78.75177778**]

   Note: If you get an answer of 78.75167778, what you did is enter into your calculator "78.450604" instead of "78.45064" before you hit the DMS → DD key. Anything after the " symbol, in this case 06"4, should be treated as 6.4 seconds, therefore, entering a 0 before the 4 would be incorrect.

8. 180°04’07”

[**180.0686111**]

9. 97°09’45”7

[**97.162269444**]

Note: If you get an answer of 97.16251944, what you did is enter into your calculator “97.094507” instead of “97.09457” before you hit the DMS → DD key.

10. 54°57’27”4

[**54.95761111**]

Note: If you get an answer of 54.95751111, what you did is enter into your calculator “54.572704” instead of “54.57274” before you hit the DMS → DD key.

S3 LESSON: y^x EXPONENTS

y^x means y times itself x times

y is called the base,

x is called the exponent

Examples:

$2^3 = 8$; $3^2 = 9$; $5^4= 625$; $10^5 = 100,000$

The y^x key is the east way to calculate this.

Clear the calculator

Enter 2 and press the y^x

Enter 3 and press the = key

Answer: 8

Do all of the above.

y can be any positive number

x can be any number

$^x\sqrt{y}$ means the x^{th} root of y

same as $y^{(1/x)}$ $[^x\sqrt{y}]^x = y = {^x\sqrt{(y^x)}}$

$^3\sqrt{8} = 2 = 8^{1/3}$

$1.7^{2.7} = 4.19$

$2^{10} = 1024$ Kilo $^{10}\sqrt{1024} = 2 = 1024^{1/10}$

Metric		Digital
$10^3 = 1000$	Kilo	$2^{10} = 1024$
$10^6 = 1{,}000{,}000$	Mega	$2^{20} = 1{,}048{,}576$
$10^9 = 1{,}000{,}000{,}000$	Giga	$2^{30} = 1{,}073{,}741{,}824$
$10^{12} = 1{,}000{,}000{,}000{,}000$	Tera	$2^{40} = 1{,}099{,}511{,}627{,}776$

Compound interest at 5% for 40 years:

$1.05^{40} = 7.04$

$1.06^{40} = 10.3$

$1.25^{25} = 265$ Kmart growth rate 25%/yr

$1.56^{25} = 67{,}315$ Walmart growth rate 56%/yr

$(1 + 1/1{,}000{,}000)^{1{,}000{,}000} = 2.718 = e$

Negative exponents

$y^{-x} = 1/y^x$

$9^{-2} = 1/9^2 = 1/81 = .012345679$

$9^{-1/2} = 1/3 = 1/9^{1/2}$

$5.7^{-1.3} = .104$

$.58^{-3.2} = 5.715$

$-3^{.5} = \text{Error}$

Exponents are very common in many situations. The calculator makes it very easy to deal with them. Just follow the rules.

Of course, Wolfram Alpha also will deal with them very easily.

S3E

y^x EXPONENTS

Use your calculator to solve the following exercises.

1. 4^7 =

2. 10^9 =

3. $4.2^{3.6}$ =

4. 8 √256 =

5. 6 √1,000,000 =

6. $^{3.2}\sqrt{8.3}$ =

7. 7^{-2} =

8. $56^{-2.4}$ =

9. $0.47^{-3.1}$ =

10. If production increases at a rate of 6.5%/year, what is your production after 15 years?

11. If production increases at a rate of 7.5%/year, what is your production after 15 years?

12. For the following exponents, match them with their name:

1. 10^3 = 1,000	a. Giga (Digital)
2. 10^6 = 1,000,000	b. Tera (Digital)
3. 10^9 = 1,000,000,000	c. Giga (Metric)
4. 10^{12} = 1,000,000,000,000	d. Tera (Metric)
5. 2^{10} = 1,024	e. Mega (Metric)
6. 2^{20} = 1,048,576	f. Kilo (Metric)
7. 2^{30} = 1,073,741,824	g. Mega (Digital)
8. 2^{40} = 1,099,511,627,776	h. Kilo (Digital)

S3EA

y^x EXPONENTS Answers: [ ]'s

Use your calculator to solve the following exercises.

1. 4^7 = [**16,384**]

2. 10^9 = [**1,000,000,000**]

3. $4.2^{3.6}$ = [**175.266**]

4. 8 √256 = [**2**]

5. 6 √1,000,000 = [**10**]

6. $^{3.2}\sqrt{8.3}$ = [**1.937**]

7. 7^{-2} = [**0.020**]

8. $56^{-2.4}$ = [**0.0000637**]

9. $0.47^{-3.1}$ = [**10.387**]

10. If production increases at a rate of 6.5%/year, what is your production after 15 years?
    [**1.065^{15} = 2.572**]

11. If production increases at a rate of 7.5%/year, what is your production after 15 years?
    [**1.075^{15} = 2.959**]

12. For the following exponents, match them with their name:
    [**1f, 2e, 3c, 4d, 5h, 6g, 7a, 8b**]

S4 LESSON: Density = Weight/Volume

How much does 55 gallons of water weigh (in lbs)?

How much does 55 gallons of gasoline weigh?

How much does 55 gallons of cement weigh?

How much does 55 gallons of mulch weigh?

Weight is measured in units such as:
Grams (gm), pound (lb), ounce (oz),
kilograms (kg), stone (st), etc

Volume is measured in such units as:
gallons(gal), quarts (qt), fluid ounces (fl oz),
liters (ltr), cubic inches (cu in or in^3),
cubic feet (cu ft or ft^3), or in general cubic U (cu U or U^3) where
U is a linear length, etc.

Suppose 1 gallon of water weighs 8.345 lbs

Then, 55 gallons would weigh 55x8.345 = 459 lbs

How do you find out what 1 gallon of water weighs?

Well, you could weigh a quart of water and multiply by 4, since 4 quarts equals one gallon.

Or, you could weigh 1 oz of water and multiply by 128 since one gallon is 128 oz.

Or, you could weigh a container full of water whose volume is 12 oz and then multiply by 128/12

Of course, you must subtract the weight of the empty container!

The Density of water is what you are computing.

Density = Mass/Volume = Weight/Volume

D = W/V or W = DV or V = W/D

So, if you know any two of these, then you always can calculate the third.

The units must always match up.

If W is lb and V is ft^3, the D must be lb/ft^3

D could be lb/gal, or oz/quart, or gm/liter, etc.

Above we determined a W and V in an experiment and calculated D, and then used this D to calculate the W when we were given the V.

What you always want to do first is learn the D for a substance.

For example, D for gasoline is 6.06 lb/gal

So, 55 gallons of gasoline would weigh:

55x6.06 = 333 lbs VxD = W galx(lb/gal) = lb

BUT, how do we know D for gasoline?

1. We could look it up in some table of densities.
2. We could find out on the Internet. My favorite is www.wolframalpha.com
3. We could do the experiment by weighing a known volume, usually pretty small.

WA1 **density of gasoline in lb/gal**

Answer: 6.06 lb/gal

But, suppose you did the experiment and found that 24.7 cu in of gasoline weighed 10.4 oz?

10.4/24.7 = .42 oz/in^3

WA2 **convert .42 oz/in^3 to lb/gal**

convert this to lb/gal

Answer: 6.06 lb/gal as it should be.

Note: Do you think I actually did this experiment?

Of course not, I just used WA backwards

WA3 **convert 6.06 lb/gal to oz/in^3**

Answer: .42 oz/in

But, in many cases, you won't be able to find the Density of a substance in any handbook, or even on Wolfram Alpha. So then, you simply must do the experiment with a convenient container.

1. Compute its volume.
2. Fill it up with the substance.
3. Calculate the Density of this substance.

Then you can find either V or W if you know the other one.

For example, how many cubic yards will one ton of insulation material fill up?

Suppose we do the experiment and find that the density of some insulation material is 2.5 lbs/gal. (I have no idea what it really would be.)

Then, WA tells us the density would be:

WA4 **convert 2.5 lbs/gal to lbs/yd^3**

Answer: 505 lbs/cu yd

So, V = W/D yields 2000/505 = 4 yd^3 as the answer.

How much does 55 gallons of cement weigh?

WA5 **density of cement in lb/gal**

Answer: 16.8 lb/gal

So 55 gallons weighs 55x16.8 = 924 lbs

If in doubt, actually do the experiment and weigh a small amount and then do the calculations.

How much does 55 gallons of mulch weigh?

WA6 **density of mulch in lb/gal**

WA doesn't know. You will probably just have to do the experiment and calculate the density.

So now, you can do a bunch of problems.

Sometimes, WA will give you the density.

Sometimes you will have to find it by experiment.

Use some handy container whose volume you know or can compute. And, fill it up and weight it. Subtract the empty container weight. Then, use WA to convert it to the Units you want.

S4E

Density = Weight/Volume

Use your calculator to solve the following exercises.

1. 1 quart of seawater (salt water) weighs 2.138 lb. What is the density of seawater (lb/gal)?

2. The density of propane is 0.0156843 lb/gal. A residential tank holds 250 gal. of propane. What is the weight (lb) of the propane in that tank?

3. The density of gold is 11.2 oz/ in^3. What is the volume (in^3) of 16 oz. (or 1 lb) of gold?

4. A quart of whole milk weighs 2.3 lb. What is the density (gal) of whole milk in lb/gal?

5. An adult is recommended to limit their salt intake to no more than 2300 mg per day. If the density of salt is 10,600 mg/tsp (teaspoons), what is the volume of salt (tsp) an adult should not exceed per day?

6. A grass catcher for a mower holds 4.4 ft^3 of grass. If the density of grass is 17.4 lb/ ft^3, what is the weight (lb) of the grass in the catcher?

7. You buy a pool which is 24 ft in diameter and fills with water to 4 ft deep. The density of water is 8.345 lb/gal. How much does the water in your pool weigh (lb)? Useful information: 1 ft^3 = 7.481 gal.

8. A ream (500 sheets) of 8.5 in x 11 in standard office paper is 2 in thick, and weighs 5 lb. What is the density of the paper (oz/in^3)? Useful information: 1 lb = 16 oz.

9. If 1 lb of feathers has a density of 0.0025 g/cm^3, what is the volume of those feathers (cm^3 and ft^3)? Useful information: 1 lb = 453.6 g; 1ft^3 = 28,317 cm^3

10. A bag of concrete mix weighs 80 lb. and has a dry volume of 0.53 ft^3. If 4 liters (L) of water are added to the mix, what is the final weight (lbs.) of the concrete? Also, what is the final volume (ft^3) that the bag will fill once mixed with water? Use these numbers to calculate the density (lb/ft^3). Useful information: Density of water: 1000 g/L (grams/liter); 1 lb = 453.6 g; 1 L = 0.03531 ft^3

S4EA

Density = Weight/Volume Answers: [ ]'s

1. D = W/V

   D = 2.138 lb/1 quart

   D = (2.138 lb/quart)x(4 gal/quart)

   D = 8.552 lb/gal

2. W = VD

   W = (250 gal)x(0.0156843 lb/gal)

   W = 3.92 lb

3. V = W/D

   V = (16 oz)/(11.2 oz/in^3)

   V = 1.43 in^3

4. D = W/V

   D = 2.3 lb/1 quart

   D = (2.3 lb/quart)x(4 gal/quart)

   D = 9.2 lb/gal

5. V = W/D

   V = (2300 mg)/(10,600 mg/tsp)

   V = 0.217 tsp

6. W = VD

   W = (4.4 ft^3)x(17.4 lb/ ft^3)

   W = 76.6 lb

7. W = VD

V = Height x Area

V = Height x πRadius^2 or Height x πx(1/2 Diameter)^2

$V = (4 \text{ ft}) \times (\pi \times (1/2 \times 24 \text{ ft})^2)$

$V = 1809.557 \text{ ft}^3$

$V = (1809.557 \text{ ft}^3) \times (7.481 \text{ gal/ft}^3)$

V = 13,537.299 gal

W = (13537.299 gal)x(8.354 lb/gal)

W = 113,091 lb

8. D = W/V

V = (8.5 in)x(11 in)x(2 in)

$V = 187 \text{ in}^3$

W = (5 lb)x(16 oz/lb)

W = 80 oz

$D = (80 \text{ oz})/(187 \text{ in}^3)$

$D = 0.4 \text{ oz/in}^3$

9. V = W/D

$V = (453.6 \text{ g})/0.0025 \text{ g/cm}^3)$

$V = 181{,}440 \text{ cm}^3$

$V = (181{,}440 \text{ cm}^3)(1/28{,}317 \text{ ft}^3/\text{cm}^3)$

$V = 6.4 \text{ ft}^3)$

10. Weight:

Concrete mix: 80 lb (given)

Water:

(4 L) x (1000 g/L) x (1/453.6 lb/g) = 8.82 lb

Total:

80 lb + 8.82 lb = 88.82 lb

Volume:

Concrete mix: 0.53 ft^3 (given)

Water:

(4 L) x (0.03531 ft3/L) = 0.14 ft^3

Total:

0.53 ft^3 + 0.14 ft^3 = 0.67 ft^3

Density:

D = W/V

D = 88.82 lb/0.67 ft^3

D = 132.57 lb/ft^3

S5 LESSON: FLO SCI ENG Formats

Numbers can be expressed in three different formats:

FLO or Floating Point is the format you are familiar with.

64327.59 is an example.

Of course you know this is the same as:
$6x10^4+4x10^3+3x10^2+2x10^1+7x10^0+5x10^{-1}+9x10^{-2}$

And, $10^0 = 1$, $10^{-n} = 1/10^n$

Now we can also express this number is what is called **SCI or scientific format**

$64327.59 = 6.432759 \times 10^4$

Or in **ENG or engineering format**

$64327.59 = 64.32759 \times 10^3$

In the ENG format you will always have 10 to an exponent that is a multiple of 3. You'll see why this is when we study Prefixes in another lesson.

SCI and ENG notations are sometimes used in documentation and you can always convert from one to the other with our calculator or to FLO if the number is not too large.

However, for very large or very small numbers, SCI or ENG formats are necessary.

Frankly, if you are going to be working with very large or very small numbers you will probably be using a computer and much more powerful tools than a calculator.

It is easy to use scientific notation with a tool like Wolfram Alpha.

However, you may occasionally see them with the calculator if you multiply or divide large numbers or use the y^x key with large exponents.

$12^{21} = 4.6x10^{22}$

Now multiply by 9^{13}

$1.169x10^{35} = 1.169388422x10^{35}$

Also, the largest exponent of 10 the calculator will accept is 99.

109^85   error

But, WA handles it just fine.

S5E

FLO SCI ENG Formats

Using your calculator, convert the following numbers to both SCI and ENG.

1. 640873.26 =

2. 2347168.002 =

3. 0.0002547 =

Using your calculator, convert the following numbers to both SCI and ENG, fixing each to the number digits past the decimal point as indicated.

4. 54178962.3 (3 digits past the decimal point) =

5. 214697.0045 (2 digits past the decimal point) =

6. 145879125 (4 digits past the decimal point) =

Using your calculator, calculate the following numbers. If you receive an error message, use Wolfram Alpha.

7. $15^{26}x2^{23}$ =

8. $26^{56}x32^{54}$ =

9. $45^{-23}x16^{-13}$ =

10. $18.45^{-56}x46.78^{-24}$ =

S5EA

FLO SCI ENG Formats Answers: [ ]'s

1. 640873.26 = [**SCI = 6.4087326×10^5; ENG = 640.87326×10^3**]

2. 2347168.002 = [**SCI = 2.347168002×10^6; ENG = 2.347168002×10^6**]

3. 0.0002547 = [**SCI = 2.547×10^{-4}; ENG = 254.7×10^{-6}**]

4. 54178962.3 (3 digits past the decimal point) =
   [**SCI = 5.418×10^7; ENG = 54.179×10^6**]

5. 214697.0045 (2 digits past the decimal point) =
   [**SCI = 2.15×10^5; ENG = 214.70×10^3**]

6. 145879125 (4 digits past the decimal point) =
   [**SCI = 1.4588×10^8; ENG = 145.8791×10^6**]

7. $15^{26} x 2^{23}$ = [**4.0331166×10^{34}**]

8. $26^{56} x 32^{54}$ = [**Error**
   WA **26^56x32^54**
   $3.28553665 \times 10^{160}$]

9. $45^{-23} x 16^{-13}$ = [**$2.101611366 \times 10^{-54}$**]

10. $18.45^{-56} x 46.78^{-24}$ =
   [**Interestingly, the calculator says "0" instead of "Error"**
   WA **18.45^-56x46.78^-24**
   $1.053799609 \times 10^{-111}$]

S5A LESSON: FLO SCI ENG Formats Addendum

As we learned in S5, numbers can be expressed in three different formats.

FLO or Floating Point is the format you are familiar with.
64327.59 is an example.

SCI or scientific format
$64327.59 = 6.432759 \times 10^4$

ENG or engineering format
$64327.59 = 64.32759 \times 10^3$

What we haven't learned yet is how to enter a number in a SCI or ENG format into the calculator.

It is very easy. You just use the EE Key.

To enter 6.432759×10^4:

Just enter 6.432759 and Press the EE key,

Then enter 4, and you are done.

Now you can change it into any other format, and also you can save it in memory and the recall it in this format.

Similar for ENG format:

Just enter 64.32759 and Press EE, and then enter 3

You can also enter negative numbers.

Just press the + <-> - key before you press the EE Key.

6.432759 + <-> - EE 3

Enters the negative of this number

You can also enter a negative exponent by just pressing the + <-> - key before entering the exponent.

6.432 EE + <-> - 4

Enters 6.432×10^{-4} or .00006432

Of course, you could also enter

-6.432×10^{-5} or -.00006432

6.432 + <-> - EE 5 + <-> -

C13 LESSON: DEG RAD GRAD THREE ANGLE MEASURES

There are three measures of an angle acceptable by the TI-30Xa calculator.

Degree **DEG** 1/360 of a circle

Gradian **GRAD** 1/400 of a circle

Radian **RAD** 1/2π of a circle with radius 1. (57.3 DEG)

In our Practical Math Foundation we will only use the **DEG** which is what automatically comes up when you turn on the calculator.

The DRG Key changes the choice of unit.

If you enter a number in the **DEG** mode and then press the **2nd** **DRG** Keys, you will transform the number to the new unit.

For example, enter 180 as **DEG**, then transform into **RAD** (3.1416) and **GRAD** (200)

Or; enter 1 in **RAD** mode, and transform into 57.3 Degrees.

We will only use **DEG** in the Foundation training.

RAD will also be used in Tiers 4 and up. It is the "natural" measurement of an angle for trig and calculus.

C13E

DEG RAD GRAD THREE ANGLE MEASURES

1. DEG stands for?

2. What fraction of a circle is one degree?

3. What are the other two angle measures on the TI-30Xa calculator?

4. Which measure comes up when you turn on the calculator?

5. How do you switch to the other two measures?

6. How do you convert Degrees to **RAD**s and **GRAD**s?

7. How many **RAD**s are 90 degrees?

8. How many **GRAD**s are 90 degrees?

9. What will we use exclusively in the Foundations Course to measure angles?

Answers are on C13EA, page 46.

Take the C13 Quiz.

C13EA

DEG RAD GRAD THREE ANGLE MEASURES

Answers: [ ]'s

1. DEG stands for? [**Degree º**]

2. What fraction of a circle is one degree? [**1/360**]

3. What are the other two angle measures on the TI-30Xa calculator? [**RAD and GRAD**]

4. Which measure comes up when you turn on the calculator? [**DEG**]

5. How do you switch to the other two measures? [**Press the DRG key once for RAD again for GRAD and again for DEG**]

6. How do you convert Degrees to RADs and GRADs? [**Enter the degrees and press the 2nd DEG key for RADs and press 2nd DEG key again for GRADs**]

7. How many RADs are 90 degrees? [**1.57**]

8. How many GRADs are 90 degrees? [**100**]

9. What will we use exclusively in the Foundations Course to measure angles? [**DEG Degrees**]

Take the C13 Quiz or review.

C14 LESSON: SIN SIN^{-1}

These two keys are used to compute the Sine of an angle, and the angle, if you know its SIN.

This is used in Trigonometry, and also for some interesting formulas in Geometry.

We will always use the Degree, **DEG**, measure of an angle in the Foundation course.

Enter the angle, say, θ, and press SIN

Example: 45 SIN yields .707

SIN (θ) is always between -1 and 1.

SIN^{-1} is the "inverse" of the SIN, 2nd SIN

If SIN (θ) = N, then SIN^{-1}(N) = θ

Example: SIN^{-1}(.707) = 45°

SIN^{-1}(N) only works for N between -1 and 1.

NOTE: SIN 135 = 0.707…in general, SIN (180°- θ) = SIN (θ)

C14E

SIN **SIN^{-1}** Answers: [ ]'s

1. **SIN** (45°) = ? [**0.707**]
2. **SIN** (0°) = ? [**0**]
3. **SIN** (10°) = ? [**0.174**]
4. **SIN** (30°) = ? [**0.500**]
5. **SIN** (60°) = ? [**0.866**]
6. **SIN** (75°) = ? [**966**]
7. **SIN** (85°) = ? [**0.996**]
8. **SIN** (90°) = ? [**1**]
9. **SIN** (95°) = ? [**0.996**]

11. **SIN** (120°) = ? [**0.866**]

12. **SIN^{-1}**(0.5) = ? [**30 degrees**]

13. What angle X, has **SIN** (X) = 0.4 ? [**23.58 degrees**]

14. **SIN^{-1}**(0.4) = ? [**23.58 degrees**]

15. **SIN^{-1}[SIN(50°)]** = ? [**50 degrees**]

Take C14 Quiz or do more exercises, C14ES.

C14ES

SIN **SIN^{-1}** Answers: [ ]'s

1. **SIN** (30° + 90°) = ? [**SIN(120°) = 0.866**]
2. **SIN** (45° + 90°) = ? [**SIN(135°) = 0.707**]
3. **SIN** (60° + 90°) = ? [**SIN(150°) = 0.5**]
4. **SIN** (90° + 90°) = ? [**SIN(180°) = 0**]
5. **SIN^{-1}** (0.866) = ? [**59.99° ~ 60°**]
6. Why the discrepancy in #5? [**Round off error SIN(60°) = .866025404 SIN(59.99°) = .865938124**]
7. **SIN^{-1}**(0.5) = ? [**30°**]
8. What angle X, has **SIN**(X) = .3? [**17.5°**]
9. **SIN^{-1}**(0.3) = ? [**17.5°**]
10. **SIN^{-1}**[SIN(x°)] = ? [**x°**]
11. **SIN**[**SIN^{-1}**(x) = ? [**x**]
12. **SIN** (θ) is always between? [**-1 and 1**]
13. SIN^{-1}(1.5) = ? [**Error**]

Take C14 Quiz or review.

C15 LESSON: COS $\quad$ COS^{-1}

These two keys are used to compute the Cosine of an angle, and the angle, if you know its COS.

This is used in Trigonometry and also for some interesting formulas in Geometry.

We will always use the Degree, **DEG**, measure of an angle in the Foundation course.

Enter the angle, say, θ, and Press COS

Example: 45 COS yields .707

COS (θ) is always between -1 and 1.

COS^{-1} is the "inverse" of the COS, 2nd COS

If COS (θ) = N, then COS^{-1}(N) = θ N between -1 and 1

Example: COS-1(.707) = 45°

NOTE: COS 135 = -.707 In general, COS (180° - θ) = - COS(θ)

You could verify: COS(90 - θ) = **SIN** (θ) for example.

SIN and COS are intimately related as you will learn in the Trigonometry section of Tier 2, and even more in Tier 4.

C15E

COS **COS^{-1}** Answers: [ ]'s

1. **COS** (45°) = ? [**0.707**]
2. **COS** (0°) = ? [**1**]
3. **COS** (10°) = ? [**0.985**]
4. **COS** (30°) = ? [**0.866**]
5. **COS** (60 °) = ? [**0.500**]
6. **COS** (75°) = ? [**0.259**]
7. **COS** (85°) = ? [**0.087**]
8. **COS** (90°) = ? [**0**]
9. **COS** (95°)= ? [**-0.087**]
10. **COS^{-1}** (0.5) = ? [**60 degrees**]
11. What angle X, has **COS** (X) = .4? [**66.4 degrees**]
14. **COS^{-1}**(.4) = ? [**66.4 degrees**]
15. **COS^{-1}**[**SIN**(50°)] = ? [**40 degrees**]

Take the C15 Quiz or do some more exercise, C15ES.

C15ES

COS **COS^{-1}** Answers: [ ]'s

1. **COS** ($30^o + 90^o$) = ? [**COS(120^o) = -0.5**]
2. **COS** ($45^o + 90^o$) = ? [**COS(135^o) = -0.707**]
3. **COS** ($60^o + 90^o$) = ? [**COS(150^o) = -0.866**]
4. **COS** ($90^o + 90^o$) = ? [**COS(180^o) = -1**]
5. **COS^{-1}** (0.866) = ? [**30^o**]
6. **COS^{-1}**(0) = ? [**90^o**]
7. **COS^{-1}**(.5) = ? [**60^o**]
8. What angle X, has **COS** (X) = .3? [**72.5^o**]
9. **COS^{-1}**(0.3) = ? [**72.5^o**]
10. **COS^{-1}** [**COS**(x^o)] = ? [**x^o**]
11. **COS**[**COS^{-1}**(x) = ? [**x**]
12. **COS** (θ) is always between? [**-1 and 1**]
13. **COS^{-1}**(1.5) = ? [**Error**]

Take the C15 Quiz or review.

C16 LESSON: TAN TAN^{-1}

These two keys are used to compute the Tangent of an angle, and the angle, if you know its **TAN**

This is used in Trigonometry.

We will always use the Degree, **DEG**, measure of an angle in the Foundation course.

Enter the angle, say, θ, and Press **TAN**

Example: 45 **TAN** yields 1

TAN (θ) can be any size

TAN^{-1} is the "**inverse**" of the **TAN**, **2nd** **TAN**

If **TAN** (θ) = N, then **TAN^{-1}**(N) = θ

Example: **TAN^{-1}**(1) = 45°

NOTE: We will not use **TAN** in the Foundation Course.

TAN is also intimately related to **SIN** and **COS**.

C16E

TAN **TAN^{-1}** Answers: [ ]'s

1. **TAN** (45°) = ? [**1**]
2. **TAN** (0°) = ? [**0**]
3. **TAN** (10°) = ? [**0.176**]
4. **TAN** (30°) = ? [**0.577**]
5. **TAN** (60°) = ? [**1.732**]
6. **TAN** (75°) = ? [**3.732**]
7. **TAN** (85°) = ? [**11.43**]
8. **TAN** (90°) = ? [**Error**]
9. **TAN** (95°) = ? [**-11.430**]
10. **TAN^{-1}** (0.05) = ? [**26.57 degrees**]
11. What angle X, has **TAN** (X) = 0.4? [**21.8°**]
12. **TAN^{-1}**(0.4) = ? [**21.8 degrees**]
13. **TAN^{-1}**[**TAN**(50°)] = ? [**50 degrees**]

Take the C16 Quiz or do more exercise, C16ES.

C16ES

TAN **TAN-1** Answers: [ ]'s

1. **TAN** (90^o) = ? [**Error**]
2. **TAN** (89.99^o) = ? [**5730**]
3. **TAN** (-89.99^o) = ? [**-5730**]
4. **TAN** (88^o) = ? [**29**]
5. **TAN** ($80\ ^o$) = ? [**6**]
6. **TAN** (60^o) = ? [**2**]
7. **TAN** (30^o) = ? [**1**]
8. **TAN** (10^o) = ? [**0.176**]
9. **TAN^{-1}** (0.577) = ? [**30^o**]
10. What angle X, has **TAN** (X) = 1 ? [**45^o**]
11. **TAN^{-1}**(1) = ? [**45^o**]
12. **TAN^{-1}**[**TAN**(150^o)] = ? [**-30^o**]
13. **TAN^{-1}**[**TAN**(-30^o)] = ? [**-30^o**]

Take the C16 Quiz or review.

A9 LESSON: (1) SIN X^O = A, -1 ≤ A ≤ 1, OR (2) $SIN^{-1}X = A^O$, 0 ≤A^O≤ 180^O

NOTE: Contrary to the audio, you cannot defer this lesson.

Two easy equations. (Apply correct **Inverse** to both sides)

Note: **X** is **angle** measured degrees (°) in the first equation
A is **angle** measured in degrees (°) in the second equation

Note: You don't need to even know what **SIN** means to solve the equation using the calculator.

Example: **SIN** X^o = .548 — Apply **SIN^{-1}** to both sides
$X^o = SIN^{-1}(.548) = 33.2^0$ — **Note:** <u>2nd</u> <u>**SIN**</u> yields SIN^{-1}

Example: **SIN X** = .8765 — **X** = 61.2° [**X** is in °]

Example: **SIN^{-1}X** = 28^0 — Apply **SIN** to both sides and get **X** = **SIN**(**SIN^{-1}X**) = **SIN**(28°) = .469

Example: (.75 + **COS**49°)**SIN^{-1}X** = (14.23 + **SIN**35°)^2
(Looks bad, but is really easy. Just do the numbers first.)

COS49° = .656; **so** .75 + .656 = 1.41 **and**

SIN35° = .574; **so** $(14.23 + .574)^2$ = 219 and so we get

1.41**SIN^{-1}X** = 219, **or** **SIN^{-1}X** = 219/1.41 = 155°

Thus, **X** = **SIN** 155^0 = .416

Check: 1.41x**SIN^{-1}**.416 = 1.41x24.6 = 34.7, not 219.

Something wrong. Must wait until **Trig Lesson T2** to understand.

Preview hint: **SIN**155° = **SIN** 25^0

A9 (1) SIN Xo = A, -1 ≤ A ≤ 1, or (2) SIN-1X = A^o ,0 ≤ A^o≤ 180^o

Two easy equations. (Apply correct **Inverse** to both sides)

Note: **X** is **angle** measured degrees (^{o}) in the first equation

A is **angle** measured in degrees (^{o}) in the second equation

Note: You don't need to even know what **SIN** means to solve the equation using the calculator.

Example: **SIN** $X^o = .548$ — Apply $\mathbf{SIN}^{-1}$ to both sides

$X^o = \mathbf{SIN}^{-1}(.548) = 33.2^0$ — **Note:** 2nd SIN yields $\mathbf{SIN}^{-1}$

Example: **SIN X** = .8765 — **X** = 61.2^o [**X** is in ^{o}]

Example: $\mathbf{SIN}^{-1}\mathbf{X} = 28^0$ — Apply **SIN** to both sides and get **X** = **SIN**($\mathbf{SIN}^{-1}$**X**) = **SIN**(28^o) = .469

Example: (.75 + **COS**49^o)$\mathbf{SIN}^{-1}$**X** = (14.23 + SIN35^o)^2

(Looks bad, but is really easy. Just do the numbers first.)

COS49^o = .656 **so** .75 + .656 = 1.41 **and**

SIN35^o = .574 **so** $(14.23 + .574)^2 = 219$ and so we get

1.41$\mathbf{SIN}^{-1}$X = 219, or $\mathbf{SIN}^{-1}$X = 219/1.41 = 155^o

Thus, **X** = **SIN** 155^0 = .416

Check: 1.41xSIN^{-1}.416 = 1.41x24.6 = 34.7, not 219.

Something wrong. Must wait until Trig **Lesson T2** to understand.

Preview hint: **SIN**155^o = **SIN** 25^o

A9E

(1) SIN X° = A, -1 ≤ A ≤ 1, or

(2) SIN^{-1}X = A° ,0 ≤A°≤ 180°

Two easy equations. (Apply correct **Inverse** to both sides)

Note: **X** is angle measured degrees (°) in the first equation
A is **angle** measured in degrees (°) in the second equation

Solve for X, the Unknown. **Note:** The Algebra is easy. The arithmetic can be complicated, but is easy with the **TI-30Xa**.

1. **SIN X°** = 0.548

2. **SIN X°** = 0.8765,

3. **SIN^{-1}X** = 28°

4. 2.3**SIN X°** = 1.92

5. **SIN X°** = 1.5

6. **SIN^{-1}**(0.8765) = **X°**

7. **SIN^{-1}**(**SIN**(56°) = **X**

8. **SIN**(**SIN^{-1}**(0.321) = **X**

9. **SIN^{-1}**(**X^2**) = 15°

10. **SIN**(3**X°**) = 0.5

A9EA

(1) SIN $X^o = A, -1 \le A \le 1$, or

(2) $SIN^{-1}X = A^o, 0 \le A^o \le 180^o$

Answers: [ ]

Two easy equations. (Apply correct **Inverse** to both sides)

Note: **X** is angle measured degrees (°) in the first equation
A is **angle** measured in degrees (°) in the second equation

Solve for **X**, the **Unknown**. Note; The Algebra is easy. The arithmetic can be complicated, but easy with the **TI-30Xa.**

1. **SIN** $X^o = 0.548$	[**33.23°**]
2. **SIN** $X^o = 0.8765$	[**61.22°**]
3. $SIN^{-1}X = 28^0$	[**0.4695**]
4. 2.3**SIN** $X^o = 1.92$	[**56.6°**]
5. **SIN** $X^o = 1.5$	[**No Solution, Impossible**]
6. $SIN^{-1}(0.8765) = X^o$	[**61.22°**]
7. $SIN^{-1}(SIN(56^o) = X$	[**56°**]
8. $SIN(SIN^{-1}(0.321) = X$	[**0.321**]
9. $SIN^{-1}(X^2) = 15^o$	[**0.5087**]
10. $SIN(3X^o) = 0.5$	[**10°**]

A9ES

(1) $\sin X^\circ = A, -1 \le A \le 1$, or

(2) $\sin^{-1} X = A^\circ, 0 \le A^\circ \le 180^\circ$

Answers: [ ]

1. $\sin X^\circ = 0.765$ [X = 49.9°]
2. $\sin X^\circ = 0.278$ [X = 16.14°]
3. $\sin^{-1}(0.254) = X^\circ$ [X = 14.71°]
4. $\sin^{-1}(X) = 45^\circ$ [X = 0.707]
5. $\sin X^\circ = 2.89$ [NO Solution]
6. $\sin(\sin^{-1}(0.5)) = X$ [X = 0.5]
7. $\sin(125^\circ) = X$ [X = 0.8191]
8. $64\sin(X^\circ) = 38.99$ [X = 37.53°]
9. $\sin(\sin^{-1}(0.75)) = X$ [X = 0.75]
10. $\sin^{-1}(\cos(60^\circ)) = X^\circ$ [X = 30°]
11. $\sin(X^{\circ 2}) = 0.171$ [X = ± 3.14°]
12. $\sin^{-1}(\cos(115)) = X$ [X = -25]

A10 LESSON: (1) COS X^o = A, $-1 \le A \le 1$, OR (2) $COS^{-1}X = A^o$, $0 \le A \le 180^o$

Two easy equations. (Apply **Inverse** to both sides)

Note: X is **angle** measured degrees (^{o}) first equation and
A is **angle** measured in degrees (^{o}) in second equation

Note: You don't need to even know what **COS** means to solve the equation using the calculator.

Example: **COS** $X^o = .548$ Apply COS^{-1} to both sides

$X^o = COS^{-1}(.548) = 56.7^o$ [X was understood to be in ^{o}] **Note**: 2nd COS yields COS^{-1}

Example: $COS^{-1}X = 28^o$ Apply **COS** to both sides

$X = COS(COS^{-1}X) = COS(28^o) = .883$

Example: $(.75 + COS49^o)COS^{-1}X = (14.23 + SIN35^o)^2$

(Looks bad, but is really easy. Just do the numbers first.)

$COS49^o = .656$ **so** $.75 + .656 = 1.41$ **and**
$SIN35^o = .574$ **so** $(14.23 + .574)^2 = 219$

So we have: $1.41COS^{-1}X = 219$ **or** $COS^{-1}X = 219/1.41 = 155$

Thus: $X = COS155^0 = -.906$

Check: $1.41xCOS^{-1}(-.906) = 1.41x155 = 219$

Note: We didn't have the same problem we had with the **SIN**. Why not? Have to wait until **Trig Lesson T3** for explanation.

A10E

(1) $COS\ X^o = A, -1 \le A \le 1$, or

(2) $COS^{-1}X = A^o, 0 \le A \le 180^o$

Two easy equations. (Apply **Inverse** to both sides)

Note: X is **angle** measured degrees (o) first equation and

A is angle measured in degrees (o) in second equation

Solve for **X**, the **Unknown**. **Note**: The Algebra is easy. The arithmetic can be complicated, but easy with the **TI-30Xa.**

1. $COS\ X^o = 0.548$

2. $COS^{-1}X = 28^o$

3. $COS\ X^o = 0.982$

4. $COS\ X^o = SIN\ 79^o$

5. $COS^{-1}X = SIN^{-1}(0.435)$

6. $4COS(3X^o) = 2.56$

7. $2.3COS^{-1}(SIN\ X^o) = 45^o$

8. $(0.75 + COS49^o)COS^{-1}X = (14.23 + SIN35^o)^2$

9. $SIN^{-1}(SIN(125^o) = X^o$

10. $COS^{-1}(COS(125^o) = X^o$

A10EA

(1) COS X^O = A, $-1 \le A \le 1$, OR

(2) $COS^{-1}X = A^O$, $0 \le A \le 180^O$

Answers: []

Two easy equations. (Apply Inverse to both sides)

Note: **X** is **angle** measured degrees (°) first equation and
A is angle measured in degrees (°) in second equation

Solve for **X**, the **Unknown. Note**: The Algebra is easy. The arithmetic can be complicated, but easy with the **TI-30Xa.**

1. **COS** X^o = 0.548 [**56.8°**]

2. $COS^{-1}X = 28^o$ [**0.8829**]

3. **COS** X^o = 0.982 [**10.9°**]

4. **COS** X^o = **SIN** 79^o [**11°**]

5. $COS^{-1}X = SIN^{-1}(0.435)$ [**0.9004**]

6. $4COS(3X^o) = 2.56$ [**16.7°**]

7. $2.3COS^{-1}(SIN\ X^o) = 45^o$ [**70.4°**]

8. $(0.75 + COS49^o)COS^{-1}X = (14.23 + SIN35^o)^2$ [**-0.9125**]

9. $SIN^{-1}(SIN(125^o) = X^o$ [**55°**]

10. $COS^{-1}(COS(125^o) = X^o$ [**125°**]

A10ES

(1) $\cos X^{O} = A$, $-1 \le A \le 1$, OR

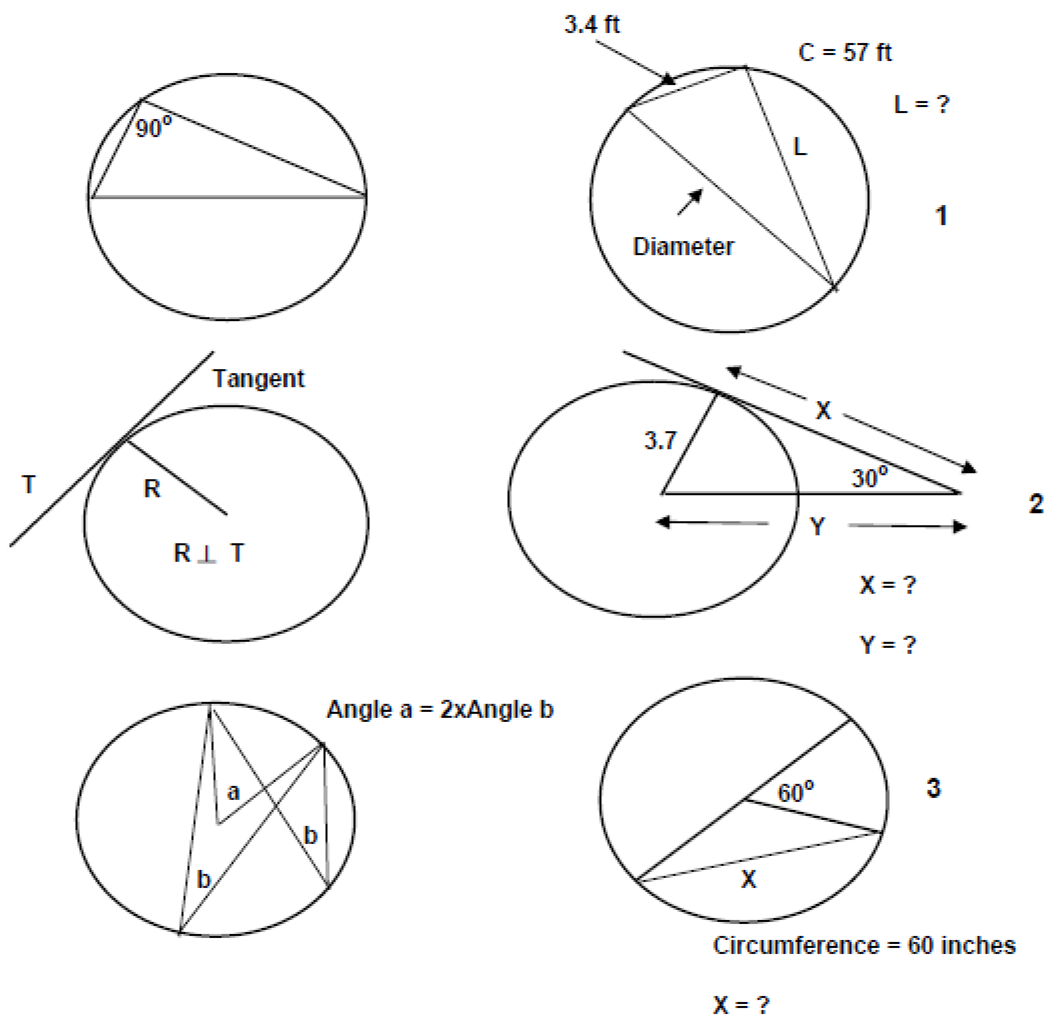
(2) $\cos^{-1} X = A^{O}$, $0 \le A \le 180^{O}$

Answers: []

1. $\cos X° = 0.267$ [X = 74.5°]
2. $\cos X° = 0.6565$ [X = 48.97°]
3. $\cos^{-1}(0.125) = X°$ [X = 82.82°]
4. $\cos^{-1}(X) = 45°$ [X = 0.707]
5. $\cos X° = -0.725$ [X = 136.47°]
6. $\cos X° = -1.76$ [NO solution]
7. $-3.75\cos(11°) = X$ [X = -3.681]
8. $\cos^{-1}(X) = 115°$ [X = -0.4226]
9. $\cos^{-1}(\sin(48°)) = X°$ [X = 42°]
10. $\cos(3X°) = -0.49$ [X = 39.78°]
11. $\cos^{-1}(X/3) = 75°$ [X = 0.7765]
12. $\sin(16.5°)\cos(X°) = 0.119$ [X = 65.23°]

There are three facts about **circles** that I find useful sometimes in a practical problem.

I will present them to you with examples below:



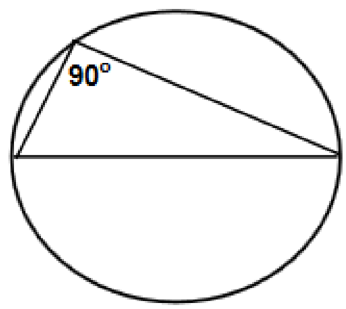
Answers: 1. 17.8 2. Y = 7.4, X = 6.4 3. X = 16.5

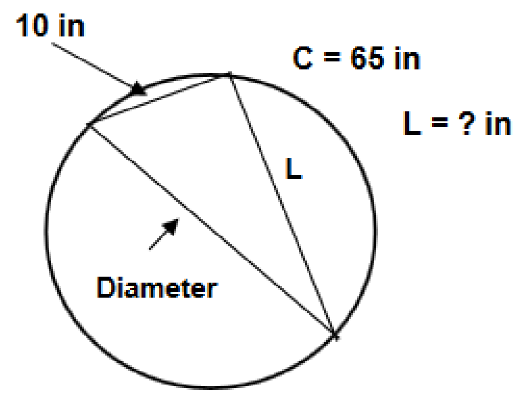
G12E

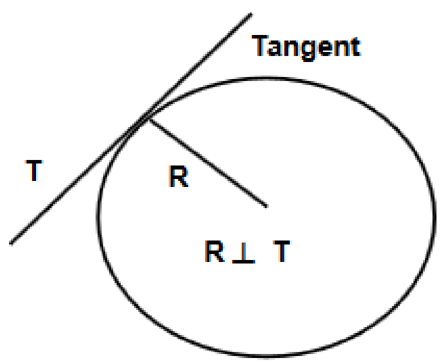
CIRCLES SPECIAL PROPERTIES

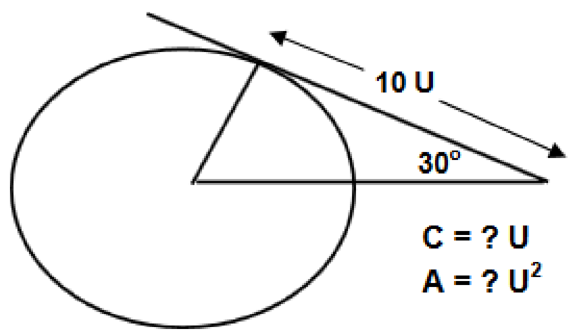
There are three facts about **circles** that I find useful sometimes in a practical problem.

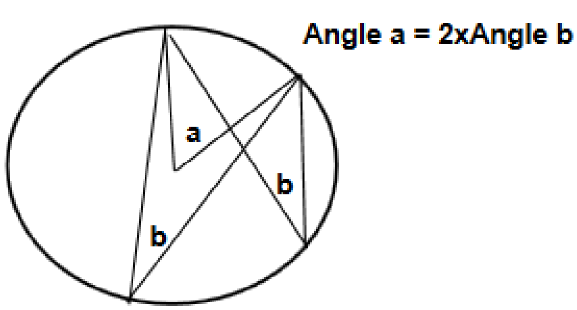
Find the Unknowns.

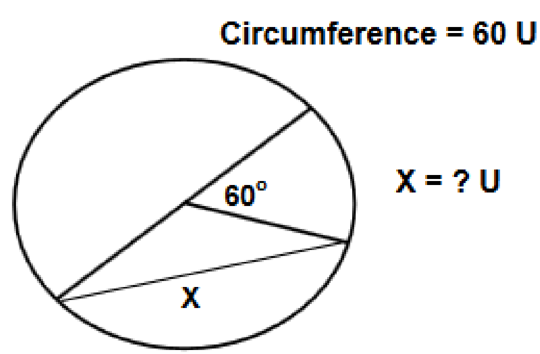










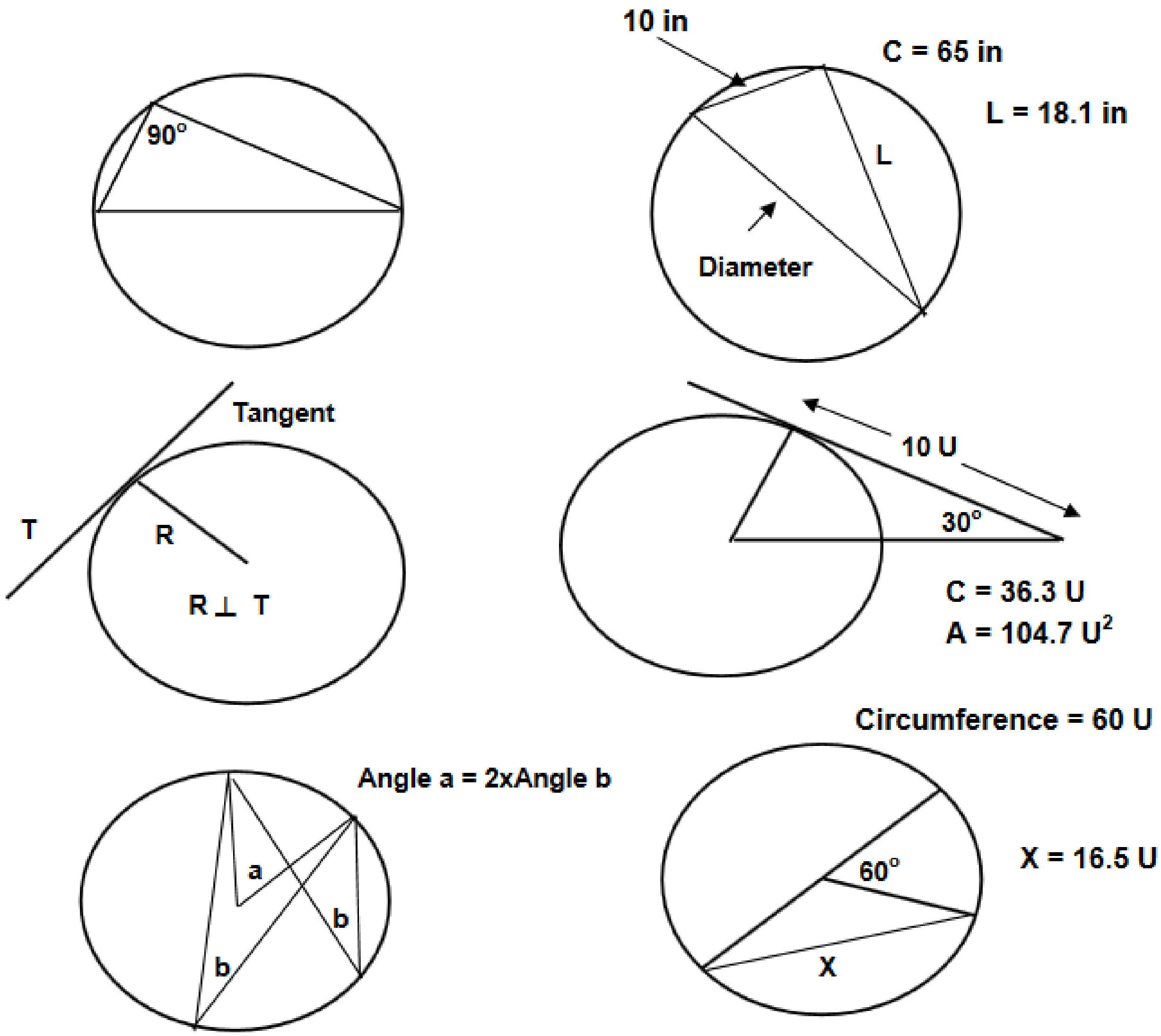


G12EA

CIRCLES SPECIAL PROPERTIES

There are three facts about **circles** that I find useful sometimes in a practical problem.

Find the Unknowns.

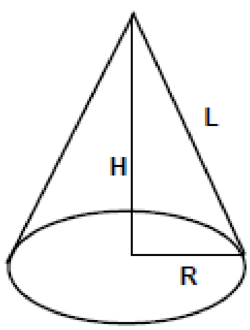


G14 LESSON: SURFACE AREAS CONES

If a **Cone** has **Radius, R,** for its **Base** and has **Height, H,** and **Length, L,** then its **Surface Area** consist of the area of the **Base** plus its **Lateral Area.**

Base Area = πR^2 and **Lateral Area** = $\pi RL = \pi R\sqrt{R^2 + H^2}$

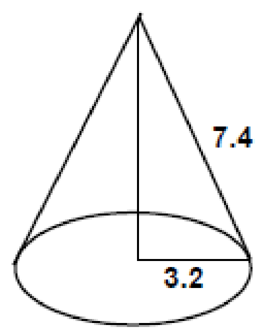
Total Area = $\pi R(R + L)$ measured in U^2



Base Area = πR^2

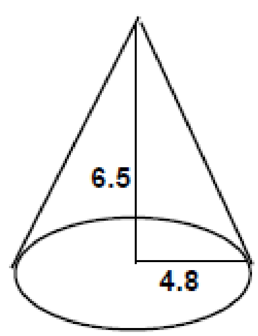
Lateral Area = πRL

Total Area = $\pi R(R + L)$



1

Find Base, Lateral, Total Areas



2

Find Base, Lateral, Total Areas

Answers: 1. 32.2, 74.4, 106.6 U2 **2. 72.4, 121.8, 194.2 U^2**

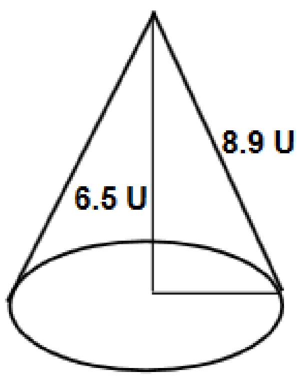
G14E

SURFACE AREAS   CONES

Find the **Base Area, BA**

Find the **Lateral Area, LA**

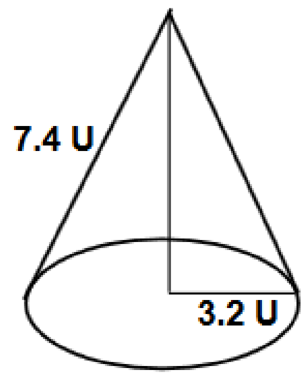
Find the **Total Area, TA**



BA = ? U^2

LA = ? U^2

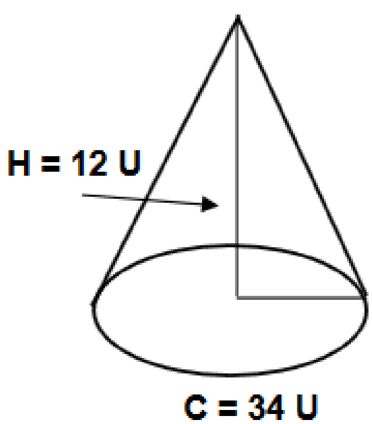
TA = ? U^2



BA = ? U^2

LA = ? U^2

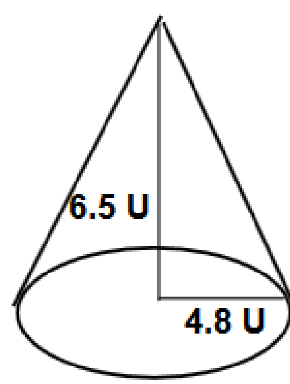
TA = ? U^2



BA = ? U^2

LA = ? U^2

TA = ? U^2



BA = ? U^2

LA = ? U^2

TA = ? U^2

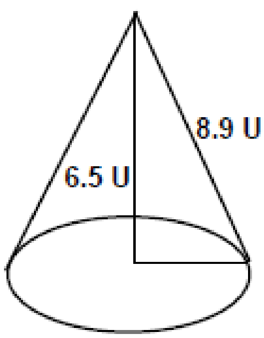
G14EA

SURFACE AREAS CONES

Find the **Base Area, BA**

Find the **Lateral Area, LA**

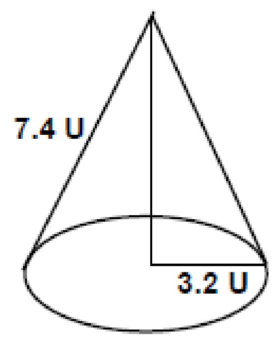
Find the **Total Area, TA**



$BA = 116\ U^2$

$LA = 170\ U^2$

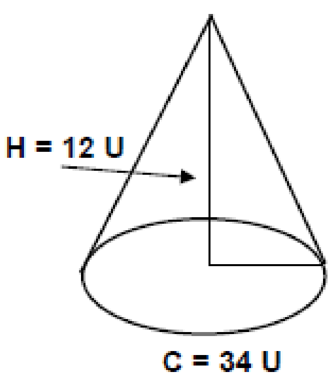
$TA = 286\ U^2$



$BA = 32.2\ U^2$

$LA = 74.4\ U^2$

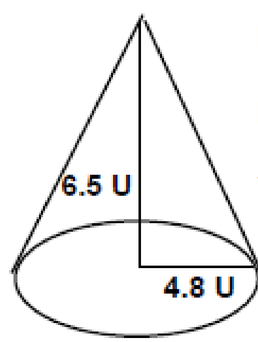
$TA = 107\ U^2$



$BA = 92\ U^2$

$LA = 224\ U^2$

$TA = 316\ U^2$



$BA = 72.4\ U^2$

$LA = 121.8\ U^2$

$TA = 194\ U^2$

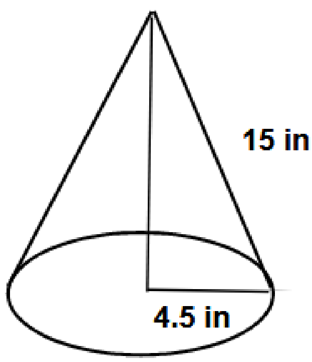
G14ES

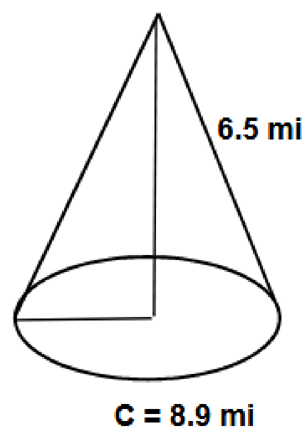
SURFACE AREAS CONES

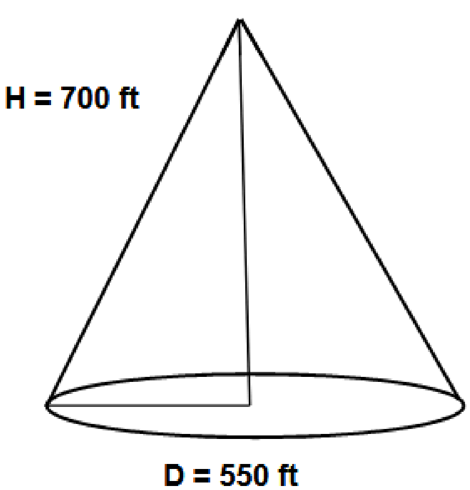
Find the **Base Area, BA**

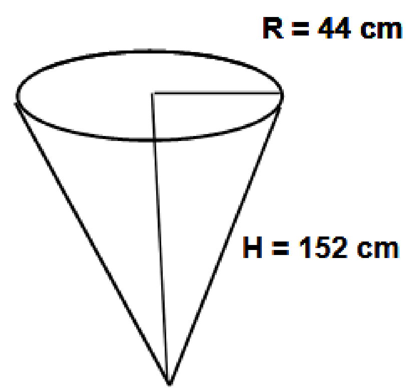
Find the **Lateral Area, LA**

Find the **Total Area, TA**









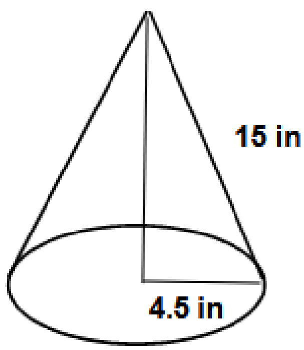
G14ESA

SURFACE AREAS CONES

Find the **Base Area, BA**

Find the **Lateral Area, LA**

Find the **Total Area, TA**



BA = 63.6 in^2

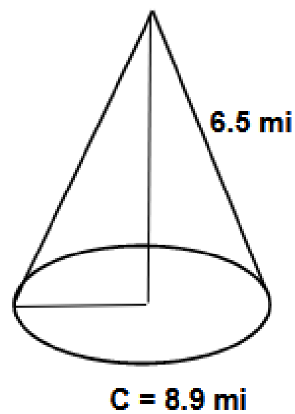
LA= 212.1 in^2

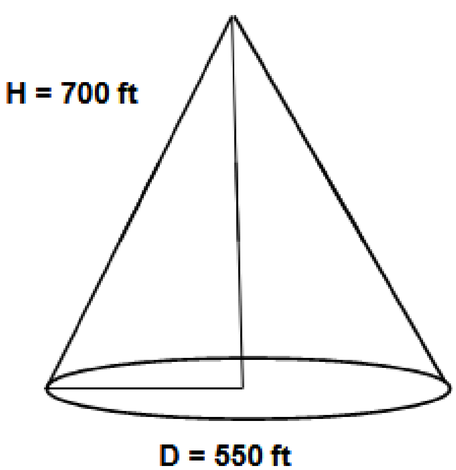
TA = 275.7 in^2

BA = 6.3 mi^2

LA= 28.9 mi^2

TA = 35.2 in^2

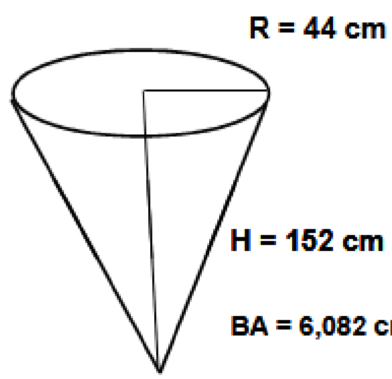




BA = 159,043 ft^2

LA= 635,229 ft^2

TA = 794,272 ft^2



BA = 6,082 cm^2

LA= 21,874 in^2

TA = 27,956 cm^2

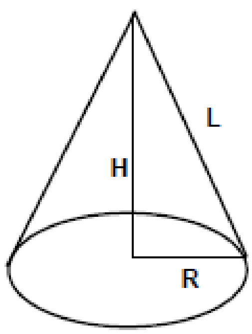
G16 LESSON: VOLUME CONES

If a **Cone** has **Radius**, **R**, for its **Base** and has **Height**, **H**, and **Length**, **L**, then its **Volume**, **V**, is:

Base Area = πR^2 and **V** = $(1/3)\pi R^2 H = (1/3)\pi R^2\sqrt{L^2 - R^2}$

Volume is measured in **Cubic Units**, U^3, where **U** is a **linear measure.**

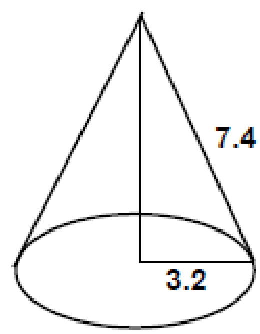
For example: cubic inches, in^3



Volume = $(1/3)\pi R^2 H$

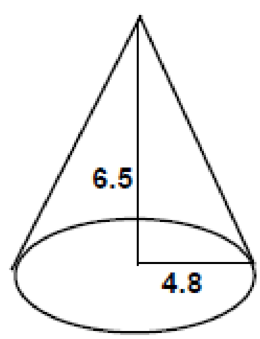
Volume = $(1/3)\pi R^2\sqrt{L^2 - R^2}$

1



Find Volume

2



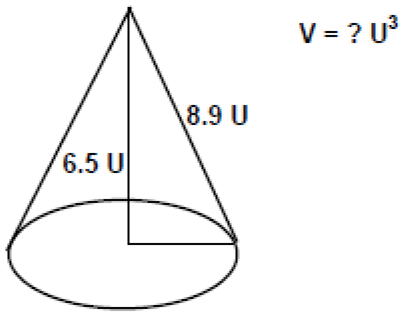
Find Volume

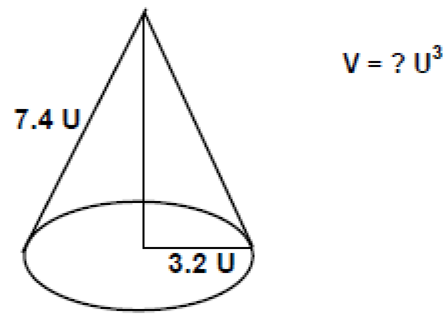
Answers: 1. 71.5 U3 **2. 156.8 cubic units or U^3**

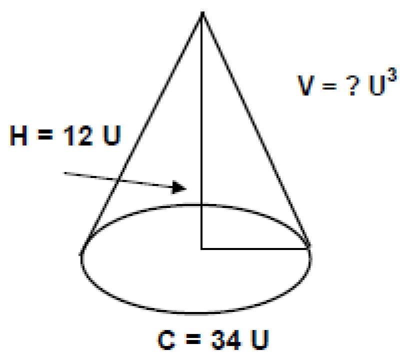
G16E

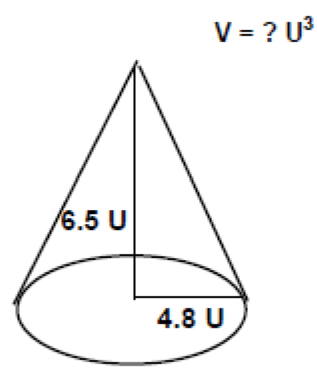
VOLUMES CONES

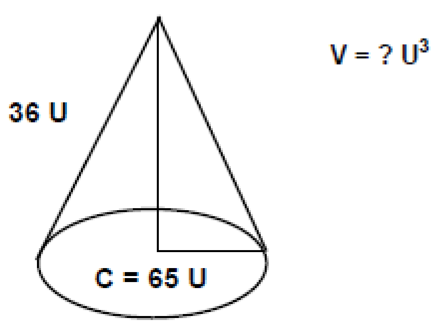
Find the **Volume**, in U^3

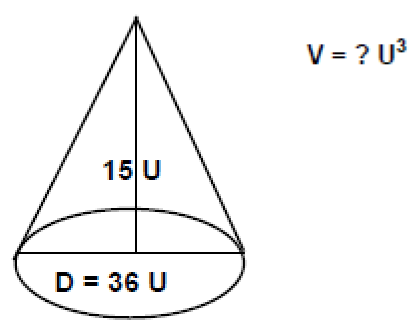








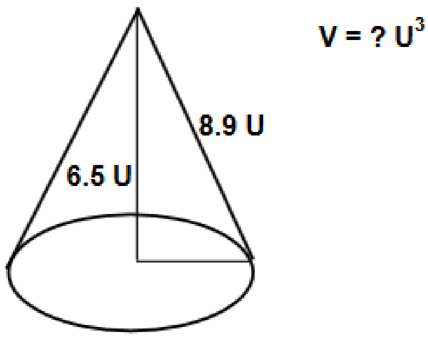


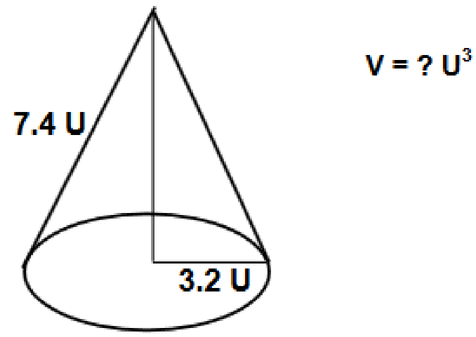


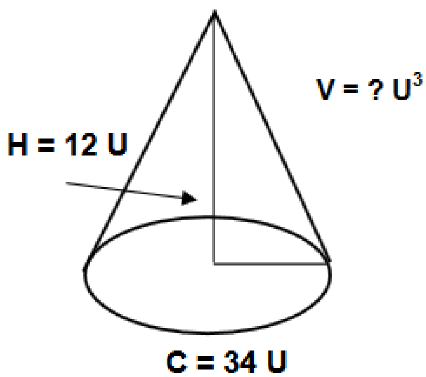
G16EA

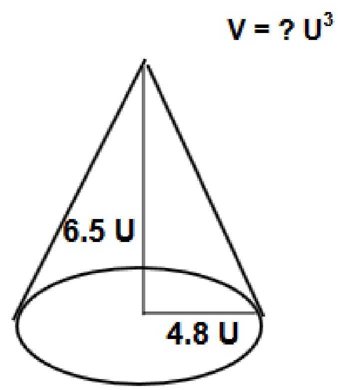
VOLUMES CONES

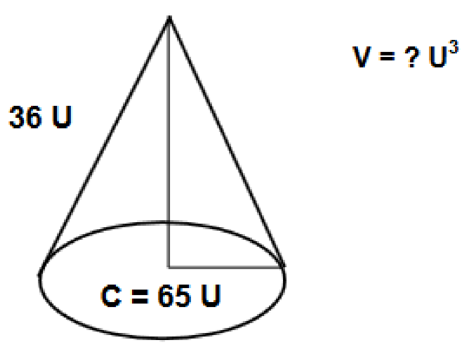
Find the **Volume**, in U^3

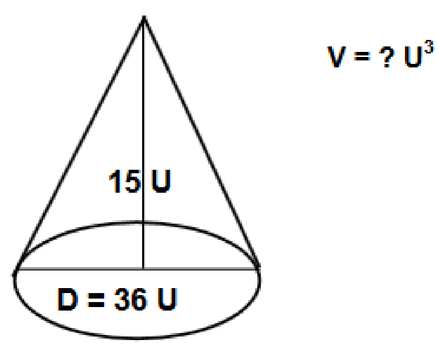








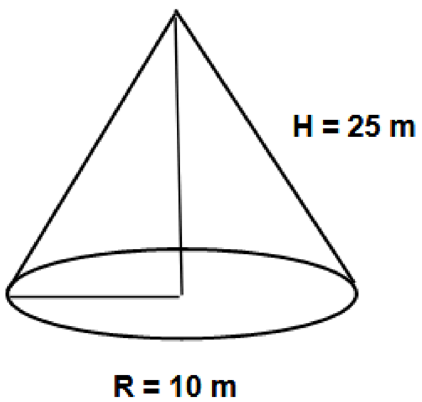


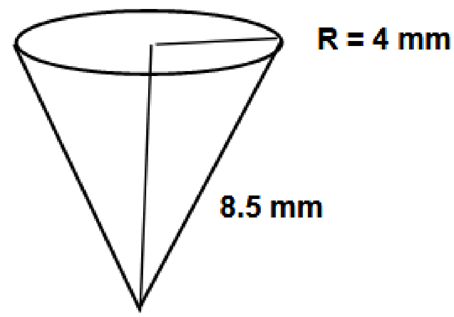


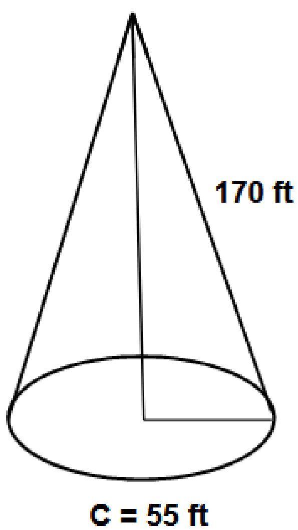
G16ES

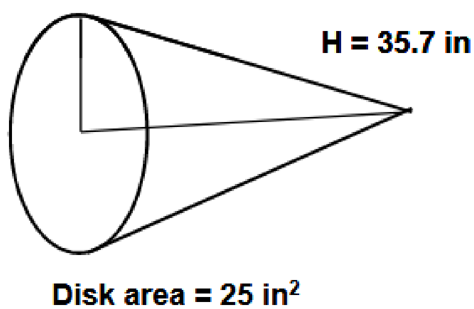
VOLUMES CONES

Find the volume.





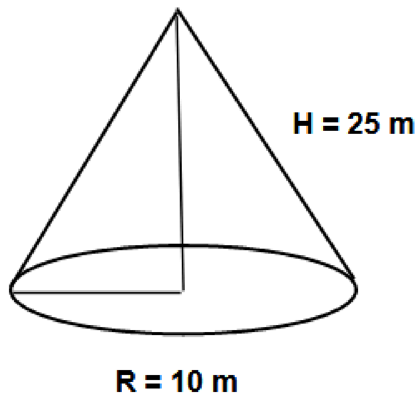




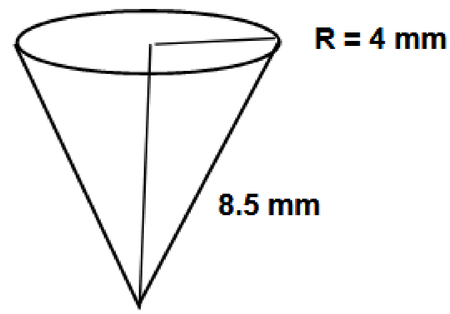
G16ESA

VOLUMES CONES

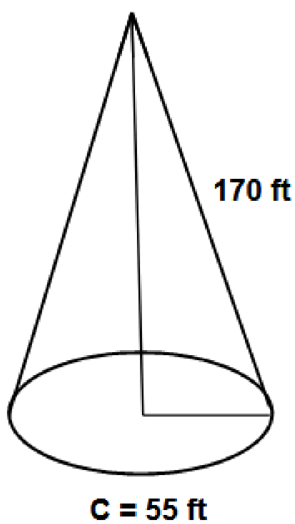
Find the volume.



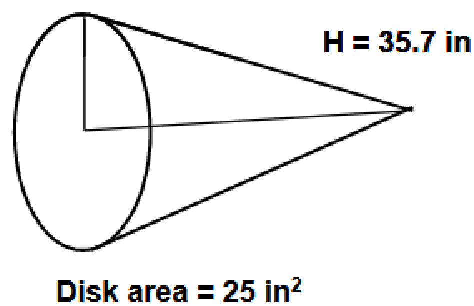
V = 2618 m^3



V = 125.7 mm^3



V = 13,623 ft^3



V = 297.5 in^3

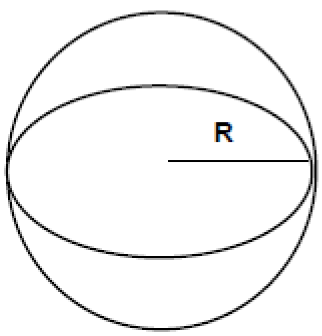
G17 LESSON: SURFACE AREA BALL OR SPHERE

The **Surface Area** of a **Sphere** with **Radius**, **R**, in **Linear Units, U**, is:

$$A = 4\pi R^2 \text{ Square Units, } U^2$$

This is very difficult to prove and we will defer that proof until Tier 4. However, I remember it as follows:

The **Area** of the **circle** of the **cross section** of the **Sphere** through its center is πR^2. I imagine it is rubber and we blow it up like a domed tent. Then its **Area** doubles and that is a **hemisphere** of **Surface Area** $2\pi R^2$. So, the whole **Sphere** is double this, or $4\pi R^2$ U^2.



$A = 4\pi R^2$

Problems

R = 5.2 ft	A =	1
R = 150 mi	A =	2
R = .035 cm	A =	3
R = 1 3/4 ft	A =	4

If the Surface Area of a Ball is to be 36 sq. in., what should its Radius be?

$4\pi R^2 = 36$ in^2, then $R = \sqrt{36/4\pi} = 1.7$ inches

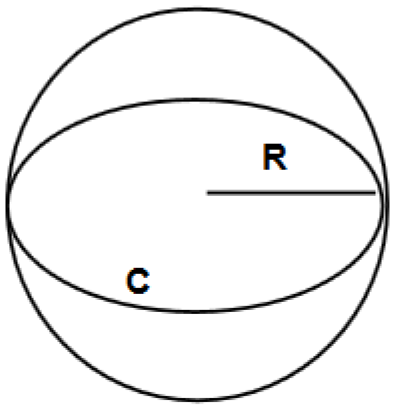
Answers: 1. 340 ft2 2. 282,750 mi^2 3. .0154 cm^2 4. 38.5 ft^2

SURFACE AREA BALL OR SPHERE

Find the **Surface Area** of the **Spheres** or **Balls**.

What is the formula for the **Surface Area** of a **Sphere** with **Radius** R?

How do you remember it?



R = Radius
D = Diameter
C = Circumference

Exercises

R = 5.2 ft	**A = ?**
R = 150 mi	**A = ?**
R = .035 cm	**A = ?**
R = 1 3/4 ft	**A = ?**
C = 36 ft	**A = ?**
C = 120 mi	**A = ?**
C = 45/8 in	**A = ?**
D = .025 cm	**A = ?**
D = 68 in	**A = ?**

If the Surface Area of a Ball is to be 36 sq. in., what should its Radius be?

G17EA

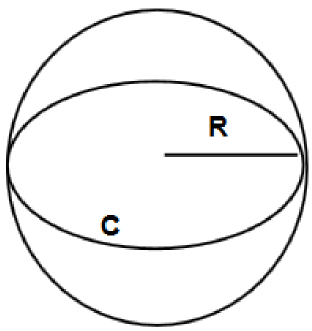
SURFACE AREA   BALL OR SPHERE   Answers: []

Find the **Surface Area** of the **Spheres** or **Balls**.

What is the formula for the Surface Area of a Sphere with Radius R? [**$4\pi R^2$**]

What's one way you can remember it? [**The Cross Section of the Ball is a circle of Radius R and Area πR^2.**]

Now, imagine blowing this up like it's rubber until each point is R from the center. [**Turns out the surface area is exactly …thus, Hemisphere area is $2\pi R^2$**]



R = Radius
D = Diameter
C = Circumference

Exercises

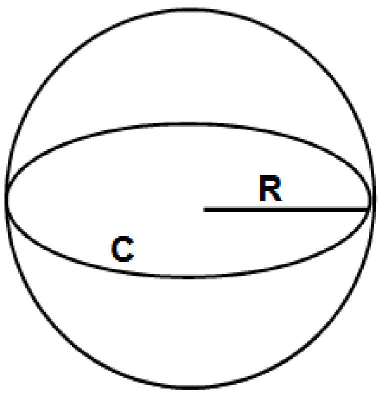
R = 5.2 ft	**A = 340 ft^2**
R = 150 mi	**A = 282,743 mi^2**
R = .035 cm	**A = .015 cm^2**
R = 1 3/4 ft	**A = 38.5 ft^2**
C = 36 ft	**A = 412.5 ft^2**
C = 120 mi	**A = 4,584 mi^2**
C = 45/8 in	**A = 6.8 in^2**
D = .025 cm	**A =002 cm^2**
D = 68 in	**A = 14,527 in^2**

If the Surface Area of a Ball is to be 36 sq. in., what should its Radius be? **1.7 in**

SURFACE AREA BALL OR SPHERE

Find the surface area for the spheres with the dimensions below.

1.) Recall the formula for surface area for a sphere, $SA = 4\pi R^2$. If the radious doubled, how much would the SA change? What about if the radius was halved?



2.) R = 35 cm

3.) R = 389 mi

4.) D = 12.6 mm

5.) C = 200,209 km

6.) C = 4π ft

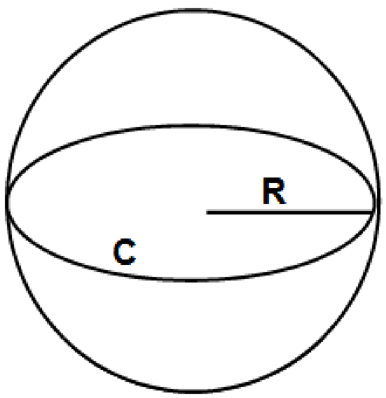
G17ESA

SURFACE AREA   BALL OR SPHERE

Find the surface area for the spheres with the dimensions below.

1.) Recall the formula for surface area for a sphere, $SA = 4\pi R^2$. If the radious doubled, how much would the SA change? What about if the radius was halved?

Answer: Because the radius is squared, doubling it would cause a 4x increase in surface area. Conversely, halving the radius would result in 4x less surface area.



2.) R = 35 cm
$SA = 15{,}394\ cm^2$

3.) R = 389 mi
$SA = 1{,}901{,}556\ mi^2$

4.) D = 12.6 mm
$SA = 498.8\ mm^2$

5.) C = 200,209 km
$SA = 12{,}759{,}020{,}060\ km^2$

6.) $C = 4\pi$ ft
$SA = 50.3\ ft^2$

G18 LESSON: VOLUME BALL OR SPHERE ARCHIMEDE TOMBSTONE

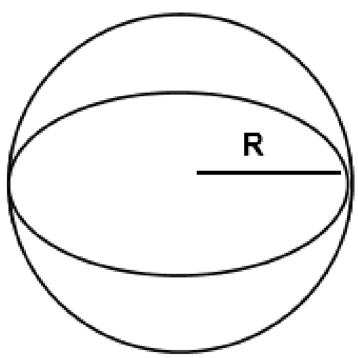
The **Volume** of a **Sphere** with **Radius**, **R**, in **linear units U**, is:

$$V = (4/3)\ \pi R^3 \text{ Cubic Units, } U^3$$

This is very difficult to prove and we will defer that proof until Tier 4. However, I remember it as follows:

> **Archimedes Tombstone:** Imagine a **Sphere** inscribed inside a **Cylinder**. The **Ratio** of the **Volume** or the **Sphere** to the **Volume** of the **Cylinder** is 2:3
>
> The **Cylinder** will have **Base Radius R** and Height **2R**.
> Thus, its **Volume** will be $\pi R^2 x 2R = 2\pi R^3$
> The **Volume** of the **Sphere** is thus, $(2/3)x2\pi R^3 = (4/3)\ \pi R^3$

Note: I say "**triangle**" three times instead of "**tombstone.**"



$A = 4\pi R^2$

$V = (4/3)\pi R^3$

Problems

R = 5.2 ft	**V =**	**1**
R = 150 mi	**V =**	**2**
R = .035 cm	**V =**	**3**
R = 1 3/4 ft	**V =**	**4**

If the Volume of a Ball is to be 36 cu. in., what should its Radius be?

$(4/3)\pi R3 = 36$ in3, then $R = \sqrt[3]{36x3/4\pi} = 2.05$ inches

Answers **1. 589 ft^3** **2. 14,137,000 mi^3** **3. .00018 cm^3** **4. 22.4 ft^3**

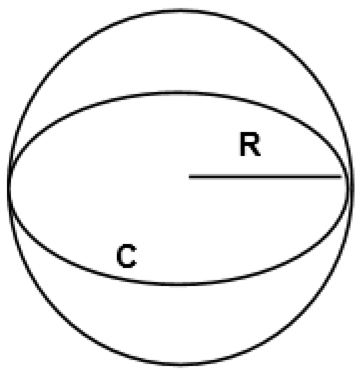
G18E

VOLUME BALL OR SPHERE

Find the **Volume** of the **Spheres** or **Balls**.

What is the formula for the **Volume** of a **Sphere** with **Radius R**?

What's one way you can remember it?



R = Radius
D = Diameter
C = Circumference

Exercises

R = 5.2 ft V = ?

R = 150 mi V = ?

R = .035 cm V = ?

R = 1 3/4 ft V = ?

C = 36 ft V = ?

C = 120 mi V = ?

C = 4 5/8 in V = ?

D = .025 cm V = ?

D = 68 in V = ?

If the Volume of a Ball is to be 100 cu. in., what should its Radius be?

G18EA

VOLUME BALL OR SPHERE Answers: []

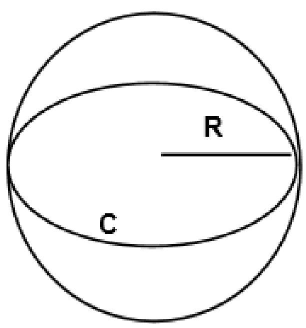
Find the **Volume** of the **Spheres** or **Balls**.

What is the formula for the **Volume** of a **Sphere** with **Radius R**?

[$(4/3)\,\pi R^3$]

What's one way you can remember it?

[Archimedes Tombstone formula whereby the Volume of the Sphere is 2/3 the Volume of a Cylinder the Sphere is inscribed in $(2/3) x \pi R^2 x 2R = (4/3)\,\pi R^3$]



R = Radius
D = Diameter
C = Circumference

Exercises

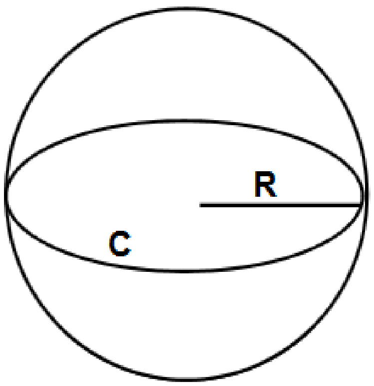
R = 5.2 ft	V = 589 ft^3
R = 150 mi	V = 14,137,167 mi^3
R = .035 cm	V = .00018 cm^3
R = 1 3/4 ft	V = 22.4 ft^3
C = 36 ft	V = 788 ft^3
C = 120 mi	V = 29,181 mi^2
C = 4 5/8 in	V = 1.67 in^3
D = .025 cm	V = .0000082 cm^3
D = 68 in	V = 164,636 in^3

If the Volume of a Ball is to be 100 cu. in., what should its Radius be? 2.88 in

G18ES

VOLUME BALL OR SPHERE

1.) Recall the formula for volume of a sphere, $V = (4/3)\pi R^3$. If the radious doubled, how much would the V change? What about if the radius was halved?



2.) R = 17 in

3.) R = 2.5 mm

4.) D = 25000 mi

5.) C = 40 km

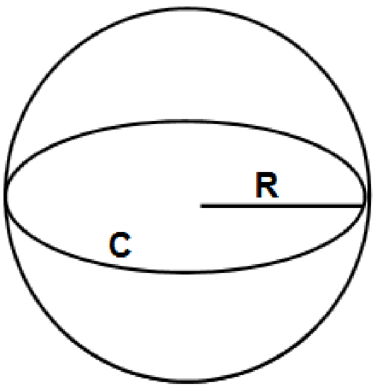
6.) C = 2π

G18ESA

VOLUME   BALL OR SPHERE

1.) Recall the formula for volume of a sphere, $V = (4/3)\pi R^3$. If the radious doubled, how much would the V change? What about if the radius was halved?

Answer: Because the radius is cubed, increasing it by a factor of 2 would increase the volume by a factor of 8. Conversely, halving the radius would reduce the volume by a factor of 8.



2.) R = 17 in
$V = 20{,}579.5\ in^3$

3.) R = 2.5 mm
$V = 65.4\ mm^3$

4.) D = 300 mi
$V = 113{,}097{,}336\ mi^3$

5.) C = 40 km
$V = 1{,}039{,}030\ km^3$

6.) $C = 2\pi$ U
$V = (4/3)\pi U^3 = 4.19 U^3$

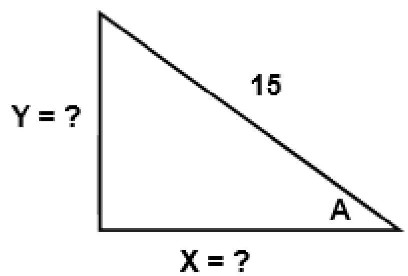
G19 LESSON: WHEN GEOMETRY IS NOT ENOUGH FOR TRIANGLES

We have learned to solve many practical problems using a combination of geometry and algebra. **Triangles** are the most common geometric figure we use in our models.

Yet, there are many practical problems involving **triangles** we still cannot solve with our current knowledge. This Lesson will point out some of these.

That's the "bad news." The "good news" is that we will be able to solve all of these problems using the tools we will learn in the last Section of the Foundation, Trigonometry.

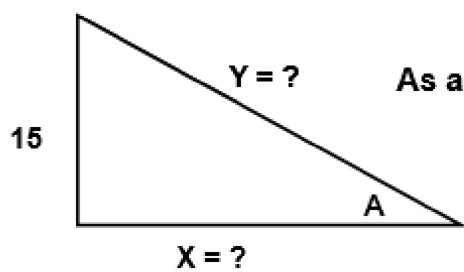
NOTE: See if you can catch the three times I use the word triangle instead of tombstone.



Problem: Find X and Y

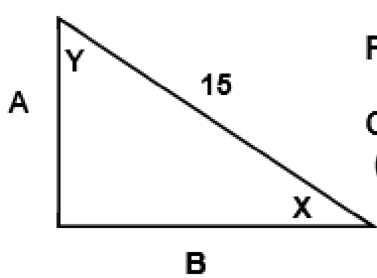
If A = 30^o or 45^o or 60^o we can solve this

With the tool of Trig, we can solve for any angle A.



As above, we can solve for A = 30^o or 45^o or 60^o

Trig will solve for all other angle A's

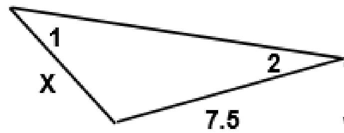


Find Angles X and Y given values of A or B.

Can Solve if A or B equals15 times (1/2) or ($\sqrt{2.2}$) of ($\sqrt{3}/2$), not otherwise, so far,

Trig solves for any A or B

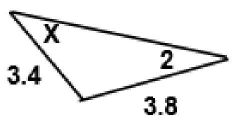
G19 When Geometry is Not Enough Problems



Find X   No Similar Triangle available

Currently can only do for special values of 1 and 2

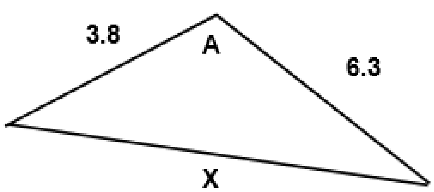
With Trig Tools can do for any angles 1 and 2



Can find X IF we have a similar triangle
with known corresponding sides

With Trig Tools Do Not need the similar triangle

Law of Sines



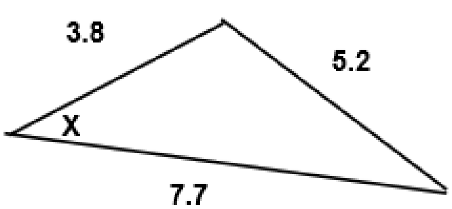
Find X

If A = 90°, OK with Pythagorean Theorem

Trig Tool for any angle A.

Generalized Pythogorean Theorem

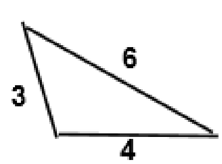
Law of Cosines



Find Angle X

Same Trig Tool as above

Useful in finding area of this triangle



Find the Area of this Triangle

Trigonometry has many profound applications beyond practical math.

G19E

WHEN GEOMETRY IS NOT ENOUGH FOR TRIANGLES

Give four examples of **triangle** "problems" we cannot yet solve with just the geometry and algebra we have learned, but which we will be able to solve with Trig.

This is an Optional Exercise.

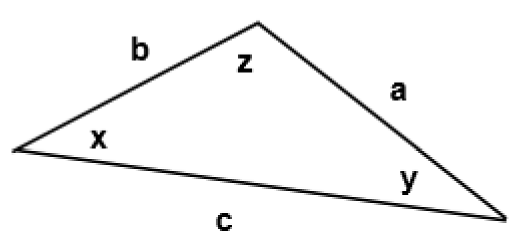
It is designed to help you appreciate the value the powerful Tool of Trigonometry will be for practical problem solving.

Before the scientific calculator was invented, Trig was pretty difficult to learn and apply to practical math.

Now, it is breeze. Aren't Power Tools wonderful?

HINT: Just imagine you know three of the variables below. Then can you find the others? With what you know now?

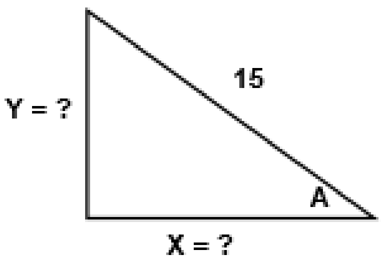
In many cases the answer will be NO. But, with Trig you will be able to solve any solvable triangle problem!



G19EA

WHEN GEOMETRY IS NOT ENOUGH FOR TRIANGLES

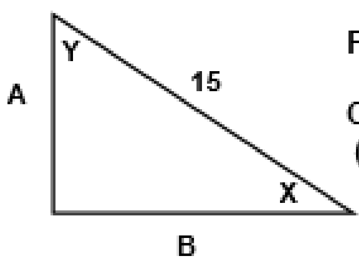
Give four examples of **triangle** "problems" we cannot yet solve with just geometry and algebra we have learned; but, which we will be able to solve with Trig.



Problem: Find X and Y

If A = 30^{o} or 45^{o} or 60^{o} we can solve this now

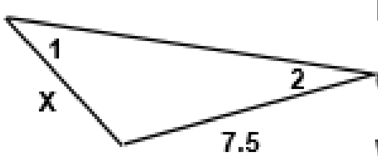
With the tool of Trig, we can solve for any angle A.



Find Angles X and Y given values of A or B.

Can Solve if A or B equals15 times
(1/2) or (√2.2) of (√3/2), not otherwise, so far.

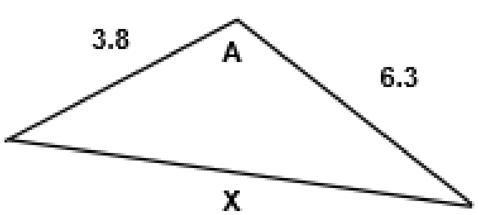
Trig solves for any A or B



Find X No Similar Triangle available

Currently can only do for special values of 1 and 2

With Trig Tools can do for any angles 1 and 2
Law of Sines



Find X or A given the other.

If A = 90^{o}, OK with Pythagorean Theorem

Trig Tool for any angle A.

Generalized Pythogorean Theorem
Law of Cosines

INTRODUCTION TO TRIGONOMETRY

Trigonometry, **Trig**, is the study of triangles.

Trig consists of several powerful tools which will empower you to solve virtually any solvable problem with triangles including the ones discussed in Lesson G19.

It begins with the basic Trig Functions, **SIN**, **COS**, and **TAN**.

These are the "**power tools**" that let us solve problems.

In the old days, there were extensive Trig Tables that were used. It was arduous to learn and apply these tables.

Today, with the power tool of the TI-30Xa, we can solve virtually any triangle problem in a matter of minutes or less.

Actually, in some ways Trig is easier than geometry.

We will learn how to use the three Trig Functions, and also, we will learn two very powerful theorems which make these tools even more valuable:

The **Law of Sines** (Lesson T6)

The Generalized **Pythagorean Theorem** commonly called: **The Law of Cosines** (Lesson T7)

Trigonometry then has many extensions into analytical geometry, complex numbers, calculus, and functional analysis which have profound effects in science, engineering and technology.

T1 LESSON: TRIG FUNCTIONS SIN COS TAN

In any **Right Triangle**, there are **Six Ratios** of side lengths. They come in sets of three where one set is just the reciprocal of the other set.

See the triangle below: a/c, b/c, and a/b are one set.

c is called the **Hypotenuse**, or **Hyp**.

b is called the **Adjacent side** (to angle 1), or **Adj**

a is called the **Opposite side** (to angle 1, or **Opp**

So, the Ratios are **Opp**/**Hyp**, **Adj**/**Hyp**, **Opp**/**Adj**

These three ratios are the three **Trig functions of angle 1**.

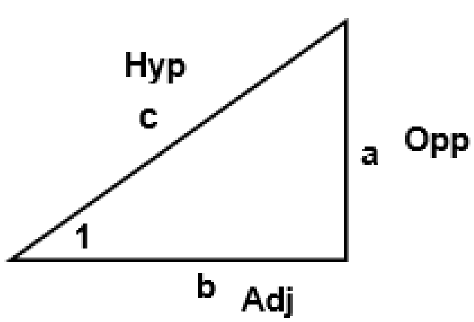
SIN(1) = Opp/Hyp

COS(1) = Adj/Hyp

TAN = Opp/Adj = **SIN**(1)/**COS**(1)

Angle 1 will always be measured in **degrees º** in this Foundation Course.

In advanced applications of Trig, angle 1 is measured in Radians, RAD.



SIN (1) = a/c = Opp/Hyp

COS (1) = b/c = Adj/Hyp

TAN (1) = a/b = Opp/Adj

When turn on the TI-30Xa, **DEG** always comes up.

SIN^{-1} COS^{-1} TAN^{-1}

Enter any number between -1 and 1, and find the angle whose **SIN** it is.

Ditto for **COS** and **TAN**. In other words, if

SIN (1) = a, then (1) = **SIN**$^{-1}$(a)

WARNING: See **T5** for some special information about **SIN**$^{-1}$

T1 Trig Functions SIN COS TAN Problems

Find **SIN**(1), **COS**(1), **TAN** (1) given **angle** (1) in **degrees** °

Find **Angle** (1), Given **SIN**(1), **COS**(1) using **SIN**$^{-1}$ and **COS**$^{-1}$

Problems: in °	Angle 1	SIN(1)	COS(1)	TAN(1)
30°	0.5	0.866	0.577	
45°	0.707	0.707	1	
60°	0.866	0.5	1.732	
17°	0.292	0.956	0.306	
38°	0.616	0.788	0.781	
52.7°	0.795	0.606	1.313	
68°	0.927	0.375	2.48	
85°	0.996	0.087	11	
90°	1	0	Error	
100°	0.985	-0.174	-5.68	
115°	0.906	-0.423	-2.15	
135°	0.707	-0.707	-1	
145°	0.574	-0.819	-0.7	
150°	0.5	-0.866	-0.577	
176°	0.07	-0.998	-0.07	

Problems: Find **angle** 1 if

	Angle (1)
SIN(1) = 0.7865	51.9°
SIN(1) = 0.5	30°
SIN(1) = -0.654	-40.8°
COS(1) = 0.7865	38.1°
COS(1) = 0.5	60°
COS(1) = -0.654	130.8°
TAN(1) = 0.7865	38.2°
TAN(1) = 0.5	26.6°
TAN(1) = -0.654	-33.2

Note: **SIN**$^{-1}$(**SIN**(120°) = 60°

COS^{-1}(**COS**(120°) = 120°

Note: These problems are repeated on the T1E page as well.

T1E

TRIG FUNCTIONS SIN COS TAN

Find **SIN**(1), **COS**(1), **TAN**(1) given **angle** (1) in degrees º

Find **Angle** (1), Given **SIN**(1), **COS**(1) using **SIN**$^{-1}$ and **COS**$^{-1}$

Angle 1	**SIN**(1)	**COS**(1)	**TAN**(1)	Evaluate
30º				**SIN**$^{-1}$[**COS**(30º)] = ?
45º				**COS**$^{-1}$[**COS**(30º)] = ?
60º				**SIN**$^{-1}$[**COS**(120º)] = ?
17º				**COS**$^{-1}$[**SIN**(120º)] = ?
38º				**COS**$^{-1}$[**SIN**(60º)] = ?
52.7º				**COS**$^{-1}$[**SIN**(45º)] = ?
68º				**TAN**$^{-1}$[**SIN**(90º)] = ?
85º				**SIN**[**COS**$^{-1}$(.5)] = ?
90º				**SIN**[**COS**$^{-1}$(.867)] = ?
100º				**COS**[**SIN**$^{-1}$(.867)] = ?
115º				**SIN**[**COS**$^{-1}$(1)] = ?
135º				**SIN**[**COS**$^{-1}$(0)] = ?
145º				**SIN**[**COS**$^{-1}$.707)] = ?
150º				**TAN**[**SIN**$^{-1}$(.707)] = ?
176º				**TAN**[**COS**$^{-1}$(.707)] = ?

Find **angle** (1) if

Angle (1)

SIN(1) = 0.7865

SIN(1) = 0.5

SIN(1) = -0.654

COS(1) = 0.7865

COS(1) = 0.5

COS(1) = -0.654

TAN(1) = 0.7865

TAN(1) = 0.5

TAN(1) = -0.654

T1EA

TRIG FUNCTIONS SIN COS TAN

Find **SIN**(1), **COS**(1), **TAN**(1) given angle (1) in **degrees** ^{o}

Find **Angle** (1), Given **SIN**(1), **COS**(1) using SIN^{-1} and COS^{-1}

Angle 1	SIN(1)	COS(1)	TAN(1)	Evaluate	
30º	0.5	0.866	0.577	SIN^{-1}[COS(30º)] = ?	60º
45º	0.707	0.707	1	COS^{-1}[COS(30º)]= ?	30º
60º	0.866	0.5	1.732	SIN^{-1}[COS(120º)] = ?	-30º
17º	0.292	0.956	0.306	COS^{-1}[SIN(120º)] = ?	30º
38º	0.616	0.788	0.781	COS^{-1}[SIN(60º)] = ?	30º
52.7º	0.795	0.606	1.313	COS^{-1}[SIN(45º)] = ?	45º
68º	0.927	0.375	2.475	TAN^{-1}[SIN(90º)] = ?	45º
85º	0.996	0.087	11.43	SIN[COS^{-1}(.5)] = ?	0.867
90º	1	0	Error	SIN[COS^{-1}(.867)] = ?	0.5
100º	0.985	-0.174	-5.68	COS[SIN^{-1}(.867)] = ?	0.5
115º	0.906	-0.423	-2.15	SIN[COS^{-1}(1)] = ?	0
135º	0.707	-0.707	-1	SIN[COS^{-1}(0)] = ?	1
145º	0.574	-0.819	-0.7	SIN[COS^{-1}.707)] = ?	0.707
150º	0.5	-0.866	-0.577	TAN[SIN^{-1}(.707)] = ?	1
176º	0.07	-0.998	-0.07	TAN[COS^{-1}(.707)] = ?	1

Find angle (1) if	Angle (1)
SIN(1) = 0.7865	51.9º
SIN(1) = 0.5	30º
SIN(1) = -.654	-40.8º
COS(1) = 0.7865	38.1º
COS(1) = 0.5	60º
COS(1) = -0.654	130.8º
TAN(1) = 0.7865	38.2º
TAN(1) = 0.5	26.6º
TAN(1) = -0.654	-33.2

T1ES

TRIG FUNCTIONS SIN COS TAN

1. $x = 30°$ Find sin(x) , cos(x), and tan(x)
2. $x = 60°$ Find sin(x), cos(x), and tan(x)
3. Find $\cos^{-1}(\sin(60°)) = ?$
4. If $\cos(x) = 0.5$ Find angle x
5. $x = 45°$ Find sin(x) cos(x) and tan(x)
6. $x = 15°$ Find sin(x), cos(x), and tan(x)
7. Find $\sin^{-1}(\cos(30°)) = ?$
8. If $\sin(x) = 0.315$ Find angle x
9. $x = 90°$ Find sin(x), cos(x), and tan(x)
10. $x = 150°$ Find sin(x), cos(x), and tan(x)
11. Find $\sin(\cos^{-1}(0.5)) = ?$
12. If $\tan(x) = 0.425$ Find angle x
13. $x = 117°$ Find sin(x), cos(x), and tan(x)
14. $x = 34.5°$ Find sin(x), cos(x), and tan(x)
15. Find $\sin^{-1}(\tan(17°)) = ?$
16. If $\sin(x) = -0.5$ Find angle x
17. $x = 100°$ Find sin(x), cos(x), and tan(x)
18. $x = 0°$ Find sin(x), cos(x), and tan(x)
19. Find $\tan^{-1}(\cos(70°)) = ?$
20. If $\tan(x) = -0.245$ Find angle x

T1ESA

TRIG FUNCTIONS SIN COS TAN Answers: [ ]

1. $x = 30°$ Find sin(x) , cos(x), and tan(x)

   [**sin(30°) = 0.5, cos(30°) = 0.866, tan(30°) = 0.577**]

2. $x = 60°$ Find sin(x), cos(x), and tan(x)

   [**sin(60°) = 0.866, cos(60°) = 0.5, tan(60°) = 1.732**]

3. Find $\cos^{-1}(\sin(60°)) = ?$

   [**30°**]

4. If cos(x) = 0.5 Find angle x

   [**x = 60°**]

5. $x = 45°$ Find sin(x) cos(x) and tan(x)

   [**sin(45°) = 0.707, cos(45°) = 0.707, tan(45°) = 1**]

6. $x = 15°$ Find sin(x), cos(x), and tan(x)

   [sin(15) = 0.259, cos(15) = 0.966, tan(15) = 0.268]

7. Find $\sin^{-1}(\cos(30°)) = ?$

   [**x = 60°**]

8. If sin(x) = 0.315 Find angle x

   [**x = 18.4°**]

9. $x = 90°$ Find sin(x), cos(x), and tan(x)

   [**sin(90°) = 1, cos(90°) = 0, tan(90°) = undefined**]

10. $x = 150°$ Find sin(x), cos(x), and tan(x)

    [**sin(150°) = 0.5, cos(150°) = -0.866, tan(150°) = -0.577**]

11. Find $\sin(\cos^{-1}(0.5)) = ?$

    [**0.866**]

12. If tan(x) = 0.425 Find angle x

[**x = 23.03°**]

13. x = 117° Find sin(x), cos(x), and tan(x)

[**sin(117°) = 0.891, cos(117°) = -0.454, tan(117°)= -1.96**]

14. x = 34.5° Find sin(x), cos(x), and tan(x)

[**sin(34.5°) = 0.566, cos(34.5°) = 0.824, tan(34.5°) = 0.687**]

15. Find $\sin^{-1}(\tan(17°))$ = ?

[**17.8°**]

16. If sin(x) = -0.5 Find angle x

[**x = 210°, 330° or -30°**]

17. x = 100° Find sin(x), cos(x), and tan(x)

[**sin(100°) = 0.985, cos(100°) = -0.174, tan(100°) = -5.67**]

18. x = 0° Find sin(x), cos(x), and tan(x)

[**sin(0°) = 0, cos(0°) = 1, tan(0°) = 0**]

19. Find $\tan^{-1}(\cos(70°))$ = ?

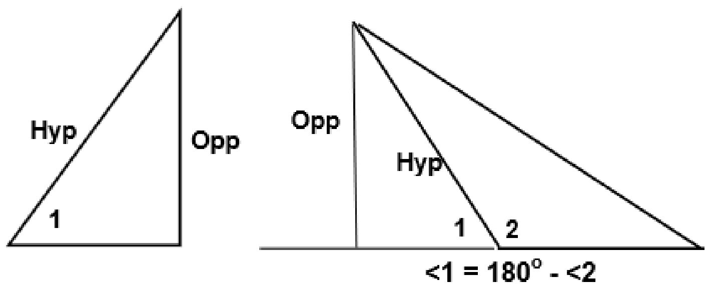
[**18.88°**]

20. If tan(x) = -0.245 Find angle x

[**x = 166.2°, 346.2°, or -13.8°**]

T2 LESSON: SIN X SINE OF X X IS AN ANGLE (DEGREES °)

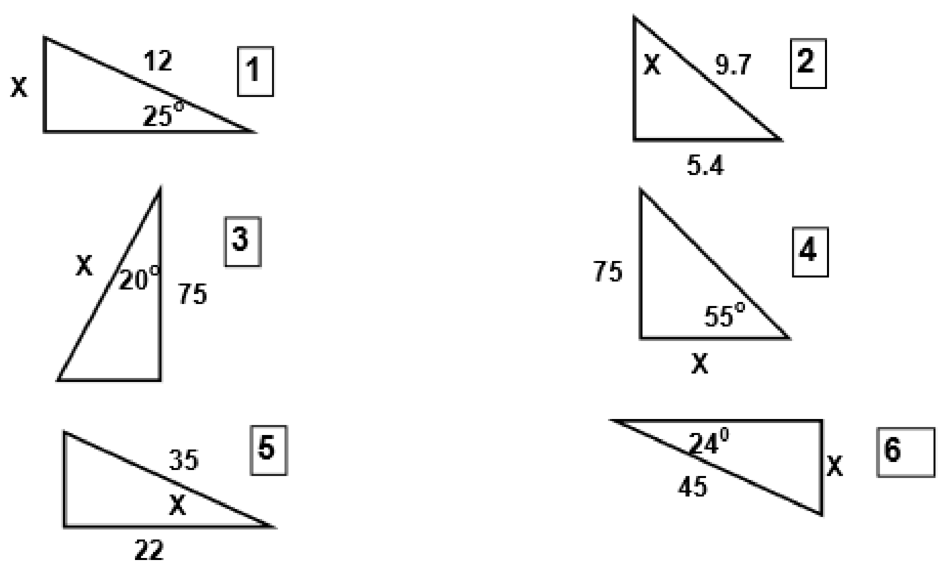
We will extend the definition of **SIN** to include all angles from 0° to 180°. In Tier 3 we will extend the definition to include all angles both positive and negative.



SIN(1) = **Opp/Hyp** **SIN**(2) = **Opp/Hyp** = **SIN**(180° - <2)

If know two out of three, find the third, **Opp**, **Hyp**, (1)

Opp = **SIN**(1)x**Hyp**	**Opp** = **SIN**(2)x**Hyp**
Hyp = **Opp/SIN**(1)	**Hyp** = **Opp/SIN**(2)
(1) = SIN^{-1}(**Opp/Hyp**)	(2) = SIN^{-1}(**Opp/Hyp**)



Answers:

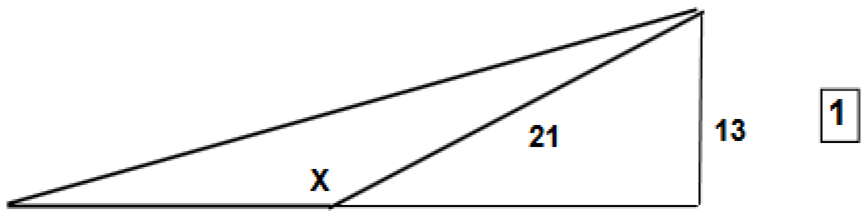
1. 5.1	4. 52.5
2. 33.8°	5. 51.1°
3. 79.8	6. 18.3

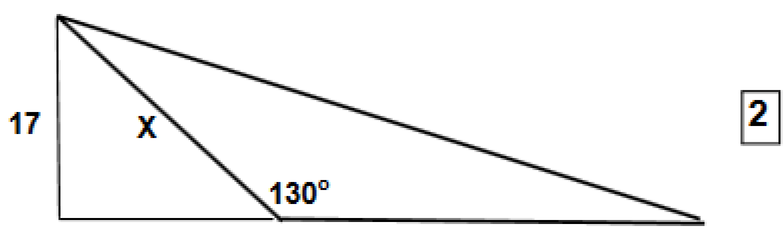
T2 SIN Problems

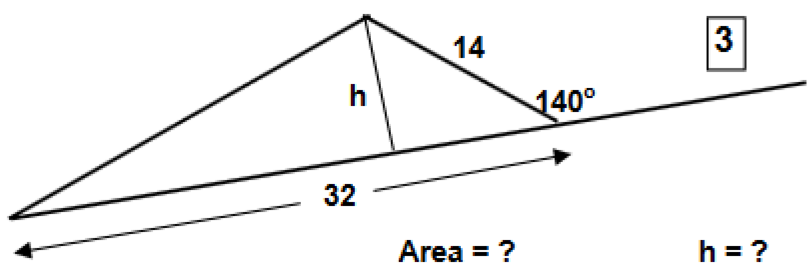
Always set up the Equation first. Then solve.

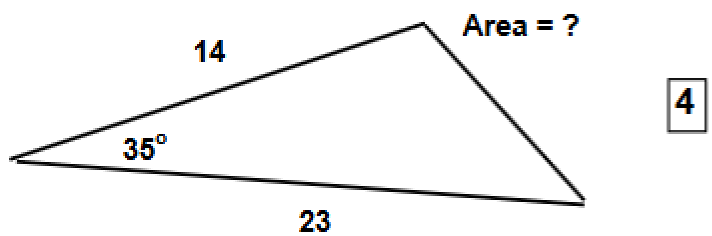
Use the **Pythagorean Theorem** if necessary.

NOTE: Why is **Area** = .5ab**SIN**(<ab) correct?









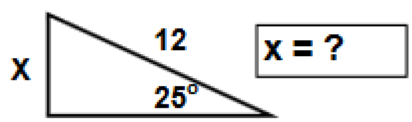
Answers:
1. 141.8°
2. 22.2
3. A =144, h = 9
4. 92.3

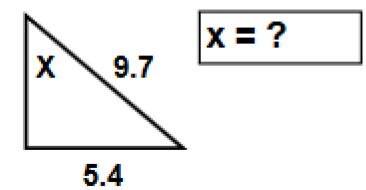
T2E

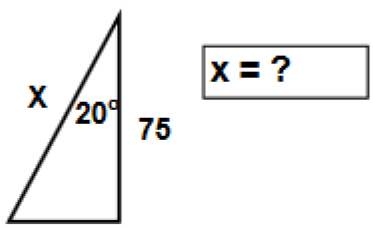
SIN X   SINE OF X

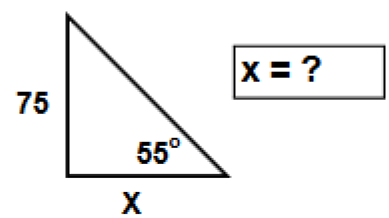
X is an **angle** (**degrees** °)

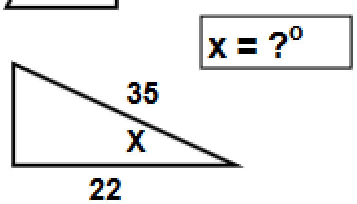
Find **x** in each of the following exercises.

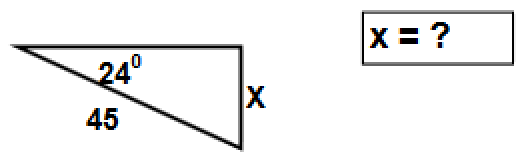


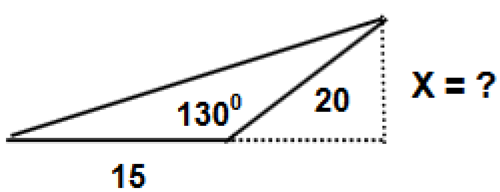






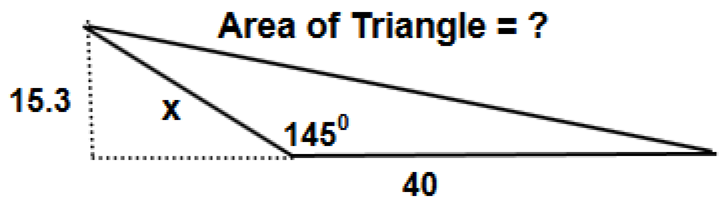






Area of Triangle = ?

Area of Triangle = ?



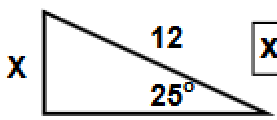
x = ?

T2EA

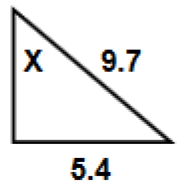
SIN X   SINE OF X

X is an **angle** (**degrees** °)

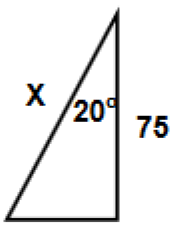
Find **x** in each of the following exercises



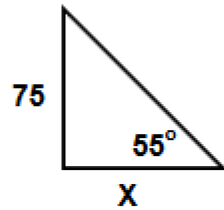
x = 5.1



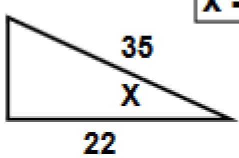
x = 33.8°



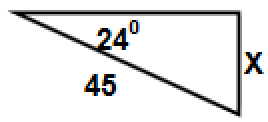
x = 79.8



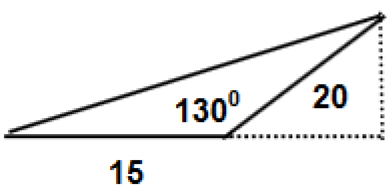
x = 52.5



x = 51°



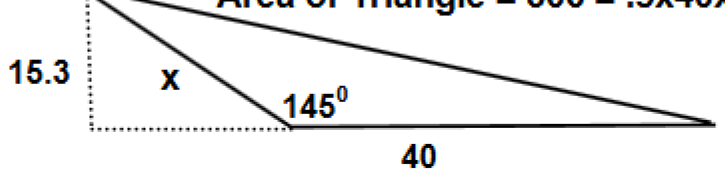
x = 18.3



X = 15.3

Area of Triangle = 115 = .5x15x20xSIN(130°) = .5x15x15.3

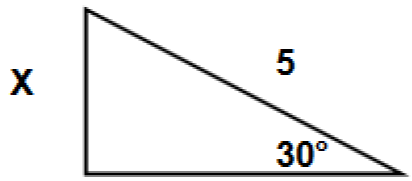
Area of Triangle = 306 = .5x40x15.3 = .5x40x26.7xSIN(145°)

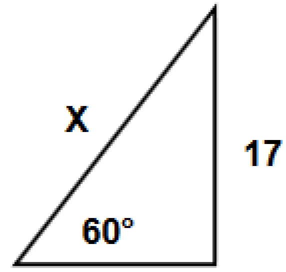


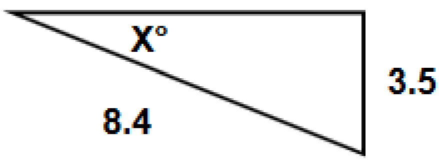
x = 26.7

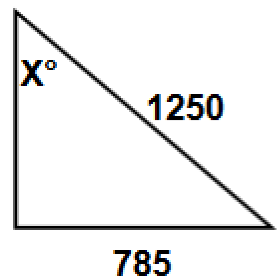
SIN X SINE OF X

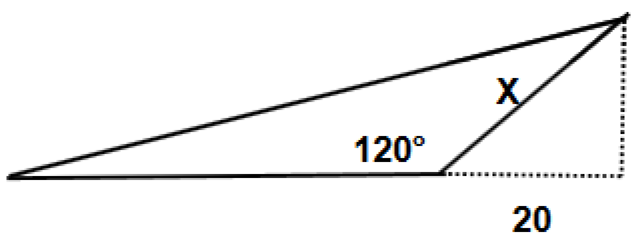
Find X in the following exercises.







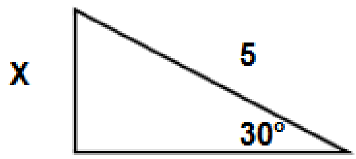




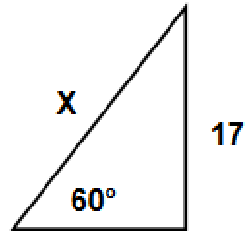
T2ESA

SIN X SINE OF X

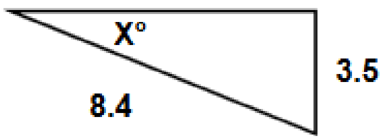
Find X in the following exercises.



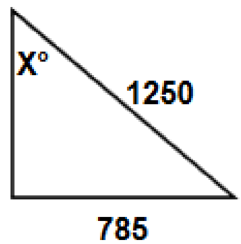
x = 2.5



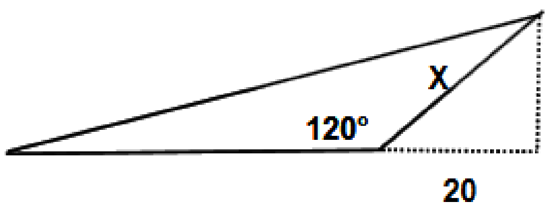
x = 19.6



x = 24.6°



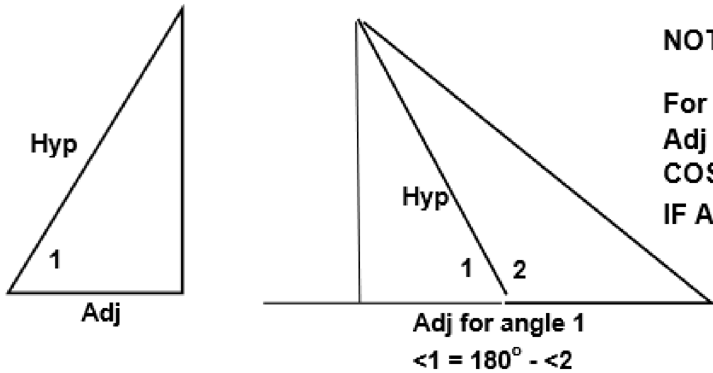
x = 38.9°



x = 40

T3 LESSON: COS X COSINE OF X. X IS AN ANGLE (DEGREES ^{o})

We will extend the definition of **COS** to include all angles from 0^{o} to 180^{o}. In Tier 3 we will extend the definition to include all angles both positive and negative.



NOTICE

For COS only
Adj is NEGATIVE
COS(2)< 0
IF Angle (2) >90°

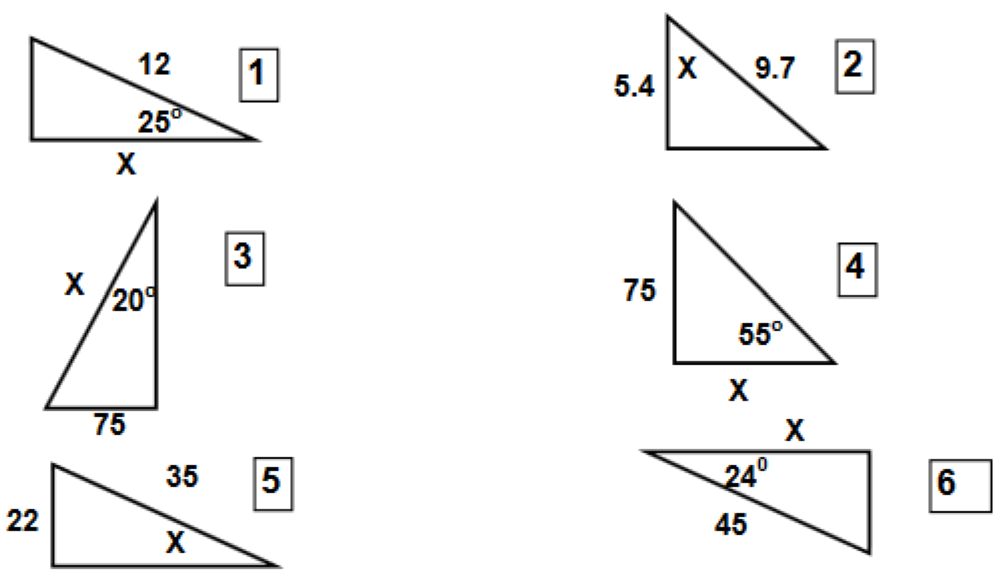
COS(1) = **Adj/Hyp** **COS**(2) = **Adj/Hyp** **COS**(2) = -**COS**(180^{o} - <1)

If you know two out of three, find the third, **Opp**, **Hyp**, (**1**)

Adj = **COS**(1)x**Hyp** **Adj** = **COS**(2)x**Hyp**

Hyp = **Adj/COS**(1) **Hyp** = **Adj/COS**(2)

(**1**) = **COS**$^{-1}$(**Adj/Hyp**) (2) = **COS**$^{-1}$(**Adj/Hyp**) **Adj** < 0



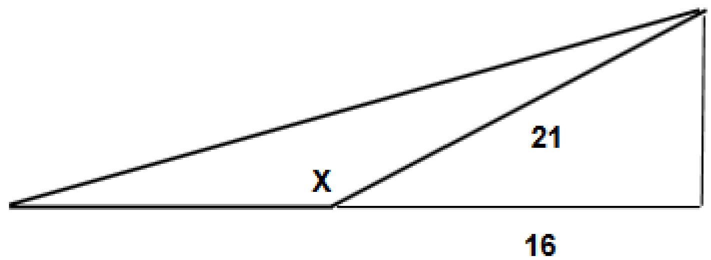
Answers: 1. 10.9 2. 56.2^{o} 3. 219
4. 52.5 5. 38.9^{o} 6. 41.1

T3 COS Problems

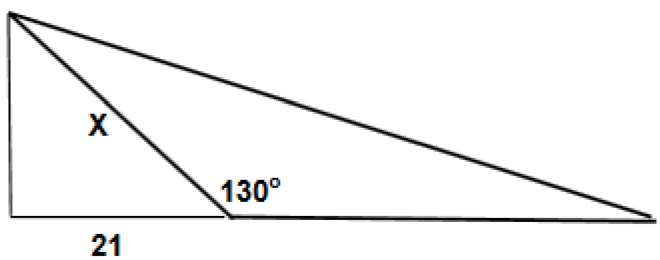
Always set up the Equation first. Then solve.

Use the **Pythagorean Theorem** if necessary.

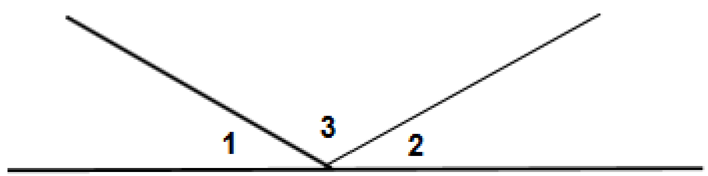
NOTE: **COS**(1) < 0 **IF** $180^\circ > (1) > 90^\circ$



1



2



<1 = <2 and <1 + <3 + <2 = 180°

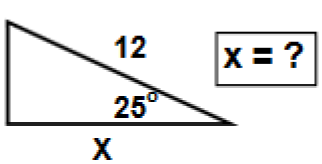
<1 = 35°	What does **COS**(<2 + <3) = ?	3
<3 = 120°	What are **COS**(1) and **COS**(2 + 3) ?	4
<2 = 30°	What are **SIN**(3) and **COS**(3) ?	5

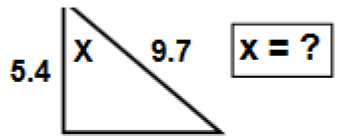
Answers: 1. 137° 2. 32.7 3. -0.819
4. 0.866, -0.866 5. 0.866, -0.5

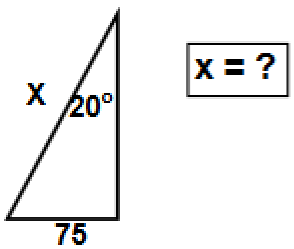
T3E

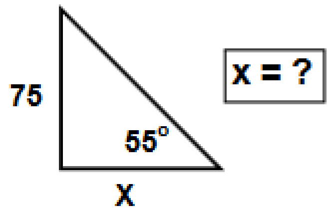
COS X COSINE OF X. X IS AN ANGLE (DEGREES °)

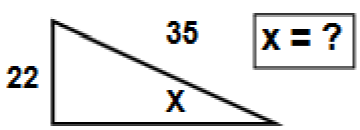
Find **x** in the following exercises.

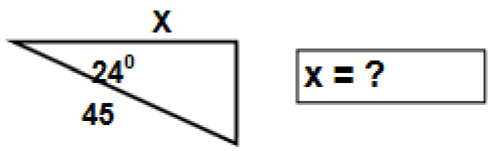


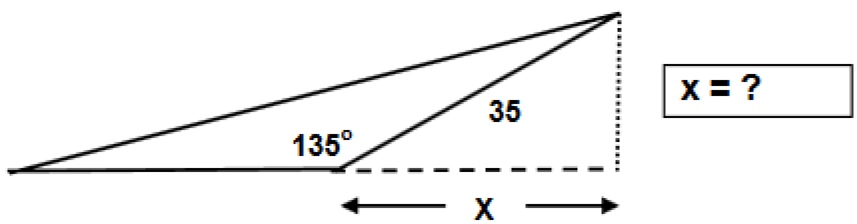








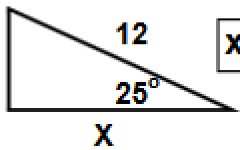




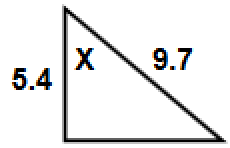
T3EA

COS X COSINE OF X. X IS AN ANGLE (DEGREES °)

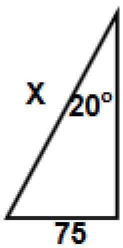
Find **x** in the following exercises.



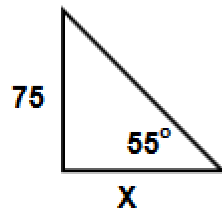
x = 10.9



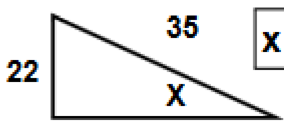
x = 56.2°



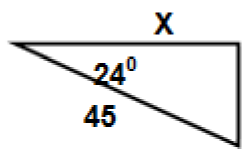
x = 219



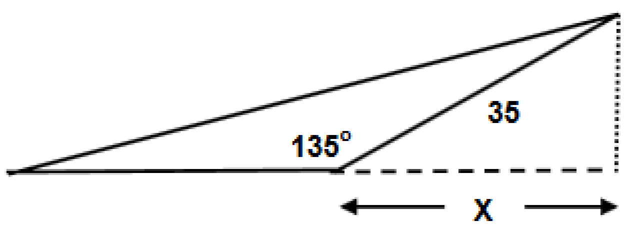
x = 52.5



x = 38.9°



x = 41.1

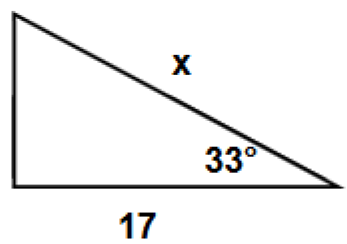


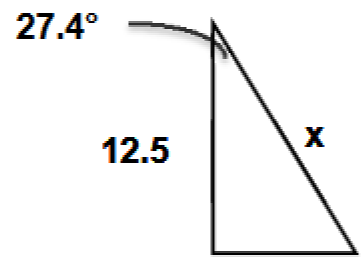
x = 24.7

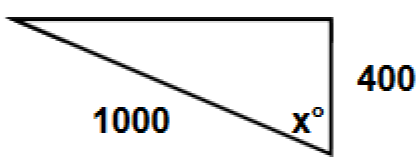
T3ES

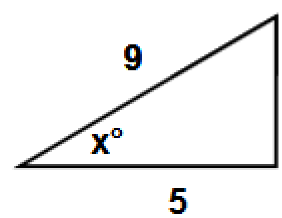
COS X COSINE OF X. X IS AN ANGLE (DEGREES °)

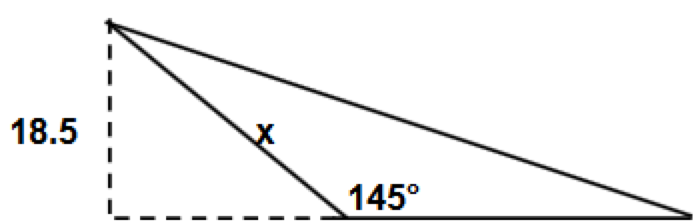
Find X in the following exercises.







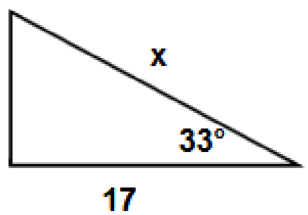




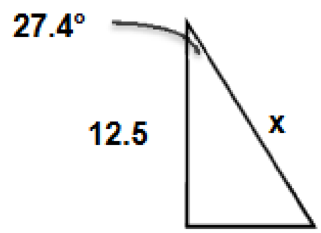
T3ESA

COS X COSINE OF X. X IS AN ANGLE (DEGREES °)

Find X in the following exercises.

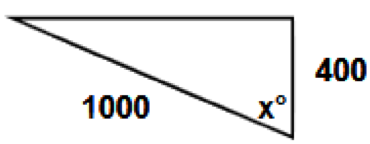


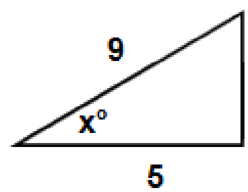
x = 20.3



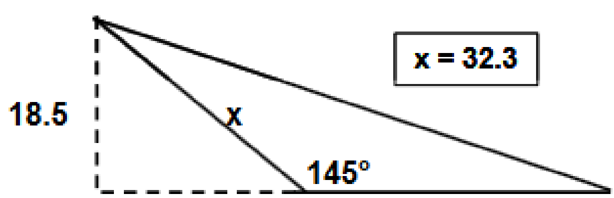
x = 14.1

x = 66.4°





x = 58.3°

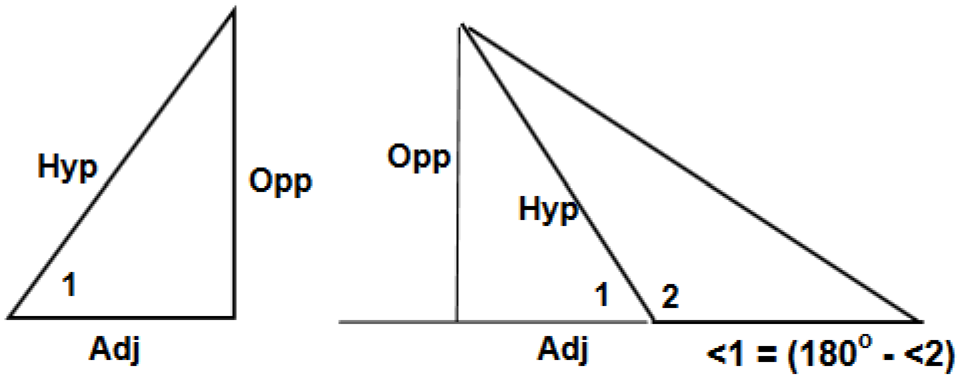


x = 32.3

T4 LESSON: TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{O})

TAN X can take on all values positive and negative

TAN X is **Not** defined at **X** = -90^{o} or 90^{o} (**Error**)

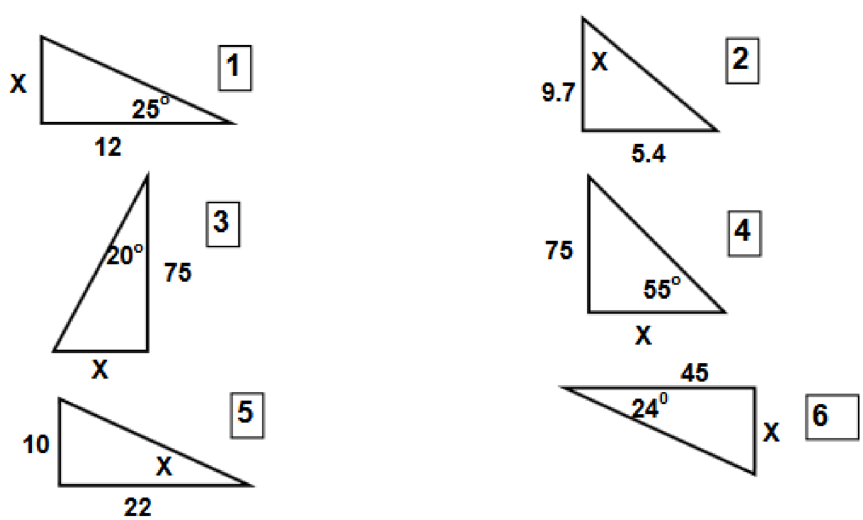


TAN(1) = **Opp/Adj** **TAN**(2) = **Opp/Adj** = -**TAN**(180^{o} - <2)

If know two out of three, find the third, **Opp**, **Adj**, (1)

Opp = **TAN**(1)x**Adj**	**Opp** = **TAN**(2)x**Adj**	**Adj** < 0
Adj = **Opp/TAN**(1)	**Adj** = **Opp/TAN**(2)	**Adj** < 0

(1) = TAN^{-1}(**Opp/Adj**) (2) = TAN^{-1}(**Opp/Adj**)



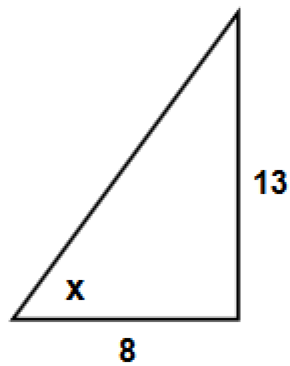
Answers:

1. 5.6	2. 29.1^{o}	3. 27.3
4. 52.5	5. 24.4^{o}	6. 20.0

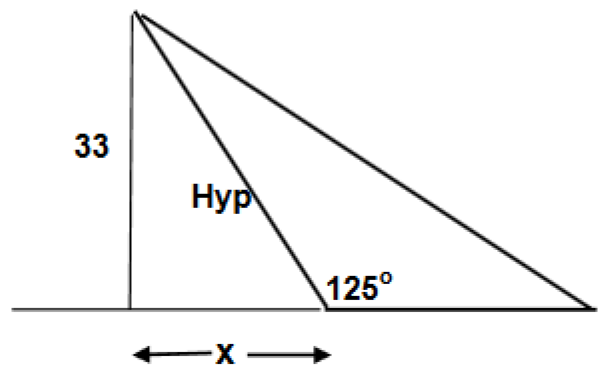
T4E

TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{O})

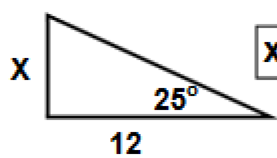
Find X in each of the following exercises.



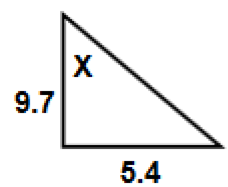
x = ?°



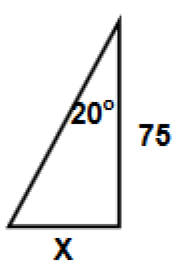
x = ?



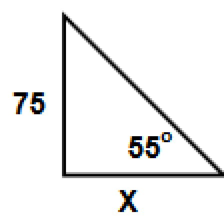
x = ?



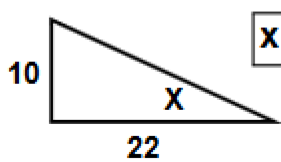
x = ?



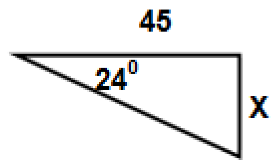
x = ?



x = ?



x = ?

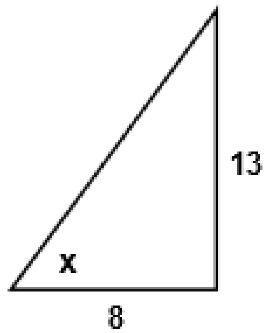


x = ?

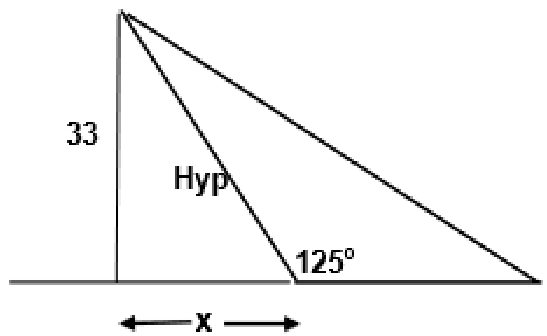
T4EA

TAN X TANGENT OF X. X IS AN ANGLE (DEGREES °)

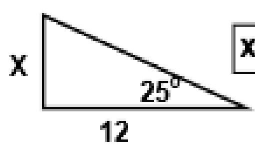
Find X in each of the following exercises.



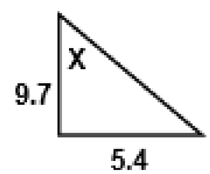
x = 58.4°



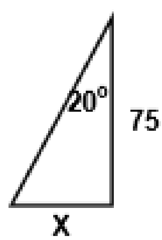
x = 23.1



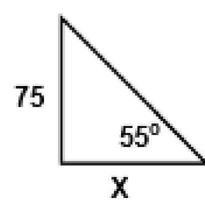
x = 5.6



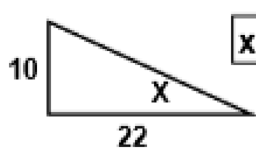
x = 29.1°



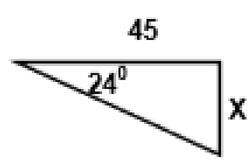
x = 27.3



x = 52.5



x = 24.4°

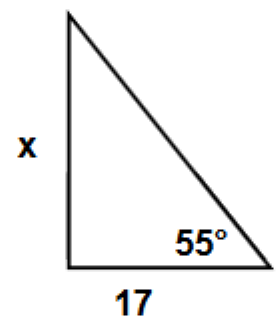


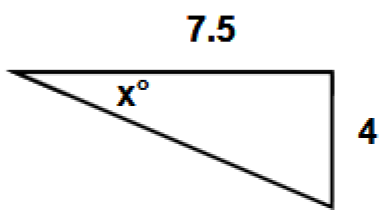
x = 20.0

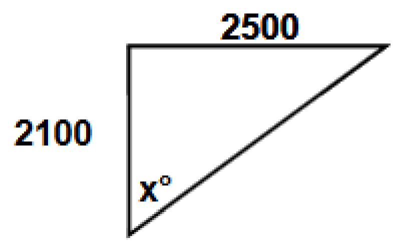
T4ES

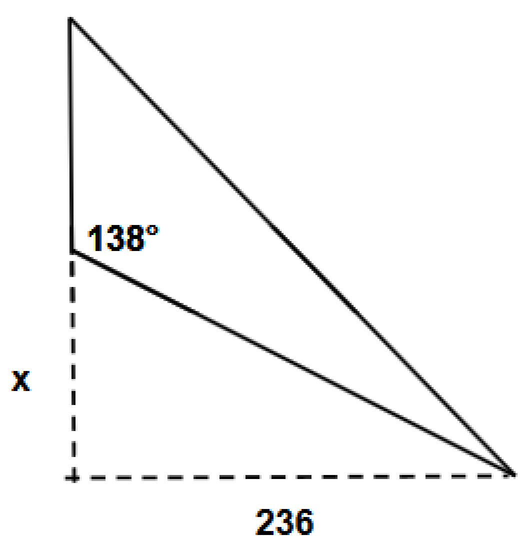
TAN X TANGENT OF X. X IS AN ANGLE (DEGREES °)

Find X in each of the following exercises.





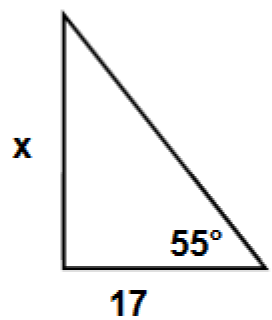




T4ESA

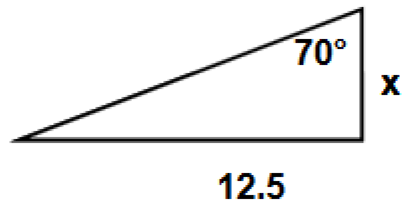
TAN X TANGENT OF X. X IS AN ANGLE (DEGREES °)

Find X in each of the following exercises.

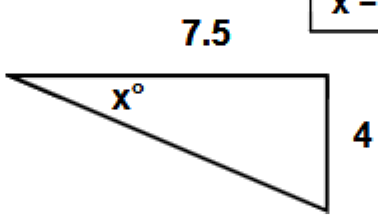


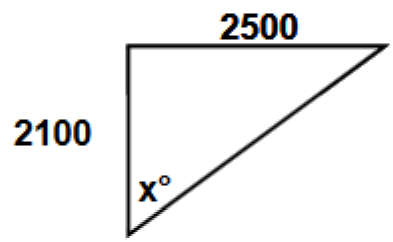
x = 24.3

x = 4.55

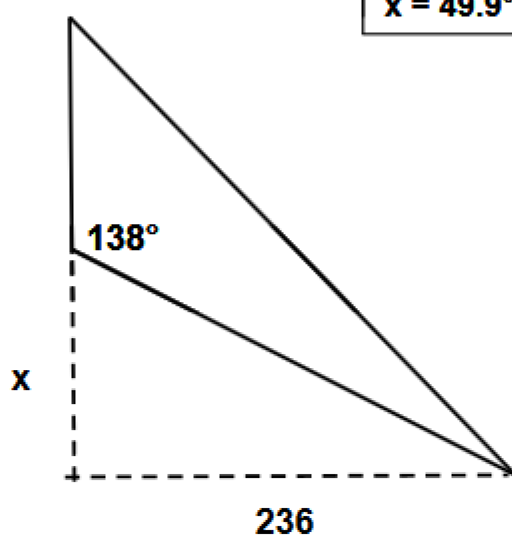


x = 28.1°





x = 49.9°



x = 262.1

T5 LESSON: WARNING ABOUT SIN^{-1}

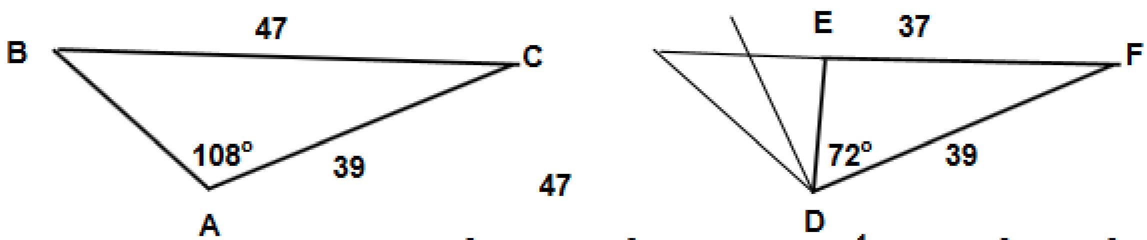
We are interested in **angles**, <A, from 0^o to 180^o

SIN(<A) = **SIN**(180^o - <A)   (see Table below)

So, if we have a **triangle** with an **angle** <A > 90^o, with **SIN**(<A), then its $\mathbf{SIN^{-1}}$ will be wrong.

See below for example:

Suppose we know **SIN**(<A) = .95105, yet $\mathbf{SIN^{-1}}$(.95105) = 72^o



$SIN(108^o) = SIN(72^o)$ $SIN^{-1}[SIN(108^o)] = 72^o$

Angle <A	SIN(<A)	SIN^{-1}	Angle <A	COS(<A)	COS^{-1}
0	0.000	0	0	1.000	0
10	0.174	10	10	0.985	10
20	0.342	20	20	0.940	20
30	0.500	30	30	0.866	30
40	0.643	40	40	0.766	40
50	0.766	50	50	0.643	50
60	0.866	60	60	0.500	60
70	0.940	70	70	0.342	70
80	0.985	80	80	0.174	80
90	1.000	90	90	0.000	90
100	0.985	80	100	-0.174	100
110	0.940	70	110	-0.342	110
120	0.866	60	120	-0.500	120
130	0.766	50	130	-0.643	130
140	0.643	40	140	-0.766	140
150	0.500	30	150	-0.866	150
160	0.342	20	160	-0.940	160
170	0.174	10	170	-0.985	170
180	0.000	0	180	-1.000	180

T5E

WARNING ABOUT SIN^{-1}

When dealing with **angles** whose measure is between 90° and 180°, what happens with the **SIN** and **COS** which can lead to confusion?

If **X** is an angle between 90° and 180° how do **SIN** and **COS** behave?

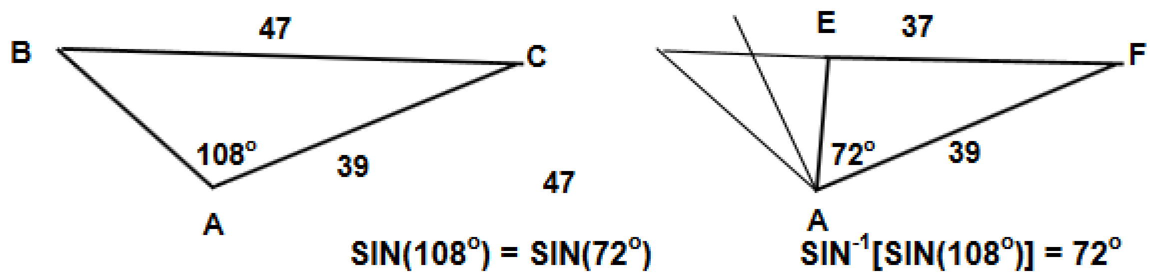
Answer: Give examples:

? For **COS**

? For **SIN**

Suppose we know **SIN**(<A) = .95105, which triangle could this apply to?

Answer: ?



Suppose we know **COS**(<A) = .3090, which triangle could this apply to?

Answer: ?

WHY?

Answer: ?

T5EA

WARNING ABOUT SIN^{-1}

When dealing with angles whose measure is between 90º and 180º, what happens with the **SIN** and **COS** which can lead to confusion?

If **X** is an angle between 90º and 180º how do **SIN** and **COS** behave?

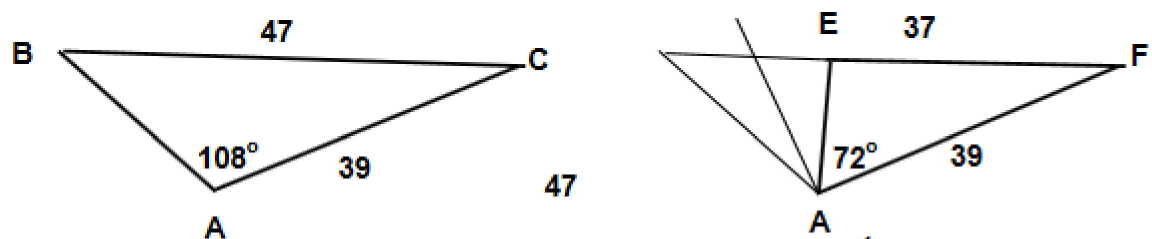
Answer: Give examples:

$\mathbf{COS}(X^{\circ}) = -\mathbf{COS}(180^{\circ} - X^{\circ})$ $\mathbf{COS}(137^{\circ}) = -\mathbf{COS}(43^{\circ})$

$\mathbf{SIN}(X^{\circ}) = \mathbf{SIN}(180^{\circ} - X^{\circ})$ $\mathbf{SIN}(137^{\circ}) = \mathbf{SIN}(43^{\circ})$

Suppose we know **SIN**(<A) = .95105, which triangle could this apply to?

Answer: Both



$SIN(108^{\circ}) = SIN(72^{\circ})$ $SIN^{-1}[SIN(108^{\circ})] = 72^{\circ}$

Suppose we know **COS**(<A) = .3090, which triangle could this apply to?

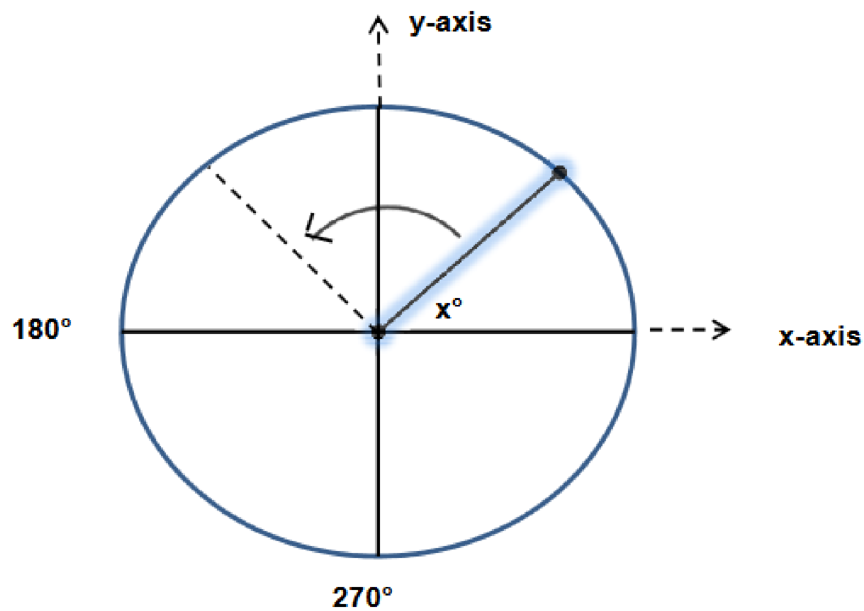
Answer: Only Triangle **AEF**

WHY?

Answer: $\mathbf{COS}\ (108^{\circ}) = -.3090$

T5ES

WARNING ABOUT SIN^{-1}

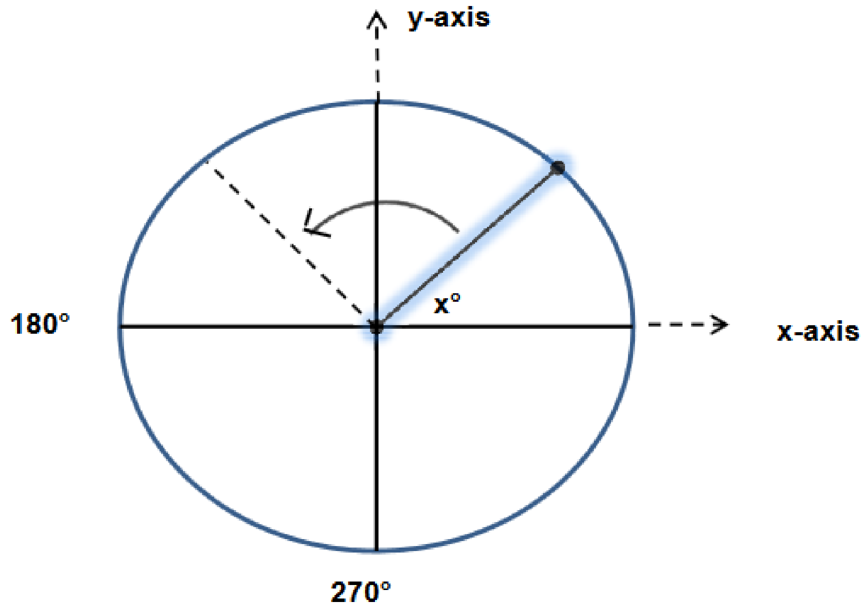


In the diagram above, the highlighted line rotates around in a circle in the xy-plane, starting at 0° and ending at 360°. Fill in the + or -

	0-90°	90-180°	180-270°	270-360°
sin(x)				
cos(x)				
tan(x)				

T5ESA

WARNING ABOUT SIN^{-1}



In the diagram above, the highlighted line rotates around in a circle in the xy-plane, starting at 0° and ending at 360°. Fill in the + or -

	0-90°	90-180°	180-270°	270-360°
sin(x)	positive	positive	negative	negative
cos(x)	positive	negative	negative	positive
tan(x)	positive	negative	positive	negative

T6 LESSON: LAW OF SINES

Problem: Suppose you have a triangle with two angles measuring 40° and 100° and the side opposite the 40° angle is 16 inches.

What is the length, X, of the side opposite the 100° angle? Look at the figure below.

Clearly X is larger than 16 in. Hmmm…maybe it is just proportional to the angles: How about:

$$X = (100^\circ/40^\circ)x16 = (5/2)x16 = 40 \ ?$$

Construct such a triangle and measure it, and you find it measures about 241/2 inches. SO; no, this doesn't work.

Hmmm…what could we do? How about trying some type of correction factor? How about taking the **SIN** of both angles?

SIN(100°)/**SIN**(40°)x16 = 24.5 **Eureka!** ??

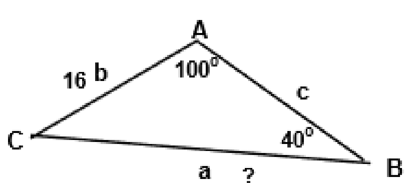
Could this always work? Answer: **YES**.

[**SIN**(<A)/**SIN**(<B)]xb = a, **ALWAYS**, for any angles.

Where **a** is opposite <A and **b** opposite <B

This is called the **Law of Sines**. We prove it in **Tier 3**.

We use it for practical problems. It makes "solving" **triangles** "child's play," especially with a **TI-30Xa**.



Law of Sines

a/**SIN**(<A) = b/**SIN**(<B) = c/**SIN**(<C)

[**SIN**<A/**SIN**<B] x b = a

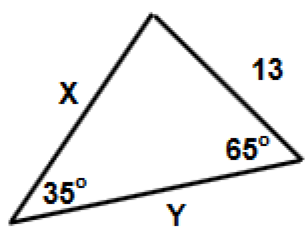
T6 Law of Sines Problems

If you know two angles and an opposite side, you can find them all.

If you know two sides and an opposite angle you can find them all. Sometimes two possibilities.

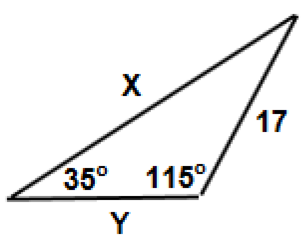
Makes solving problems "child's play."

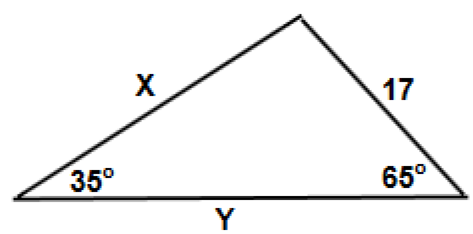
Still, if you know two sides and the included angle, we can't solve for the third side. Need one more tool.



$X/SIN(65^o) = 13/SIN(35^o)$, $X = 20.5$

$Y/SIN(80^o) = 13/SIN(35^o)$, $Y = 22.3$

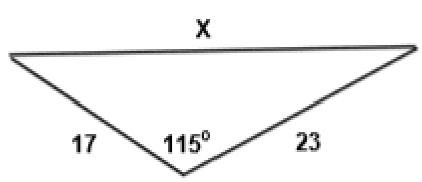




X/SIN(115°) = 17/**SIN**35°) **X** = 26.9 **Y** = 14.8	X/**SIN**(65°) = 17/**SIN**35°) **X** = 26.9 **Y** = 29.2

Observe: 115° + 65° = 180° **Thus:** SIN(115°) = SIN(65°)

Find X



Find X

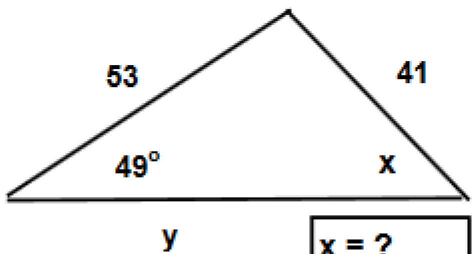
Got to recognize limitations

Need One More Tool

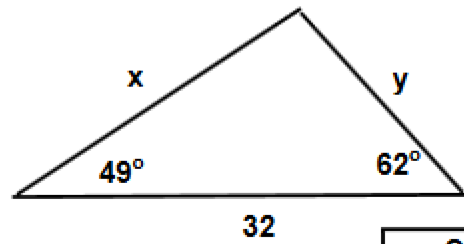
T6E

LAW OF SINES

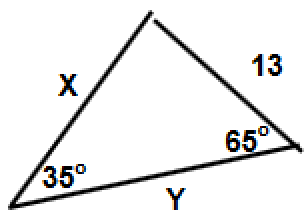
Find the Unknowns and answer questions.



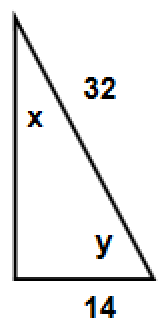
x = ?
y = ?



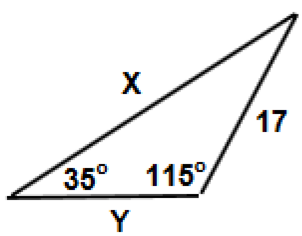
x = ?
y = ?



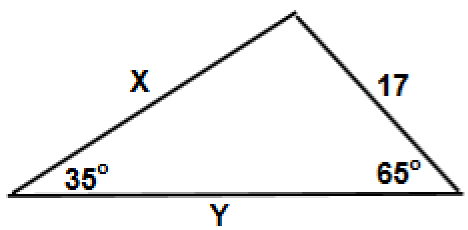
x = ?
y = ?



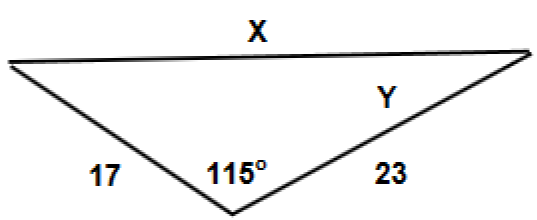
x = ?
y = ?



x = ?
y = ?



x = ?
y = ?

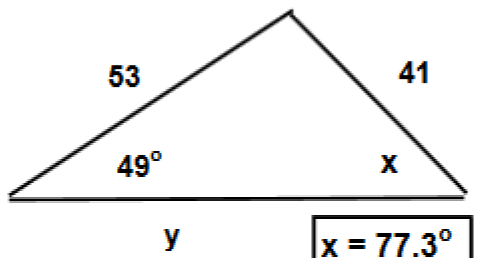


x = ?
y = ?

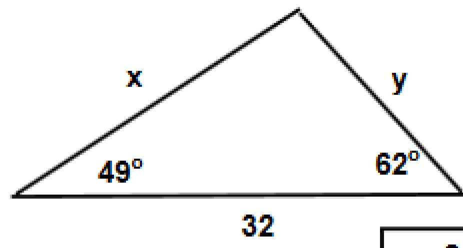
T6EA

LAW OF SINES

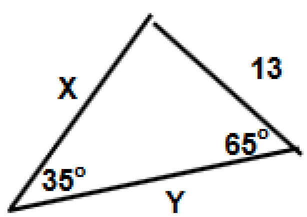
Find the unknowns and answer questions.



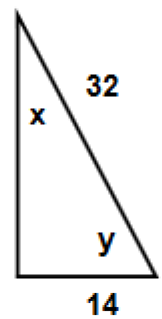
x = 77.3°
y = 43.8



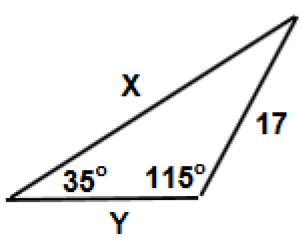
x = 30.3
y = 25.9



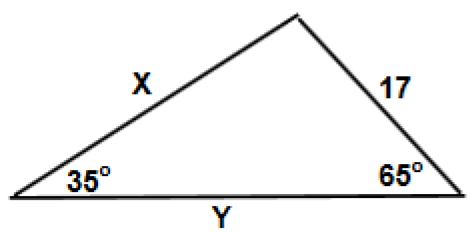
x =20.5
y = 22.3



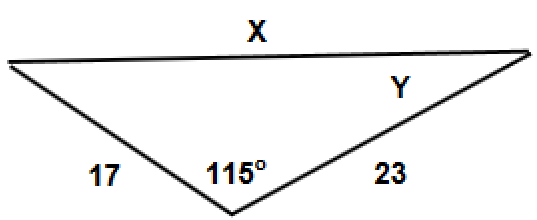
x = 25.9°
y = 64.1°



x = 26.9
y = 14.8



x = 26.9
y = 29.2

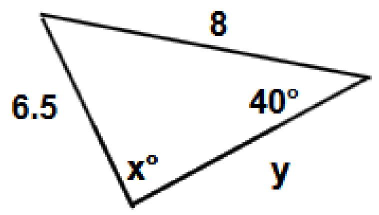


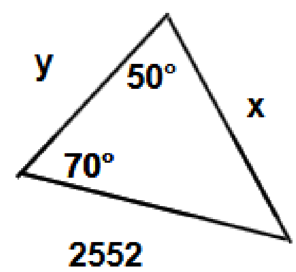
Can not find x and y with tools given so far.
See T7 for solution.

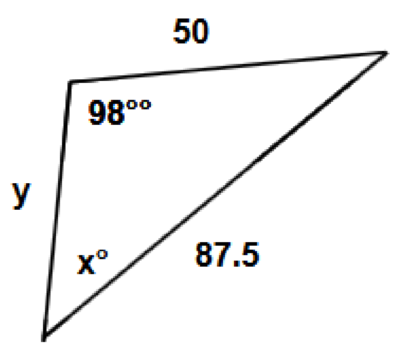
T6ES

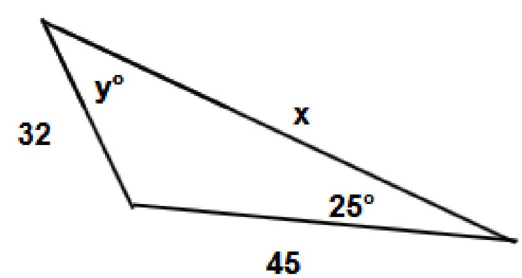
LAW OF SINES

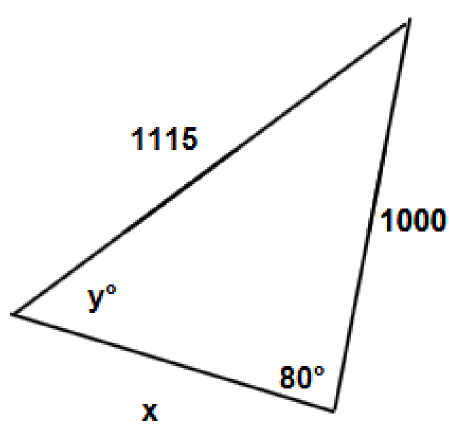
Find X and Y in the following exercises.





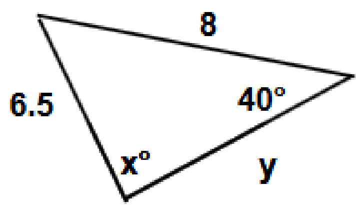




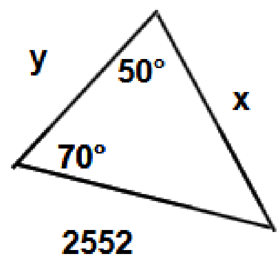


LAW OF SINES

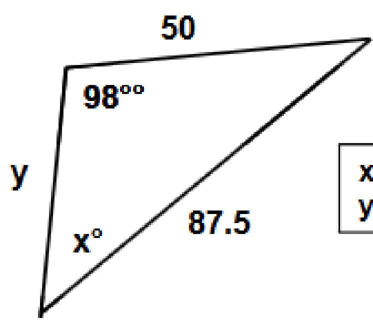
Find X and Y in the following exercises.



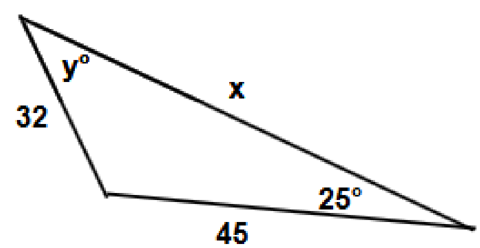
x = 52.3°
y = 6.18



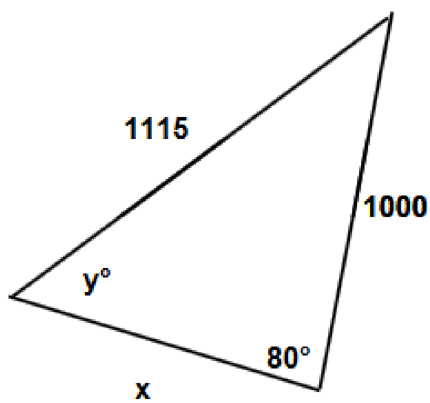
x = 3130.5
y = 2885.1



x = 34.5°
y = 72.4



x = 66.5
y = 36.5°



x = 697.1
y = 62°

T7 LESSON: LAW OF COSINES - GENERALIZED PYTHAGOREAN THEOREM

Suppose we know two sides and the included **angle** of a **triangle**. How can we calculate third side's length?

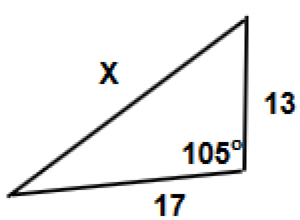
Easy if the **angle** is 90°. $c^2 = a^2 + b^2$

We need a "**correction factor**" for non-right **angles**,

$<a,b$ $c^2 = a^2 + b^2 - 2ab$**COS**$(<a,b)$, works for all **triangles**.

Also, let us find the **angles** when we only know the three sides of a **triangle**.

$<a,b =$ **COS**$^{-1}[(a^2 + b^2 - c^2)/(2ab)]$, where $<a,b$ is included angle.



$X^2 = 17^2 + 13^2 - 2 \times 13 \times 17 \times$ **COS**$105°$

Thus, **X** = 23.9 **NOTE: COS**105° = -.2589, so **CF** is +

$X^2 = 39^2 + 47^2 - 2 \times 39 \times 47 \times$ **COS**$20°$

Thus, **X** = 16.9 mi

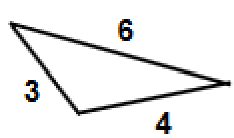
Now we can also calculate y° Use **Law of Sines**

y° = 72° or (180° - 72°) = 108°

Clearly from the diagram 108° is correct.

Find the Area of the 3, 4, 6 triangle using A = .5ab**SIN**$(<a,b)$

First, we must calculate $<a,b$ where a = 3, b = 4



$<3,4 =$ **COS**$^{-1}[(3^2 + 4^2 - 6^2)/(2 \times 3 \times 4)] = 117.3°$

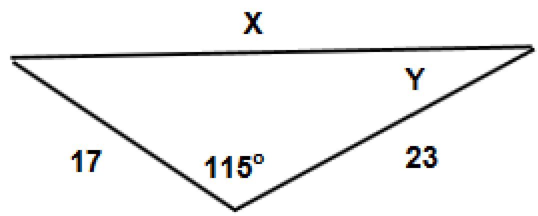
Area = .5x**SIN**(117.3°)x3x4 = 5.33 U^2

T7E

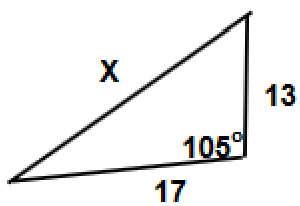
LAW OF COSINES - GENERALIZED PYTHAGOREAN THEOREM

Find the Unknowns

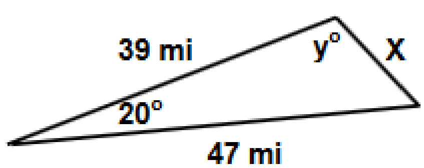
Start with the problem we could not solve in T6



x = ?
y = ?

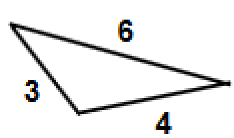


x = ?



x = ?
y = ?

Find the Area of this triangle

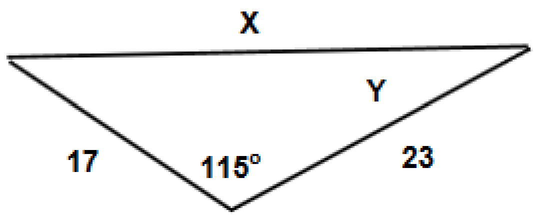


T7EA

LAW OF COSINES - GENERALIZED PYTHAGOREAN THEOREM

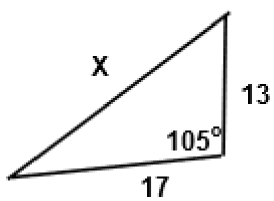
Find the Unknowns

Start with the problem we could not solve in T6



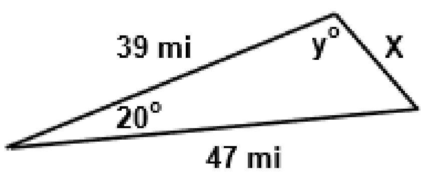
x = 33.9
y = 27°

$x^2 = 17^2 + 23^2 - 2x17x23x$**COS**$(115°)$
$y =$ **SIN**$^{-1}$[{**SIN**(115°)/33.9}x17]



$X^2 = 17^2 + 13^2$ -2x13x17x**COS**105°

Thus, **X** = 23.9 **NOTE**: **COS**105° = -.2589, so, **CF** is **+**



$X^2 = 39^2 + 47^2$ - 2x39x47x**COS**20°

Thus, **X** = 16.9 mi

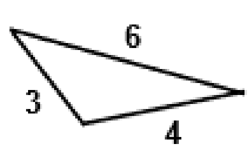
Now we can also calculate y°

Use **Law of Sines**

Clearly from the diagram 108° is correct. y° = 72° or (180° - 72°) = 108°

Find the Area of the 3, 4, 6 triangle using A = .5ab**SIN**(<a,b)

First, we must calculate <a,b where a = 3, b = 4

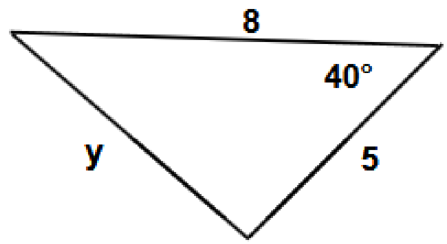


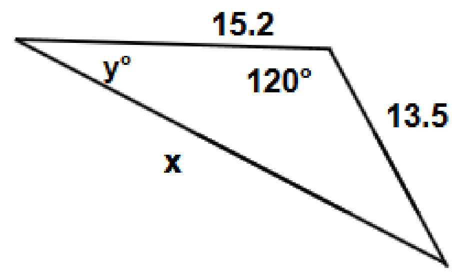
<3,4 = **COS**$^{-1}$[$(3^2 + 4^2 - 6^2)$/(2x3x4)] = 117.3°

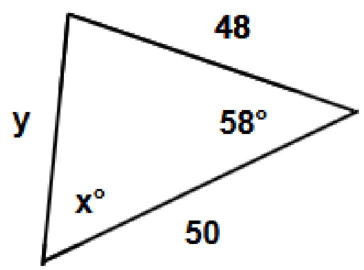
Area = .5x**SIN**(117.3°)x3x4 = 5.33 U^2

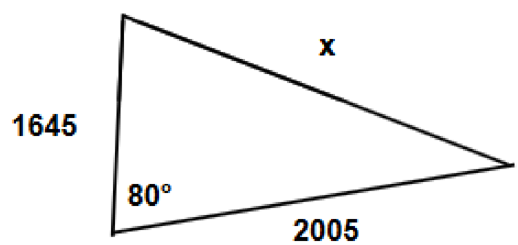
LAW OF COSINES

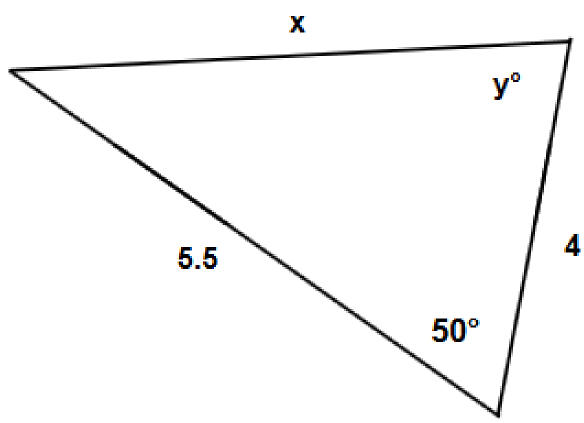
Find X and Y in the following exercises.



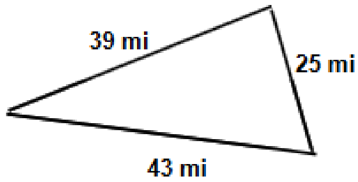


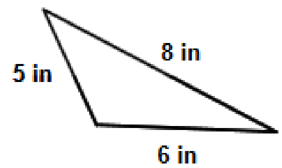


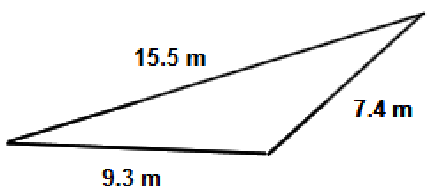


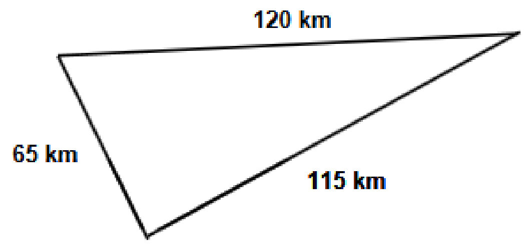


AREAS OF IRREGULAR TRIANGLES





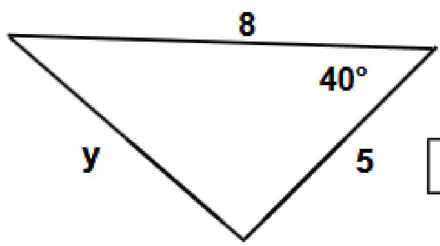




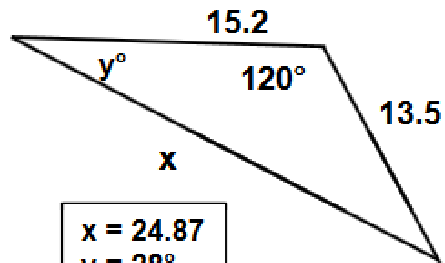
T7ESA

LAW OF COSINES

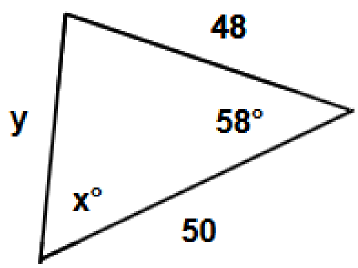
Find X and Y in the following exercises.



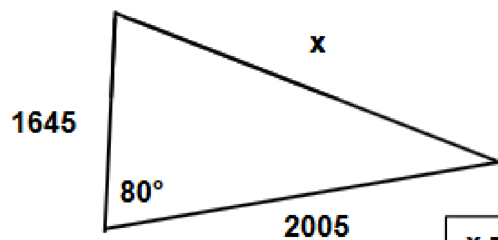
y = 5.26



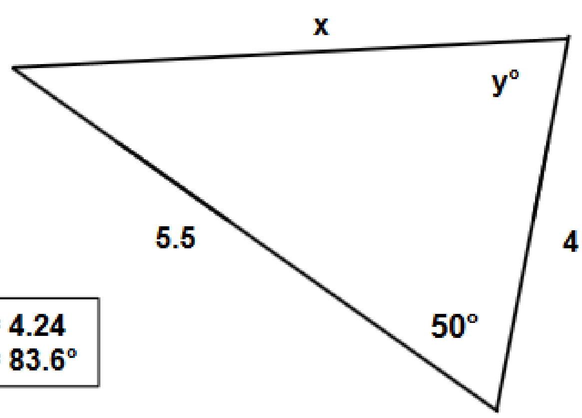
x = 24.87
y = 28°



x = 58.9°
y = 47.5

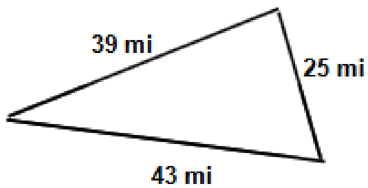


x = 2362.3

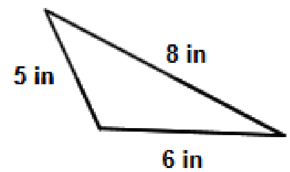


x = 4.24
y = 83.6°

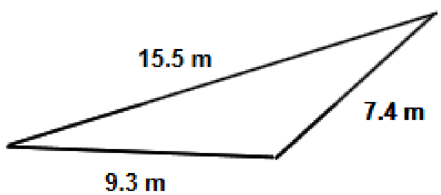
LAW OF COSINES



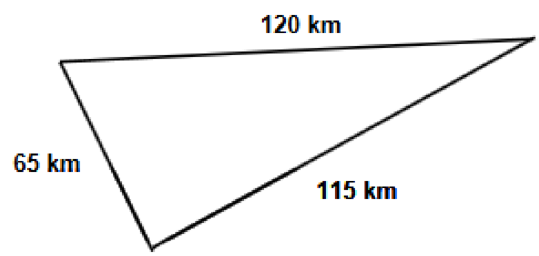
A = 482.1 mi^2



A = 14.98 in^2



A = 23.9 m^2



A = 3,659 km^2

T8 LESSON: TRIGONOMETRY BEYOND PRACTICAL MATH

Trigonometry is a huge extremely important subject with profound applications in science and engineering.

The **Trig Functions** are called the **Circle Functions** and are defined for **ALL** angles, both positive and negative.

Trig Functions are very important in **calculus**.

Trig Functions are probably best understood in the context of the Complex Number System.

Trig Functions are the basis of modern spectrometry via what is called the **Fourier Transform**.

The **Trig Functions** are periodic and that is what makes them so important in any type of **cyclical behavior** such as vibration analysis, and music.

So next, you will need to understand the **Trig functions** via graphs in analytical geometry (Tier 3).

Then one needs to learn about them in the context of the Complex Number System. That is when many of the famous **Trig Identities** will become very natural and understandable. What I consider the most important equation in all of mathematics makes this clear (Tier 4).

Then one needs to learn about their behavior utilizing the **calculus**. It is truly amazing (Tier 5).

Ultimately, they are profound in Functional Analysis and modern physics such as **Quantum Theory** (Tier 9).

T8E

TRIGONOMETRY BEYOND PRACTICAL MATH

Trigonometry is a huge extremely important subject with profound applications in science and engineering and advanced mathematics.

If you are planning to study math beyond Practical Math, then you should be aware of some of the future applications of **Trigonometry**.

List as many things you have heard about where **Trig** will be useful and applicable.

If you study other resources such as Wikipedia you will probably come up with other applications in addition to those I have pointed out.

Please accept my best wishes for your future success.

I hope mathematics will be rewarding to you in your future endeavors, and enjoyable too.

Thank you for studying this **Foundations Course**.

Dr. Del.

T8EA

TRIGONOMETRY BEYOND PRACTICAL MATH

Trigonometry is a huge extremely important subject with profound applications in science and engineering.

The **Trig Functions** are called the **Circle Functions** and are defined for **ALL** angles, both positive and negative.

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Ultimately, they are profound in **Functional Analysis** and modern physics such as **Quantum Theory** (Tier 9).

S6 Lesson: Prefixes

In science and engineering Prefixes are used to change the size of units.

For example, Kilometer, km, means 1,000 Meters

So, 1 km = 1,000m = 10^3 m

1 centimeter = .01m = (1/100)m = 10^{-2}m = 1cm

1 decimeter = .1m = (1/10)m = 10^{-1}m = 1dm

1 millimeter = .001m = (1/1000)m = 10^{-3}m = 1mm

The most common Metric Prefixes are listed below along with their exponents of 10.

milli (m)	-3	Kilo (K)	+3	Thousand
micro(μ)	-6	Mega(M)	+6	Million
nano (n)	-9	Giga (G)	+9	Billion
pico (p)	-12	Tera (T)	+12	Trillion

Examples: 27 nS = $27x10^{-9}$ S = .000000027 S

27 μS = $27x10^{-6}$ S = .000027 S

45 GH = $45x10^{9}$ H = 45000000000 H

78KB = $78x10^{3}$B = 78000B

3.5KΩ = 3500Ω

Now the laws or rules of exponents are:

$10^n \times 10^m = 10^{n+m}$ for any exponents n and m

Also, $10^0 = 1$ and $10^{-n} = 1/10^n$

So, suppose we have, for example:

$7mAx8M\Omega = 7x10^{-3}Ax8x10^{6}\Omega = 56x10^{3}V = 56KV$

Since, $1Ax1\Omega = 1V$ [This is Ohm's Law]

Thus, we see mxM = K since $10^{-3}x10^{6} = 10^{3}$

So we multiply, x, **two prefixes to get one prefix by simply adding the exponents**.

mxG = M since -3 + 9 = 6
mxm = µ since -3+ -3 = -6
nxK = µ since -9 + 3 = -6

If you are going to become an electrician or electronics technician you should learn this prefix table, and practice multiplying prefixes.

Then, you will use this along with the Technician's Triangle we will discuss in another lesson.

This will greatly simplify calculations you will be making when you troubleshoot electrical or electronic systems or equipment.

In the **Metric system** we use powers of 10

In the **Digital system** we use powers of 2.

Note: $2^{10} = 1024 \approx 1000 = 10^3$

The most common Digital Prefixes are listed below along with their exponents of 2.

milli	(m)	-10	Kilo	(K)	+10
micro	(μ)	-20	Mega	(M)	+20
nano	(n)	-30	Giga	(G)	+30
pico	(p)	-40	Tera	(T)	+40

If you are going to become a computer or communications technician, you will want to master this system as well. It works just like the metric system.

For example, mSxMH = KC since 1Sx1H = 1C

Because -10 + 20 = +10

The purpose of this Lesson is to make you aware of these Prefixes. You will want to master them IF you decide to learn a technical field where they are used a lot.

Prefix Product Table

		0	**+3**	**+6**	**+9**	**+12**
X		1	K	M	G	T
0	1	1	K	M	G	T
-3	**m**	m	1	K	M	G
-6	**μ**	μ	m	1	K	M
-9	**n**	n	μ	m	1	K
-12	**p**	p	n	μ	m	1

We will make use of this when we discuss the Technician's Triangle

Of course, this Table can be expanded, but this is what one usually uses.

For example, mxn = p

But, μxn = f

where femto stands for 10^{-15}

Some Musings.

Most of us don't really appreciate the difference between a million and a billion.

How long is one million seconds, 1 MS ?

11.57 days 1,000,000/60/60/24

How long is one billion seconds, 1GS ?

32 years 11,570/365

How long is one trillion seconds, 1 TS ?

32,000 years.

Apply similar questions about our national debt and our money supply.

One million pennies is ten thousand dollars

One billion pennies is ten million dollars.

The DNA in one human cell is about 6 ft long if it unwound. Of course, it is very thin. Similar to extending your little finger from LA to Paris.

There are about one trillion cells in your body. So how long would your DNA be if it was all strung out end to end? How about a billion miles?

S6E

Prefixes

1. Using the generic unit of measure, S, and the **metric** prefixes, calculate the new prefix for the following problems.
   a. mS x nS
   b. mS x MS
   c. KS x MS
   d. μS x μS
   e. nS x GS
   f. TS x μS
   g. GS x KS
   h. mS x μS
   i. GS x pS
   j. TS x μS

2. Using the generic unit of measure, S, and the **metric** prefixes, convert the following to numbers.
   a. 15 nS
   b. 23 KS
   c. 47 TS
   d. 28 μS
   e. 84 GS
   f. 18 MS
   g. 43 pS
   h. 98 mS
   i. 4.2 mS
   j. 3.84 GS

3. Using the generic unit of measure, S, and the **digital** prefixes, calculate the new prefix for the following problems.
   a. mS x nS
   b. mS x MS
   c. KS x MS
   d. µS x µS
   e. nS x GS
   f. TS x µS
   g. GS x KS
   h. mS x µS
   i. GS x pS
   j. TS x µS

Q4. Using the generic unit of measure, S, and the **digital** prefixes, convert the following to numbers.
   a. 15 nS
   b. 23 KS
   c. 47 TS
   d. 28 µS
   e. 84 GS
   f. 18 MS
   g. 43 pS
   h. 98 mS
   i. 4.2 mS
   j. 3.84 GS

S6EA

Prefixes

1.

a. $mS \times nS = 10^{-3}S \times 10^{-9}S = 10^{-12}S = pS$

b. $mS \times MS = 10^{-3}S \times 10^{6}S = 10^{3}S = KS$

c. $KS \times MS = 10^{3}S \times 10^{6}S = 10^{9}S = GS$

d. $\mu S \times \mu S = 10^{-6}S \times 10^{-6}S = 10^{-12}S = pS$

e. $nS \times GS = 10^{-9}S \times 10^{9}S = 10^{0}S = S$

f. $TS \times \mu S = 10^{12}S \times 10^{-6}S = 10^{6}S = MS$

g. $GS \times KS = 10^{9}S \times 10^{3}S = 10^{12}S = TS$

h. $mS \times \mu S = 10^{-3}S \times 10^{-6}S = 10^{-9}S = nS$

i. $GS \times pS = 10^{9}S \times 10^{-12}S = 10^{-3}S = mS$

j. $TS \times \mu S = 10^{12}S \times 10^{-6}S = 10^{6}S = MS$

2.

a. 15 nS = 15×10^{-9} S = 0.000000015 S

b. 23 KS = 23×10^{3} S = 23,000 S

c. 47 TS = 47×10^{12} S = 47,000,000,000,000 S

d. 28 μS = 28×10^{-6} S = 0.000028 S

e. 84 GS = 84×10^{9} S = 84,000,000,000 S

f. 18 MS = 18×10^{6} S = 18,000,000 S

g. 43 pS = 43×10^{-12} S = 0.000000000043 S

h. 98 mS = 98×10^{-3} = 0.098 S

i. 4.2 mS = 4.2×10^{-3} = 0.0042 S

j. 3.84 GS = 3.84×10^{9} S = 3,840,000,000 S

3.

a. mS x nS = 2^{-10}S x 2^{-30}S = 2^{-40}S = pS
b. mS x MS = 2^{-10}S x 2^{20}S = 2^{10}S = KS
c. KS x MS = 2^{10}S x 2^{20}S = 2^{30}S = GS
d. µS x µS = 2^{-20}S x 2^{-20}S = 2^{-40}S = pS
e. nS x GS = 2^{-30}S x 2^{30}S = 2^{0}S = S
f. TS x µS = 2^{40}S x 2^{-20}S = 2^{20}S = MS
g. GS x KS = 2^{30}S x 2^{10}S = 2^{40}S = TS
h. mS x µS = 2^{-10}S x 2^{-20}S = 2^{-30}S = nS
i. GS x pS = 2^{30}S x 2^{-40}S = 2^{-10}S = mS
j. TS x µS = 2^{40}S x 2^{-20}S = 2^{20}S = MS

4.

a. 15 nS = $15x2^{-30}$ S = 0.000000014 S
b. 23 KS = $23x2^{10}$ S = 23,552 S
c. 47 TS = $47x2^{40}$ S = $5.167704651x10^{13}$ S
d. 28 µS = $28x2^{-20}$ S = 0.000026703 S
e. 84 GS = $84x2^{30}$ S = 8,589,934,592 S
f. 18 MS = $18x2^{20}$ S = 18,874,368 S
g. 43 pS = $43x2^{-40}$ S = $3.910827218x10^{-11}$ S
h. 98 mS = $98x2^{-10}$ = 0.095703125 S
i. 4.2 mS = $4.2x2^{-10}$ = 0.004101562 S
j. 3.84 GS = $3.84x2^{30}$ S = 34,123,168,604 S

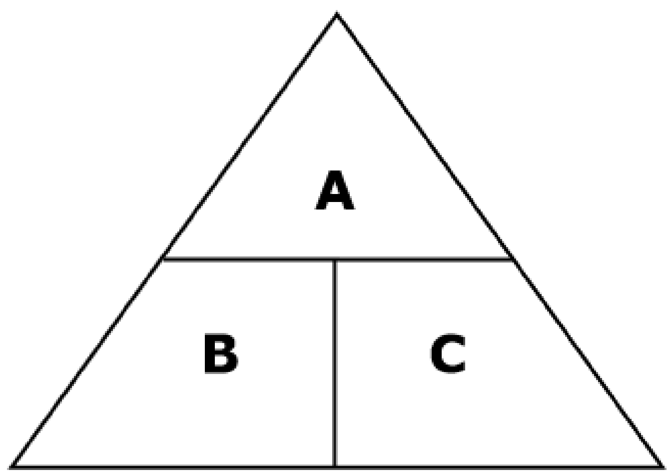
S7 Lesson: Technician's Triangle

Often one is faced with an equation A = BxC, where one must solve for one of these variables when the other two are known.

This yields three equations as you have learned.

A = BxC B = A/C C = A/B

Sometimes it is easiest to simply put this into what I call a Technician's Triangle. Then, one can "solve" the equation very easily.



Now to "solve" for any variable, just perform the calculation with the other two variables.

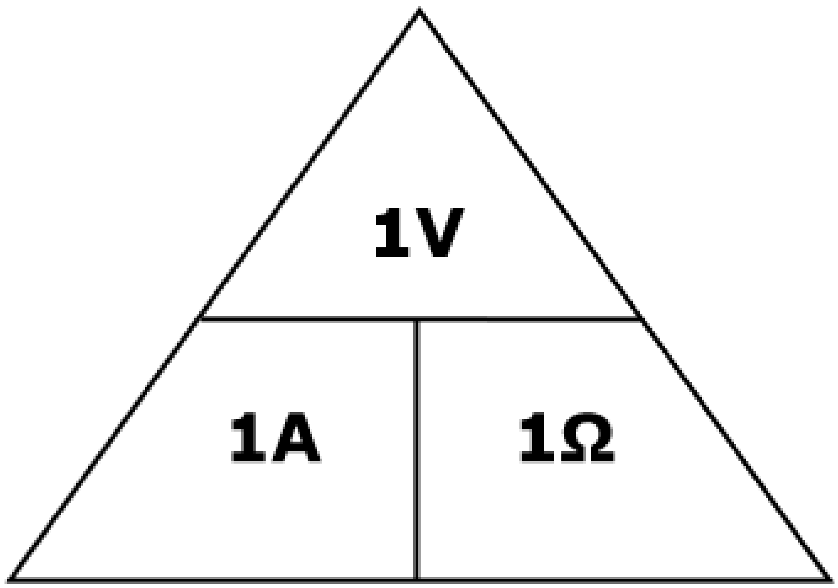
A = BxC B = A/C C = A/B

Things get interesting when the units involved have prefixes attached.

Let's look at an example from electronics.

Ohm's Law is $1V = 1A \times 1\Omega$

Where: V is Volts, A is Amps, Ω is Resistance



But, often one has to deal with prefixes attached to these units. For example, we might have:

$5\mu A \times 7K\Omega = .000005 \times 7000\ V = .035\ V = 35\ mV$

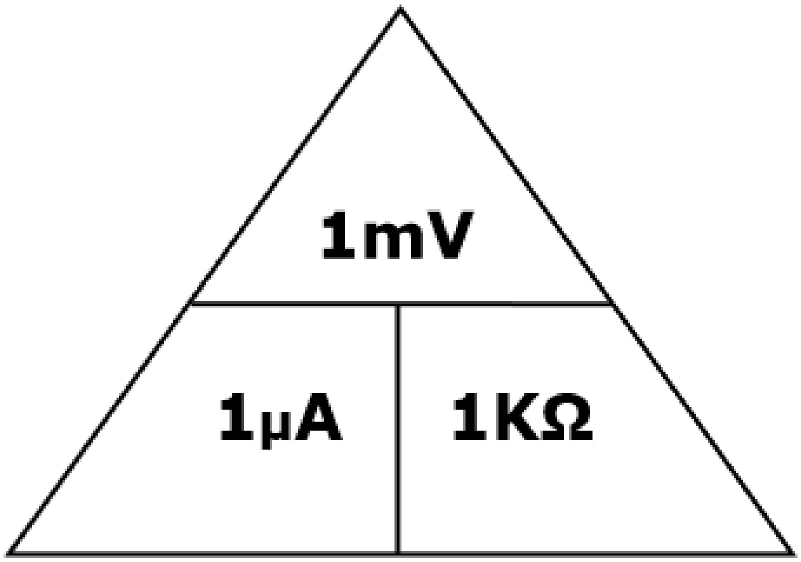
This is the way it has been dealt with classically.

There must be an easier way!

Well, there is.

We learned in the Prefixes lesson that:

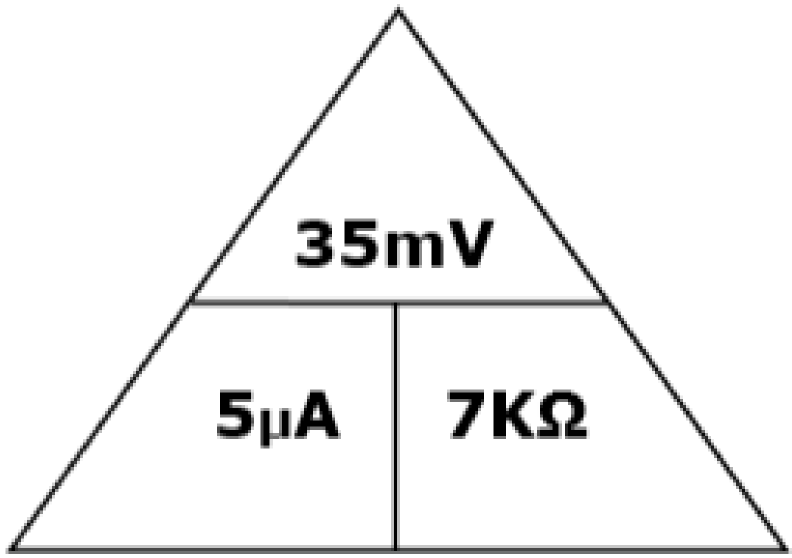
μ x K = m



This then leads us to the following Tech Triangle.

Remember we know μ x K = m

So, this then leads us to the following Tech Triangle



So, all we have to do to solve for any one of these given the other two is simply do the simple arithmetic.
This is much easier than the old-fashioned way.

5µAx7KΩ = .000005x7000V = .035V= 35mV

Or 35mV/7KΩ = .035/7000 A = .000005 A = 5µA

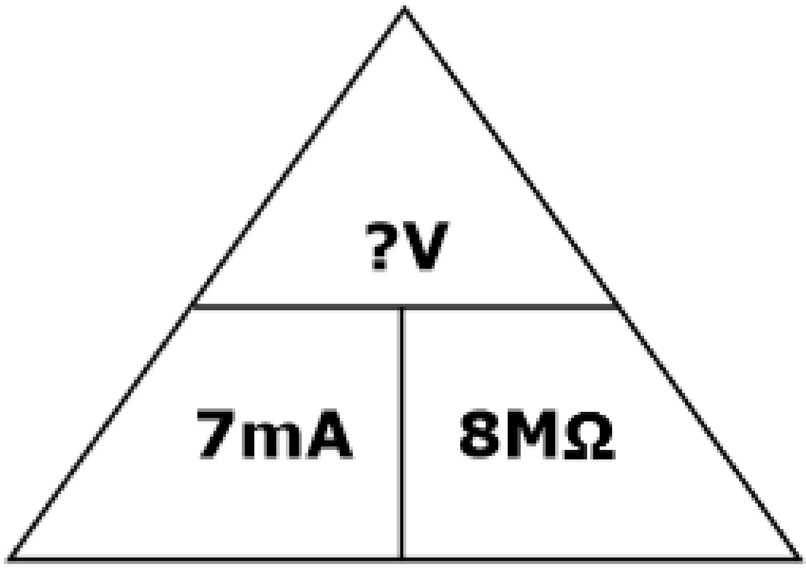
Or 35mV/5µA = .035/.000005 Ω = 7000Ω = 7 KΩ

It was amazing how many times engineers and technicians got the decimal place wrong and were off by an order of magnitude, i.e., 10x.

So, quick now, what is 7 mA times 8 MΩ ?

Remember we know m x M = K

So, this then leads us to the following Tech Triangle



Answer: 56 KV

This is much easier than the old-fashioned way.

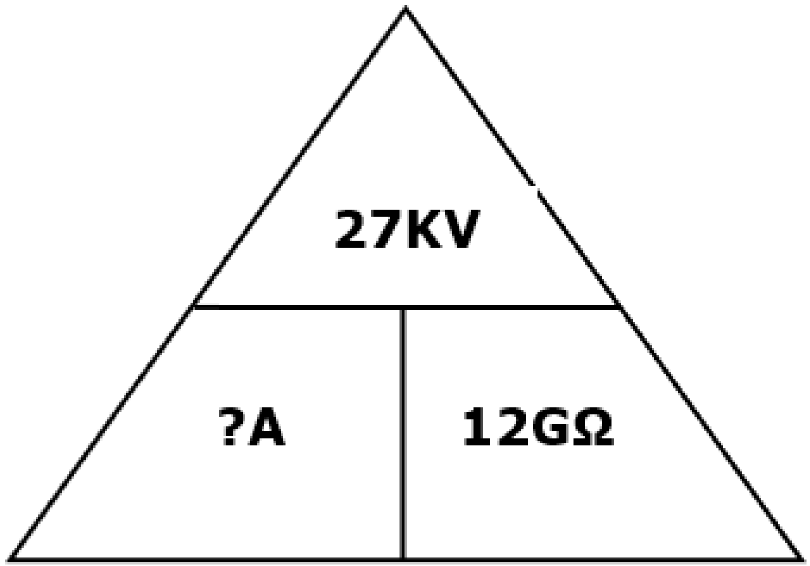
So, quick now, what is 7 mA times 8 MΩ ?

Try it the old-fashioned way if you want to experience what some of our ancestors went through. Even with slide rules and log tables it was more difficult than with a calculator. But, it is even easy to make a mistake with a calculator doing it the old-fashioned way.

Try: 2.4mA x 6.7 MΩ   Use the TT,   mxM = K

Answer: 2.4x6.7 KV = 16 KV

OK one more, quick. 27KV across a 12GΩ resistor yields how many amps, A? So, this then leads us to the following Tech Triangle



Well look in the Prefix Table.

What times G yields K?   Answer: μ

[G is +9 and K is +3, so we need a -6 since 9+(-6)= +3
  So, we need a μ and Gxμ = K]

So, the answer is 27/12 μA = 2.25 μA

This is much easier than the old-fashioned way.

Try it the old-fashioned way if you want to experience what some of our ancestors went through. They didn't even have calculators. But, our calculator won't even take this many 0's in FLO so
you would have to use SCI format.

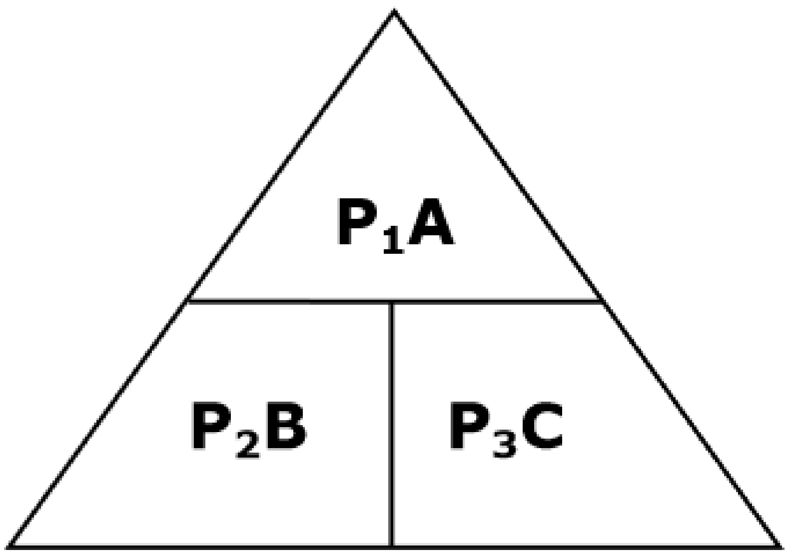
27000/12000000000 = .00000225

There are many fields where you have an equation like

1A = 1Bx1C where A,B,C are some units.

Then, a Technician's Triangle will apply.

You will need to learn the Prefixes and remember to multiply two prefixes you just add their exponents of their power of 10, or of their power of 2 in the digital case.



Where $P_1 = P_2xP_3$, from the Table of Prefixes

This is much easier than the old-fashioned way.

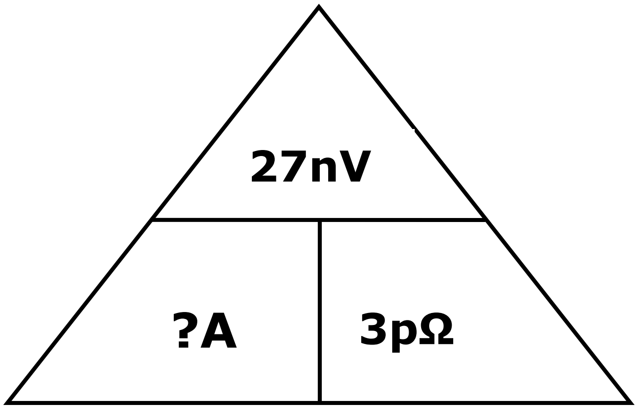
Simply practice in whatever technical field you are in with the relevant equations.

S7E

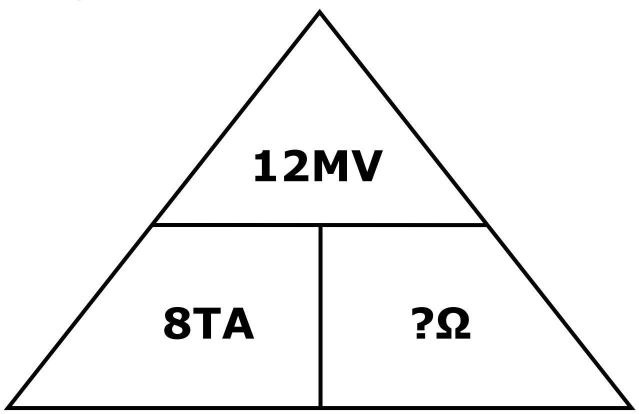
Technician's Triangle

Solve for the unknown using metric prefixes.

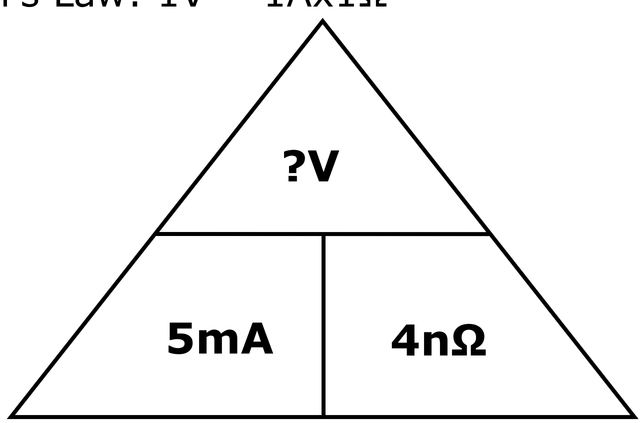
1. Ohm's Law: 1V = 1Ax1Ω



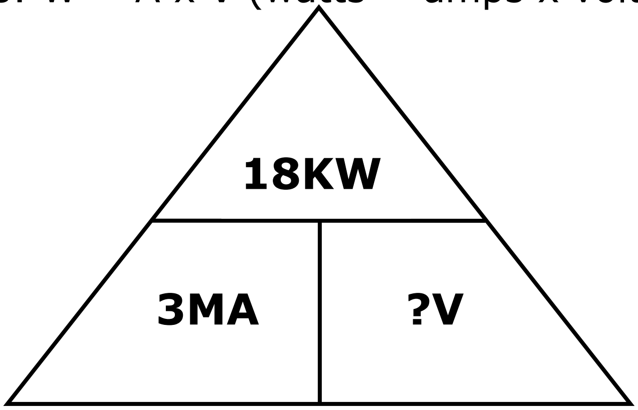
2. Ohm's Law: 1V = 1Ax1Ω



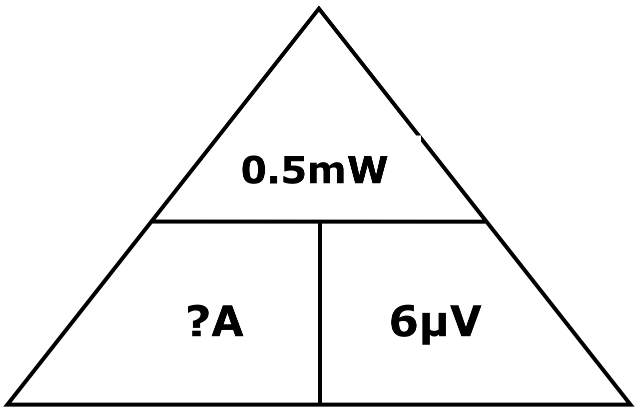
3. Ohm's Law: 1V = 1Ax1Ω



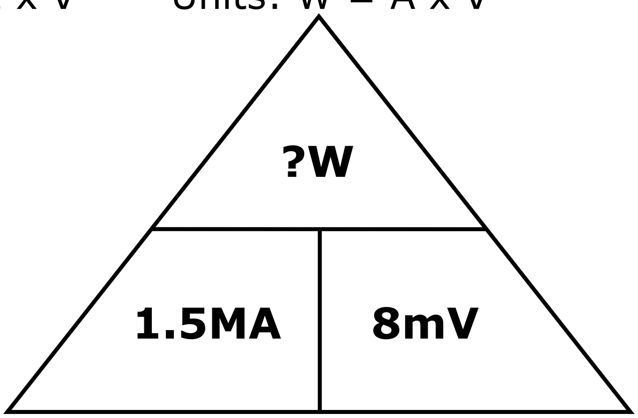
4. P = I x V (power = current x volts)
Units: W = A x V (watts = amps x volts)



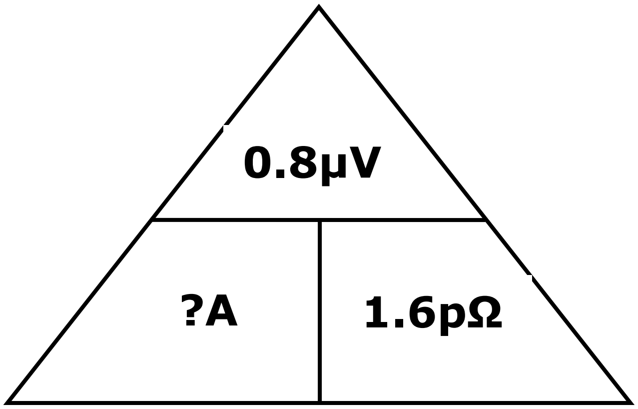
5. P = I x V   Units: W = A x V



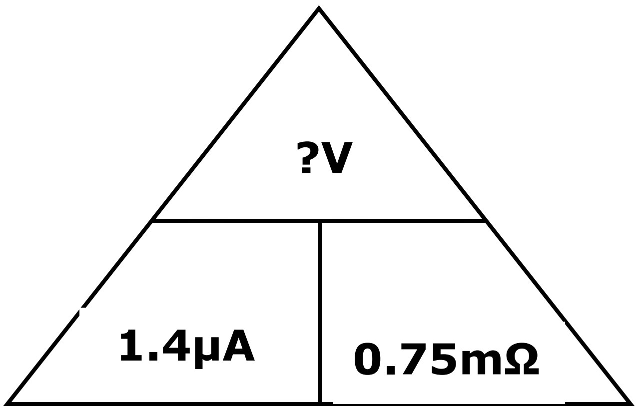
6. P = I x V   Units: W = A x V



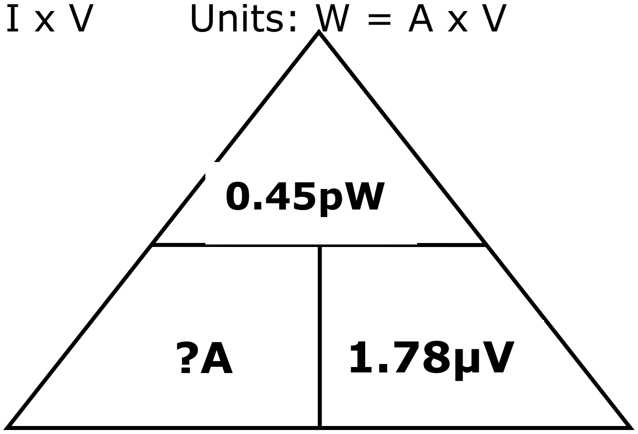
7. Ohm's Law: 1V = 1Ax1Ω



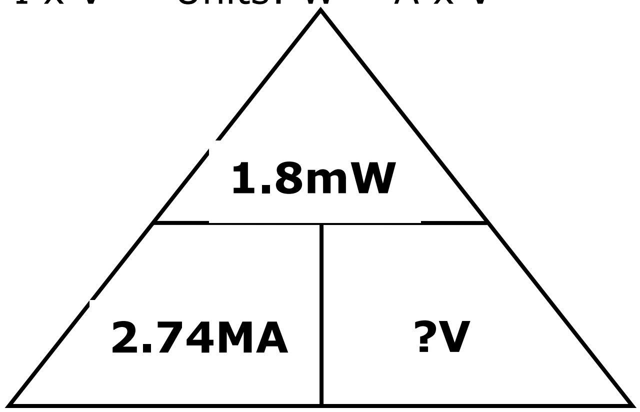
8. Ohm's Law: 1V = 1Ax1Ω



9. P = I x V   Units: W = A x V



10. P = I x V   Units: W = A x V



S7EA

Technician's Triangle

1. 27nV = ?A x 3pΩ

        n = ? + p --> -9 = ? + -12 --> ? = +3 --> K

    27nV = ?KA x 3pΩ

        27 = ? x 3 -> ? = 9

    Unknown: 9KA

        27nV = 9KA x 3pΩ

2. 12MV = 8TA x ?Ω

        M = T + ? --> +6 = +12 + ? --> ? = -6 --> μ

    12MV = 8TA x ?μΩ

        12 = 8 x ? -> ? = 1.5

    Unknown: 1.5μΩ

        12MV = 8TA x 1.5μΩ

3. ?V = 5mA x 4nΩ

        ? = m + n --> ? = (-3) + (-9) --> ? = -12 --> p

    ?pV = 5mA x 4nΩ

        ? = 5 x 4 -> ? = 20

    Unknown: 20pV

        20pV = 5mA x 4nΩ

4. 18KW = 3MA x ?V

K = M + ? --> +3 = +6 + ? --> ? = -3 --> m

18KW = 3MA x ?mV

18 = 3 x ? -> ? = 6

Unknown: 6mV

18KW = 3MA x 6mV

5. 0.5mW = ?A x 6µV

m = ? + µ --> -3 = ? + (-6) --> = +3 --> K

0.5mW = ?KA x 6µV

0.5 = ? x 6 -> ? = 1/12 or 0.083

Unknown: 0.083KA

0.5mW = 0.083KA x 6µV

6. ?W = 1.5MA x 8mV

? = M + m --> ? = +6 + (-3) --> = +3 --> K

?KW = 1.5MA x 8mV

? = 1.5 x 8 -> ? = 12

Unknown: 12KW

12KW = 1.5MA x 8mV

7. 0.8µV = ?A x 1.6pΩ

µ = ? + p --> -6 = ? + (-9) --> ? = +3 --> K

0.8µV = ?KA x 1.6pΩ

0.8 = ? x 1.6 -> ? = 0.5

Unknown: 0.5KA

0.8µV = 0.5KA x 1.6pΩ

8. ?V = 1.4µA x 0.75mΩ

?= μ + m --> ? = (-6) + (-3) --> ? = -9 --> n

?nV = 1.4μA x 0.75mΩ

? = 1.4 x 0.75 -> ? = 1.05

Unknown: 1.05nV

1.05nV = 1.4μA x 0.75mΩ

9. 0.45pW = ?A x 1.78μV

p = ? + μ --> -12 = -6 + ? --> = -6 --> μ

0.45pW = ?μA x 1.78μV

0.45 = ? x 1.78 -> ? = 0.253

Unknown: 0.253μA

0.45pW = 0.253μA x 1.78μV

10. 1.8mW = 2.74MA x ?V

m = M + ? --> -3 = +6 + ? --> = -9 --> n

1.8mW = 2.74MA x ?nV

1.8 = 2.74 x ? -> ? = 0.657

Unknown: 0.657nV

1.8mW = 2.74MA x 0.657nV

S8 Lesson: Polar Rectangular Coordinates

In the plane, there are two ways to specify a point.

Rectangular Coordinates (**x,y**)

Polar Coordinates (**r**, **θ**) where

r = $(x^2 + y^2)^{1/2}$,

θ = **tan** $^{-1}$(**y**/**x**) in Quadrants 1 and 4

and θ = **tan**-1(**y**/**x**) + 180º in Quads 2 and 3

Example 1: (4,3) = (5, 36.87º) since **tan**$^{-1}$(3/4) = 36.87º and 5 = $(4^2 + 3^2)^{1/2}$

Example 2: (-4,3) = (5, 143.13º) since **tan**$^{-1}$(-3/4) = -36.87º + 180º = 143.13º

Fortunately, the **TI-30Xa** will do this automatically with the R → P and P → R Keys.

2nd . 2 This fixes the display to two digits past.

FUNCTION	KEY	ENTER	DISPLAY
	4	4	
x <--> y	2nd π		0.00
	3	3	
R< - ->P	2nd -		5.00
x <--> y	2nd π		36.87

FUNCTION	KEY	ENTER	DISPLAY

	4	4	
+< -->-		-4	
x <--> y	2^{nd} π		0.00
	3	3	
R< - ->P	2^{nd} -		5.00
x <--> y	2^{nd} π		143.13

You can go from **P** to **R** also.

FUNCTION	KEY	ENTER	DISPLAY
	5	5	
x <--> y	2^{nd} π		0.00
		143.13	143.13
P< - ->R	2^{nd} x		-3.9999
x <--> y	2^{nd} π		3.00

Note: All of this works if you use **RAD** or **GRAD** for the degrees, for those of you who are more advanced in trigonometry.

Now just do some Exercises

(4, 9) = (9.85, 66.03°) **R** to **P**

(7, 197°) = (-6.69, -2.05) **P** to **R**

S8E

Polar Rectangular Coordinates Exercises

For the following exercises, graph the rectangular coordinates to determine quadrant, then solve for the polar coordinates.

1. (5, 12)
2. (8, 15)
3. (-8, -15)
4. (-4.5, 6.3)
5. (3.7, -8.2)
6. (-8.9, -12.5)

For the following exercises, solve for the rectangular coordinates.

7. (9, 45º)
8. (6, 32 º)
9. (12, 127º)
10. (4.7, 118.6º)
11. (5.6, 210º)
12. (7.8, 301.9º)

Using the R → P button on your calculator, convert these rectangular coordinates to polar coordinates.

13. (5, 7)
14. (8, 13)
15. (-7, 16)
16. (6.3, -8.2)

Using the P → R button on your calculator, convert these polar coordinates to rectangular coordinates.

17. (9, 27º)
18. (10, 75º)
19. (4.7, 190.5º)
20. (13.45, 347º)

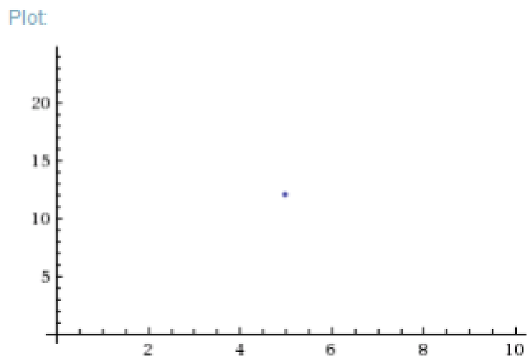
S8EA

Polar Rectangular Coordinates Exercise Answers

1. **r** $= (x^2 + y^2)^{1/2}$

$\mathbf{r} = (5^2 + 12^2)^{1/2}$

$\mathbf{r} = 13$



Quadrant 1

$\Theta = \tan^{-1}(y/x)$

$\Theta = \tan^{-1}(12/5)$

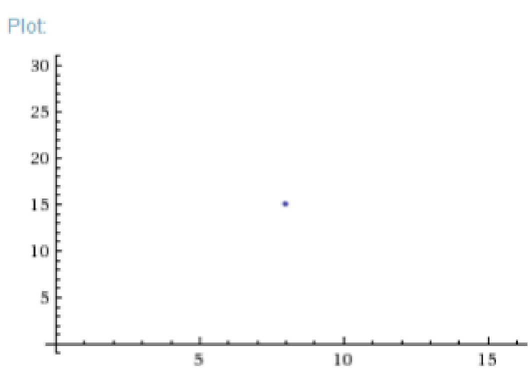
$\Theta = 67.38°$

(13, 67.38°)

2. $\mathbf{r} = (x^2 + y^2)^{1/2}$

$\mathbf{r} = (8^2 + 15^2)^{1/2}$

$\mathbf{r} = 17$



Quadrant 1

$\Theta = \tan^{-1}(y/x)$

$\Theta = \tan^{-1}(15/8)$

$\Theta = 61.93^o$

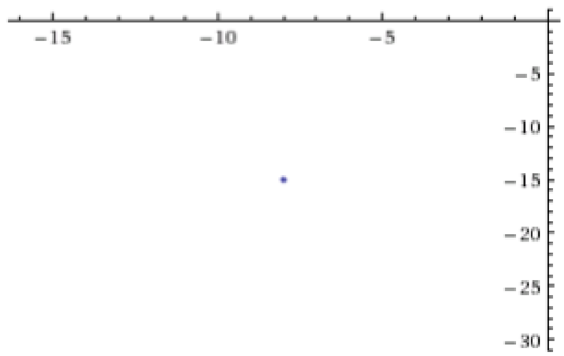
$(17, 61.93^o)$

3. $r = (x^2 + y^2)^{1/2}$

$r = ((-8)^2 + (-15)^2)^{1/2}$

$r = 17$

Plot:



Quadrant 3

$\Theta = \tan^{-1}(y/x) + 180^o$

$\Theta = \tan^{-1}(-15/-8) + 180^o$

$\Theta = 241.93^o$

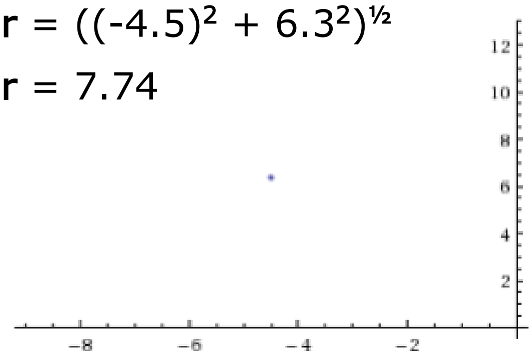
$(17, 241.93^o)$

4. $r = (x^2 + y^2)^{1/2}$

Plot:

$r = ((-4.5)^2 + 6.3^2)^{½}$

$r = 7.74$



Quadrant 2

$\Theta = \tan^{-1}(y/x) + 180^{o}$

$\Theta = \tan^{-1}(-4.5/6.3) + 180^{o}$

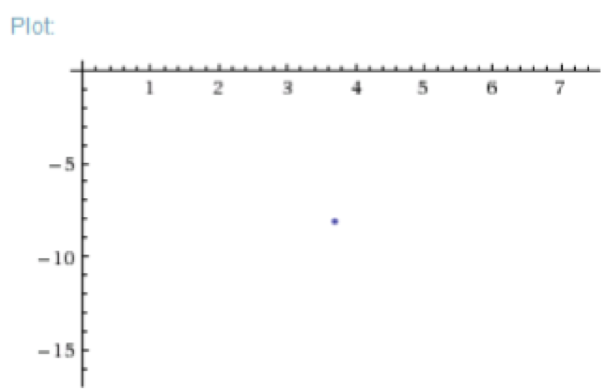
$\Theta = 144.46^{o}$

$(7.74, 144.46^{o})$

5. $r = (x^2 + y^2)^{1/2}$

$r = (3.7^2 + (-8.2)^2)^{1/2}$

$r = 9.00$



Quadrant 4

$\Theta = \tan^{-1}(y/x)$

$\Theta = \tan^{-1}(3.7/-8.2)$

$\Theta = -24.29^{o} = -24.29^{o} + 360^{o} = 335.71^{o}$

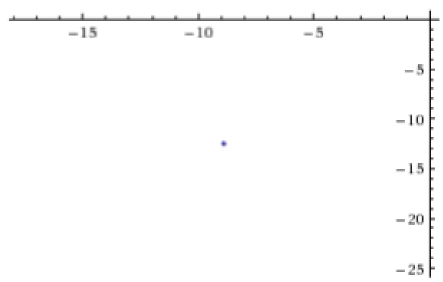
(9.00, -24.29º) or (9.00, 335.71º)

6. $r = (x^2 + y^2)^{1/2}$

$r = ((-8.9)^2 + (-12.5)^2)^{1/2}$

$r = 15.35$

Plot:



Quadrant 3

$\Theta = \tan^{-1}(y/x) + 180^\circ$

$\Theta = \tan^{-1}(-8.9/-12.5) + 180^\circ$

$\Theta = 215.45^\circ$

(15.35, 215.45º)

7. $x = r\cos(\Theta)$

$x = 9\cos(45^\circ)$

$x = 6.36$

$y = r\sin(\Theta)$

$y = 9\sin(45^\circ)$

$y = 6.36$

(6.36, 6.36)

8. $x = r\cos(\Theta)$

$x = 6\cos(32^\circ)$

$x = 5.09$

$y = r\sin(\Theta)$

$y = 6\sin(32°)$

$y = 3.18$

(5.09, 3.18)

9. $x = r\cos(\Theta)$

$x = 12\cos(127°)$

$x = -7.22$

$y = r\sin(\Theta)$

$y = 12\sin(127°)$

$y = 9.58$

(-7.22, 9.58)

10. $x = r\cos(\Theta)$

$x = 4.7\cos(118.6°)$

$x = -2.25$

$y = r\sin(\Theta)$

$y = 4.7\sin(118.6°)$

$y = 4.13$

(-2.25, 4.13)

11. $x = r\cos(\Theta)$

$x = 5.6\cos(210°)$

$x = -4.8$

$y = r\sin(\Theta)$

$y = 5.6\sin(210°)$

$y = -2.8$

(-4.8, -2.8)

12. $x = r\cos(\Theta)$

$x = 7.8\cos(301.9°)$

$x = 4.12$

y = rsin(Θ)

y = 7.8sin(301.9º)

y = -6.62

(4.12, -6.62)

13. (8.60, 54.46º)

14. (15.26, 58.39º)

15. (17.46, 113.63º)

16. (10.34, -52.47º) or (10.34, 307.53º) 52.47º + 360º = 307.53º

17. (8.02, 4.09)

18. (2.59, 9.66)

19. (-4.62, -0.86)

20. (13.11, -3.03)

Note: (13.45, -13º) will get you the same answer because 347º-360º=-13º