Craig Hane, Ph.D., Founder

Workforce Development:

Advanced Math for Industry

1.1 Lessons Abbreviation Key Table

C = Calculator Lesson
P = Pre-algebra Lesson
A = Algebra Lesson
G = Geometry Lesson
T = Trigonometry Lesson
S = Special Topics

The number following the letter is the Lesson Number.

E = Exercises with Answers: Answers are in brackets **[ ]**.
EA = Exercises Answers: (only used when answers are not on the same page as the exercises.)
ES = Exercises Supplemental: Complete if you feel you need additional problems to work.

1.2 Exercises Introduction

Why do the Exercises?

Mathematics is like a "game." The more you practice and play the game the better you will understand and play it.

The Foundation's Exercises, which accompany each lesson, are designed to reinforce the ideas presented to you in that lesson's video.

It is unlikely you will learn math very well by simply reading about it or listening to Dr. Del, or anyone else, or watching someone else doing it.

You WILL learn math by "doing math."

It is like learning to play a musical instrument, or write a book, or play a sport, or play chess, or cooking.

You will learn by practice.

Repetition is the key to mastery.

You will make mistakes. You will sometimes struggle to master a concept or technique. You may feel frustration sometimes "**WE ALL DO**."

But, as you learn and do math, you will begin to find pleasure and enjoyment in it as you would in any worthwhile endeavor. Treat it like a sport or game.

These exercises are the KEY to your SUCCESS!

ENJOY!

C13 LESSON: DEG RAD GRAD THREE ANGLE MEASURES

There are three measures of an angle acceptable by the TI 30XA calculator.

Degree **DEG** 1/360 of a circle

Gradian **GRAD** 1/400 of a circle

Radian **RAD** 1/2π of a circle with radius 1. (57.3 DEG)

In our Practical Math Foundation we will only use the **DEG** which is what automatically comes up when you turn on the calculator.

The DRG Key changes the choice of unit.

If you enter a number in the **DEG** mode and then press the **2nd** DRG Keys, you will transform the number to the new unit.

For example, enter 180 as **DEG**, then transform into **RAD** (3.1416) and **GRAD** (200)

Or; enter 1 in **RAD** mode, and transform into 57.3 Degrees.

We will only use **DEG** in the Foundation training.

RAD will also be used in Tiers 4 and up. It is the "natural" measurement of an angle for trig and calculus.

C13E

DEG   RAD   GRAD   THREE ANGLE MEASURES

1. DEG stands for?

2. What fraction of a circle is one degree?

3. What are the other two angle measures on the TI 30XA calculator?

4. Which measure comes up when you turn on the calculator?

5. How do you switch to the other two measures?

6. How do you convert Degrees to **RAD**s and **GRAD**s?

7. How many **RAD**s are 90 degrees?

8. How many **GRAD**s are 90 degrees?

9. What will we use exclusively in the Foundations Course to measure angles?

Answers are on C13EA, page 46.

Take the C13 Quiz.

C13EA

DEG RAD GRAD THREE ANGLE MEASURES

Answers: [ ]'s

1. DEG stands for? [**Degree º**]

2. What fraction of a circle is one degree? [**1/360**]

3. What are the other two angle measures on the TI 30XA calculator? [**RAD and GRAD**]

4. Which measure comes up when you turn on the calculator? [**DEG**]

5. How do you switch to the other two measures? [**Press the DRG key once for RAD again for GRAD and again for DEG**]

6. How do you convert Degrees to RADs and GRADs? [**Enter the degrees and press the 2nd DEG key for RADs and press 2nd DEG key again for GRADs**]

7. How many RADs are 90 degrees? [**1.57**]

8. How many GRADs are 90 degrees? [**100**]

9. What will we use exclusively in the Foundations Course to measure angles? [**DEG Degrees**]

Take the C13 Quiz or review.

C14 LESSON: SIN SIN^{-1}

These two keys are used to compute the Sine of an angle, and the angle, if you know its **SIN**.

This is used in Trigonometry, and also for some interesting formulas in Geometry.

We will always use the Degree, **DEG**, measure of an angle in the Foundation course.

Enter the angle, say, θ, and press **SIN**

Example: 45 **SIN** yields .707

SIN (θ) is always between -1 and 1.

SIN^{-1} is the "inverse" of the **SIN**, **2nd** **SIN**

If **SIN** (θ) = N, then **SIN^{-1}**(N) = θ

Example: **SIN^{-1}**(.707) = 45^{o}

SIN^{-1}(N) only works for N between -1 and 1.

NOTE: SIN 135 = 0.707…in general, SIN (180°- θ) = SIN (θ)

C14E

SIN **SIN^{-1}** Answers: [ ]'s

1. **SIN** (45°) = ? [0.707]

2. **SIN** (0°) = ? [0]

3. **SIN** (10°) = ? [0.174]

4. **SIN** (30°) = ? [0.500]

5. **SIN** (60°) = ? [0.866]

6. **SIN** (75°) = ? [966]

7. **SIN** (85°) = ? [0.996]

8. **SIN** (90°) = ? [1]

9. **SIN** (95°) = ? [0.996]

11. **SIN** (120°) = ? [0.866]

12. **SIN^{-1}**(0.5) = ? [30 degrees]

13. What angle X, has **SIN** (X) = 0.4 ? [23.58 degrees]

14. **SIN^{-1}**(0.4) = ? [23.58 degrees]

15. **SIN^{-1}[SIN(50°)]** = ? [50 degrees]

Take C14 Quiz or do more exercises, C14ES.

C14ES

SIN **SIN^{-1}** Answers: [ ]'s

1. **SIN** ($30^o + 90^o$) = ? [**SIN(120^o) = 0.866**]

2. **SIN** ($45^o + 90^o$) = ? [**SIN(135^o) = 0.707**]

3. **SIN** ($60^o + 90^o$) = ? [**SIN(150^o) = 0.5**]

4. **SIN** ($90^o + 90^o$) = ? [**SIN(180^o) = 0**]

5. **SIN^{-1}** (0.866) = ? [**59.99^o ~ 60^o**]

6. Why the discrepancy in #5? [**Round off error SIN(60^o) = .866025404 SIN(59.99^o) = .865938124**]

7. **SIN^{-1}**(0.5) = ? [**30^o**]

8. What angle X, has **SIN**(X) = .3? [**17.5^o**]

9. **SIN^{-1}**(0.3) = ? [**17.5^o**]

10. **SIN^{-1}**[SIN(x^o)] = ? [**x^o**]

11. **SIN**[**SIN^{-1}**(x) = ? [**x**]

12. **SIN** (θ) is always between? [**-1 and 1**]

13. SIN^{-1}(1.5) = ? [**Error**]

Take C14 Quiz or review.

C15 LESSON: COS COS^{-1}

These two keys are used to compute the Cosine of an angle, and the angle, if you know its COS.

This is used in Trigonometry and also for some interesting formulas in Geometry.

We will always use the Degree, **DEG**, measure of an angle in the Foundation course.

Enter the angle, say, θ, and Press COS

Example: 45 COS yields .707

COS (θ) is always between -1 and 1.

COS^{-1} is the "inverse" of the COS, 2nd COS

If COS (θ) = N, then **COS^{-1}**(N) **=** θ   N between -1 and 1

Example: COS-1(.707) = 45^o

NOTE: COS 135 **= -.**707 In general, COS (180^o **-** θ) **= -** COS(θ)

You could verify: COS(90 - θ) = **SIN** (θ) for example.

SIN and COS are intimately related as you will learn in the Trigonometry section of Tier 2, and even more in Tier 4.

C15E

COS **COS^{-1}** Answers: [ ]'s

1. **COS** (45°) = ? [**0.707**]
2. **COS** (0°) = ? [**1**]
3. **COS** (10°) = ? [**0.985**]
4. **COS** (30°) = ? [**0.866**]
5. **COS** (60 °) = ? [**0.500**]
6. **COS** (75°) = ? [**0.259**]
7. **COS** (85°) = ? [**0.087**]
8. **COS** (90°) = ? [**0**]
9. **COS** (95°)= ? [**-0.087**]
10. **COS^{-1}** (0.5) = ? [**60 degrees**]
11. What angle X, has **COS** (X) = .4? [**66.4 degrees**]
14. **COS^{-1}**(.4) = ? [**66.4 degrees**]
15. **COS^{-1}**[**SIN**(50°)] = ? [**40 degrees**]

Take the C15 Quiz or do some more exercise, C15ES.

C15ES

COS **COS^{-1}** Answers: [ ]'s

1. **COS** (30^o + 90^o) = ? [**COS**(120^o) = -0.5]
2. **COS** (45^o + 90^o) = ? [**COS**(135^o) = -0.707]
3. **COS** (60^o + 90^o) = ? [**COS**(150^o) = -0.866]
4. **COS** (90^o + 90^o) = ? [**COS**(180^o) = -1]
5. **COS^{-1}** (0.866) = ? [30^o]
6. **COS^{-1}**(0) = ? [90^o]
7. **COS^{-1}**(.5) = ? [60^o]
8. What angle X, has **COS** (X) = .3? [72.5^o]
9. **COS^{-1}**(0.3) = ? [72.5^o]
10. **COS^{-1}** [**COS**(x^o)] = ? [x^o]
11. **COS**[**COS^{-1}**(x) = ? [x]
12. **COS** (θ) is always between? [-1 and 1]
13. **COS^{-1}**(1.5) = ? [Error]

Take the C15 Quiz or review.

C16 LESSON: TAN $\quad$ TAN^{-1}

These two keys are used to compute the Tangent of an angle, and the angle, if you know its **TAN**

This is used in Trigonometry.

We will always use the Degree, **DEG**, measure of an angle in the Foundation course.

Enter the angle, say, θ, and Press <u>TAN</u>

Example: 45 <u>TAN</u> yields 1

<u>TAN</u> (θ) can be any size

<u>TAN^{-1}</u> is the "**inverse**" of the <u>TAN</u>, <u>**2nd**</u> <u>TAN</u>

If <u>TAN</u> (θ) = N, then TAN^{-1}(N) = θ

Example: <u>TAN^{-1}</u>(1) = 45°

NOTE: We will not use <u>TAN</u> in the Foundation Course.

<u>TAN</u> is also intimately related to **SIN** and **COS**.

C16E

TAN **TAN^{-1}** Answers: [ ]'s

1. **TAN** (45°) = ? [**1**]
2. **TAN** (0°) = ? [**0**]
3. **TAN** (10°) = ? [**0.176**]
4. **TAN** (30°) = ? [**0.577**]
5. **TAN** (60°) = ? [**1.732**]
6. **TAN** (75°) = ? [**3.732**]
7. **TAN** (85°) = ? [**11.43**]
8. **TAN** (90°) = ? [**Error**]
9. **TAN** (95°) = ? [**-11.430**]
10. **TAN^{-1}** (0.05) = ? [**26.57 degrees**]
11. What angle X, has **TAN** (X) = 0.4? [**21.8°**]
12. **TAN^{-1}**(0.4) = ? [**21.8 degrees**]
13. **TAN^{-1}[TAN**(50°)] = ? [**50 degrees**]

Take the C16 Quiz or do more exercise, C16ES.

C16ES

TAN **TAN-1** Answers: [ ]'s

1. **TAN** (90°) = ? [**Error**]
2. **TAN** (89.99°) = ? [**5730**]
3. **TAN** (-89.99°) = ? [**-5730**]
4. **TAN** (88°) = ? [**29**]
5. **TAN** (80 °) = ? [**6**]
6. **TAN** (60°) = ? [**2**]
7. **TAN** (30°) = ? [**1**]
8. **TAN** (10°) = ? [**0.176**]
9. **TAN**$^{-1}$ (0.577) = ? [**30°**]
10. What angle X, has **TAN** (X) = 1 ? [**45°**]
11. **TAN**$^{-1}$(1) = ? [**45°**]
12. **TAN**$^{-1}$[**TAN**(150°)] = ? [**-30°**]
13. **TAN**$^{-1}$[**TAN**(-30°)] = ? [**-30°**]

Take the C16 Quiz or review.

A9 LESSON: (1) SIN X^O = A, $-1 \le A \le 1$, OR (2) $SIN^{-1}X = A^O$, $0 \le A^O \le 180^O$

NOTE: Contrary to the audio, you cannot defer this lesson.

Two easy equations. (Apply correct **Inverse** to both sides)

Note: **X** is **angle** measured degrees (°) in the first equation
A is **angle** measured in degrees (°) in the second equation

Note: You don't need to even know what **SIN** means to solve the equation using the calculator.

Example: **SIN** $X^o = .548$ — Apply **SIN^{-1}** to both sides
$X^o = SIN^{-1}(.548) = 33.2^0$ — **Note:** <u>2nd</u> <u>**SIN**</u> yields SIN^{-1}

Example: **SIN X** = .8765 — **X** = 61.2° [**X** is in °]

Example: **SIN^{-1}X** = 28^0 — Apply **SIN** to both sides and get **X** = **SIN**(**SIN^{-1}X**) = **SIN**(28°) = .469

Example: $(.75 + \textbf{COS}49^o)\textbf{SIN}^{-1}\textbf{X} = (14.23 + \textbf{SIN}35^o)^2$
(Looks bad, but is really easy. Just do the numbers first.)

COS49° = .656; **so** .75 + .656 = 1.41 **and**

SIN35° = .574; **so** $(14.23 + .574)^2 = 219$ and so we get

$1.41\textbf{SIN}^{-1}\textbf{X} = 219$, **or** $\textbf{SIN}^{-1}\textbf{X} = 219/1.41 = 155^o$

Thus, **X** = **SIN** $155^0 = .416$

Check: $1.41\text{x}\textbf{SIN}^{-1}.416 = 1.41\text{x}24.6 = 34.7$, not 219.

Something wrong. Must wait until **Trig Lesson T2** to understand.

Preview hint: **SIN**155° = **SIN** 25^0

A9 (1) SIN Xo = A, $-1 \le A \le 1$, or (2) SIN-1X = A^o ,$0 \le A^o \le 180^o$

Two easy equations. (Apply correct **Inverse** to both sides)

Note: **X** is **angle** measured degrees (^{o}) in the first equation

A is **angle** measured in degrees (^{o}) in the second equation

Note: You don't need to even know what **SIN** means to solve the equation using the calculator.

Example: **SIN** $X^o = .548$	Apply **SIN**$^{-1}$ to both sides
$X^o = \mathbf{SIN}^{-1}(.548) = 33.2^0$	**Note:** <u>2nd</u> <u>**SIN**</u> yields **SIN**$^{-1}$
Example: **SIN X** = .8765	**X** = 61.2^o [**X** is in ^{o}]
Example: **SIN**$^{-1}$**X** = 28^0	Apply **SIN** to both sides and get **X** = **SIN**(**SIN**$^{-1}$**X**) = **SIN**(28^o) = .469

Example: (.75 + **COS**49^o)**SIN**$^{-1}$**X** = (14.23 + SIN35^o)^2

(Looks bad, but is really easy. Just do the numbers first.)

COS49^o = .656 **so** .75 + .656 = 1.41 **and**

SIN35^o = .574 **so** (14.23 + .574)^2 = 219 and so we get

1.41**SIN**$^{-1}$X = 219, or **SIN**$^{-1}$X = 219/1.41 = 155^o

Thus, **X** = **SIN** 155^0 = .416

Check: 1.41xSIN^{-1}.416 = 1.41x24.6 = 34.7, not 219.

Something wrong. Must wait until Trig **Lesson T2** to understand.

Preview hint: **SIN**155^o = **SIN** 25^o

A9E

(1) $\text{SIN } X^o = A, -1 \le A \le 1$, or

(2) $\text{SIN}^{-1}X = A^o, 0 \le A^o \le 180^o$

Two easy equations. (Apply correct **Inverse** to both sides)

Note: X is angle measured degrees (°) in the first equation
A is **angle** measured in degrees (°) in the second equation

Solve for X, the Unknown. **Note:** The Algebra is easy. The arithmetic can be complicated, but is easy with the **TI-30XA**.

1. $\text{SIN } X^o = 0.548$
2. $\text{SIN } X^o = 0.8765$,
3. $\text{SIN}^{-1}X = 28^o$
4. $2.3\text{SIN } X^o = 1.92$
5. $\text{SIN } X^o = 1.5$
6. $\text{SIN}^{-1}(0.8765) = X^o$
7. $\text{SIN}^{-1}(\text{SIN}(56^o) = X$
8. $\text{SIN}(\text{SIN}^{-1}(0.321) = X$
9. $\text{SIN}^{-1}(X^2) = 15^o$
10. $\text{SIN}(3X^o) = 0.5$

A9EA

(1) SIN X^o = A, -1 ≤ A ≤ 1, or

(2) $SIN^{-1}X$ = A^o ,0 ≤A^o≤ 180^o

Answers: [ ]

Two easy equations. (Apply correct **Inverse** to both sides)

Note: **X** is angle measured degrees (°) in the first equation
A is **angle** measured in degrees (°) in the second equation

Solve for **X**, the **Unknown**. Note; The Algebra is easy. The arithmetic can be complicated, but easy with the **TI-30XA.**

1.	**SIN X^o** = 0.548	[**33.23^o**]
2.	**SIN X^o** = 0.8765	[**61.22^o**]
3.	**$SIN^{-1}X$** = 28^0	[**0.4695**]
4.	2.3**SIN X^o** = 1.92	[**56.6^o**]
5.	**SIN X^o** = 1.5	[**No Solution, Impossible**]
6.	**SIN^{-1}**(0.8765) = **X^o**	[**61.22^o**]
7.	**SIN^{-1}**(**SIN**(56^o) = **X**	[**56^o**]
8.	**SIN**(**SIN^{-1}**(0.321) = **X**	[**0.321**]
9.	**SIN^{-1}**(**X^2**) = 15^o	[**0.5087**]
10.	**SIN**(3**X^o**) = 0.5	[**10^o**]

A9ES

(1) SIN $X^o = A, -1 \le A \le 1$, or

(2) $SIN^{-1}X = A^o, 0 \le A^o \le 180^o$

Answers: [ ]

1. SIN X° = 0.765 [**X = 49.9°**]
2. SIN X° = 0.278 [**X = 16.14°**]
3. $SIN^{-1}(0.254) = X°$ [**X = 14.71°**]
4. $SIN^{-1}(X) = 45°$ [**X = 0.707**]
5. SIN X° = 2.89 [**NO Solution**]
6. $SIN(SIN^{-1}(0.5)) = X$ [**X = 0.5**]
7. SIN(125°) = X [**X = 0.8191**]
8. 64SIN(X°) = 38.99 [**X = 37.53°**]
9. $SIN(SIN^{-1}(0.75)) = X$ [**X = 0.75**]
10. $SIN^{-1}(COS(60°)) = X°$ [**X = 30°**]
11. $SIN(X^{\circ 2}) = 0.171$ [**X = ± 3.14°**]
12. $SIN^{-1}(COS(115))=X$ [**X = -25**]

A10 LESSON: (1) COS X^o = A, $-1 \le A \le 1$, OR (2) $COS^{-1}X = A^o$, $0 \le A \le 180^o$

Two easy equations. (Apply **Inverse** to both sides)

Note: X is **angle** measured degrees (^{o}) first equation and
A is **angle** measured in degrees (^{o}) in second equation

Note: You don't need to even know what **COS** means to solve the equation using the calculator.

Example: **COS** $X^o = .548$ Apply $\mathbf{COS^{-1}}$ to both sides

$X^o = COS^{-1}(.548) = 56.7^o$ [X was understood to be in ^{o}] **Note**: <u>2nd</u> <u>COS</u> yields $\mathbf{COS^{-1}}$

Example: $COS^{-1}X = 28^o$ Apply **COS** to both sides

$X = COS(COS^{-1}X) = COS(28^o) = .883$

Example: $(.75 + COS49^o)COS^{-1}X = (14.23 + SIN35^o)^2$

(Looks bad, but is really easy. Just do the numbers first.)

$COS49^o = .656$ **so** $.75 + .656 = 1.41$ **and**
$SIN35^o = .574$ **so** $(14.23 + .574)^2 = 219$

So we have: $1.41COS^{-1}X = 219$ **or** $COS^{-1}X = 219/1.41 = 155$

Thus: $X = COS155^0 = -.906$

Check: $1.41xCOS^{-1}(-.906) = 1.41x155 = 219$

Note: We didn't have the same problem we had with the **SIN**. Why not? Have to wait until **Trig Lesson T3** for explanation.

A10E

(1) $\cos X^o = A$, $-1 \le A \le 1$, or

(2) $\cos^{-1}X = A^o$, $0 \le A \le 180^o$

Two easy equations. (Apply **Inverse** to both sides)

Note: X is **angle** measured degrees (o) first equation and

A is angle measured in degrees (o) in second equation

Solve for **X**, the **Unknown**. **Note**: The Algebra is easy. The arithmetic can be complicated, but easy with the **TI-30XA.**

1. $\cos X^o = 0.548$
2. $\cos^{-1}X = 28^o$
3. $\cos X^o = 0.982$
4. $\cos X^o = \sin 79^o$
5. $\cos^{-1}X = \sin^{-1}(0.435)$
6. $4\cos(3X^o) = 2.56$
7. $2.3\cos^{-1}(\sin X^o) = 45^o$
8. $(0.75 + \cos 49^o)\cos^{-1}X = (14.23 + \sin 35^o)^2$
9. $\sin^{-1}(\sin(125^o) = X^o$
10. $\cos^{-1}(\cos(125^o) = X^o$

A10EA

(1) COS X^O = A, -1 ≤ A ≤ 1, OR

(2) $COS^{-1}X = A^O$, 0≤A≤180^O

Answers: []

Two easy equations. (Apply Inverse to both sides)

Note: **X** is **angle** measured degrees (º) first equation and
A is angle measured in degrees (º) in second equation

Solve for **X**, the **Unknown.** **Note**: The Algebra is easy. The arithmetic can be complicated, but easy with the **TI-30XA.**

1. **COS X^o** = 0.548 [**56.8º**]

2. **COS^{-1}X** = 28º [**0.8829**]

3. **COS X^o** = 0.982 [**10.9º**]

4. **COS X^o** = **SIN** 79º [**11º**]

5. **COS^{-1}**X = **SIN^{-1}**(0.435) [**0.9004**]

6. 4**COS**(3**X**º) = 2.56 [**16.7º**]

7. 2.3**COS^{-1}**(**SIN X**º) = 45º [**70.4º**]

8. (0.75 + **COS**49º)**COS^{-1}X** = (14.23 + **SIN**35º)^2 [**-0.9125**]

9. **SIN^{-1}**(**SIN**(125º) = **X**º [**55º**]

10. **COS^{-1}**(**COS**(125º)= Xº [**125º**]

A10ES

(1) $\cos X^{O} = A, -1 \le A \le 1$, OR

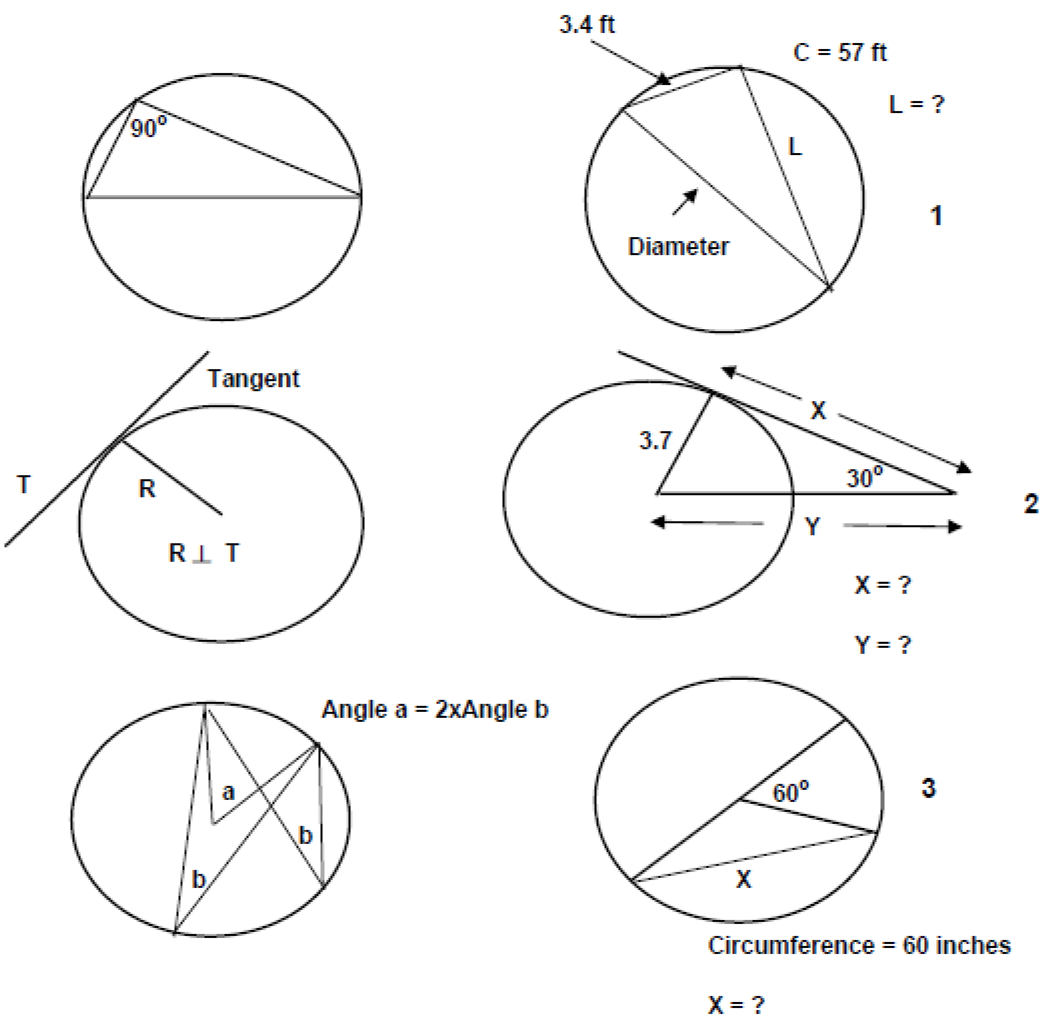
(2) $\cos^{-1} X = A^{O}, 0 \le A \le 180^{O}$

Answers: []

1. $\cos X° = 0.267$ [X = 74.5°]
2. $\cos X° = 0.6565$ [X = 48.97°]
3. $\cos^{-1}(0.125) = X°$ [X = 82.82°]
4. $\cos^{-1}(X) = 45°$ [X = 0.707]
5. $\cos X° = -0.725$ [X = 136.47°]
6. $\cos X° = -1.76$ [NO solution]
7. $-3.75\cos(11°) = X$ [X = -3.681]
8. $\cos^{-1}(X) = 115°$ [X = -0.4226]
9. $\cos^{-1}(\sin(48°)) = X°$ [X = 42°]
10. $\cos(3X°) = -0.49$ [X = 39.78°]
11. $\cos^{-1}(X/3) = 75°$ [X = 0.7765]
12. $\sin(16.5°)\cos(X°) = 0.119$ [X = 65.23°]

There are three facts about **circles** that I find useful sometimes in a practical problem.

I will present them to you with examples below:



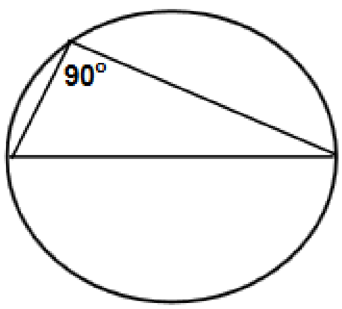
Answers: 1. 17.8 2. Y = 7.4, X = 6.4 3. X = 16.5

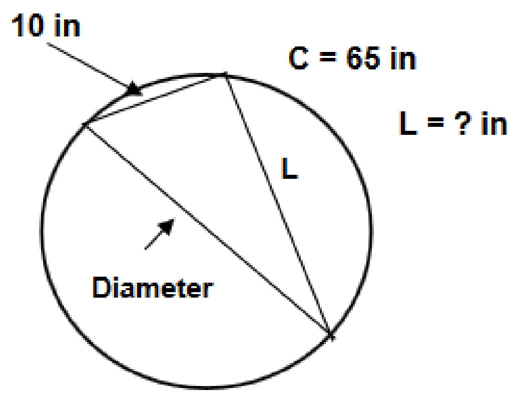
G12E

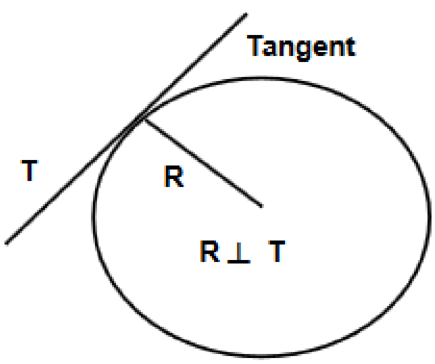
CIRCLES   SPECIAL PROPERTIES

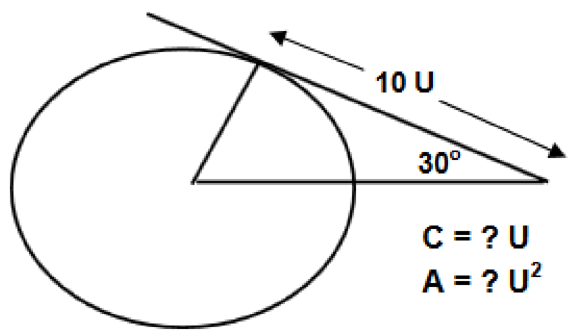
There are three facts about **circles** that I find useful sometimes in a practical problem.

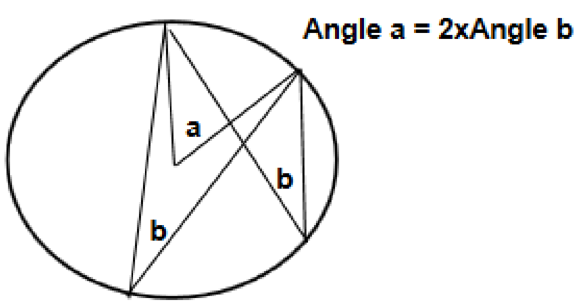
Find the Unknowns.

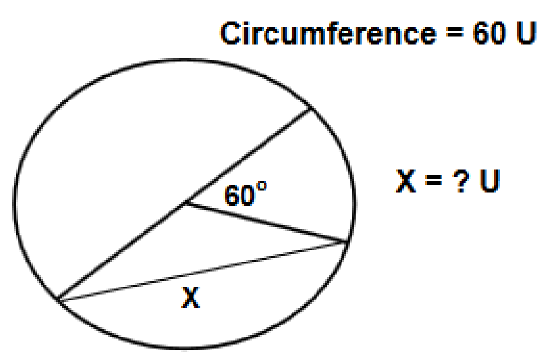










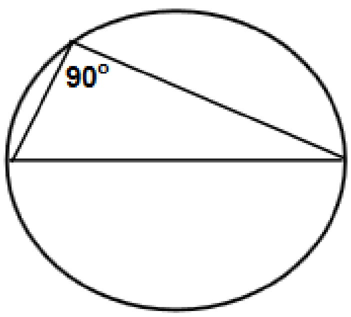


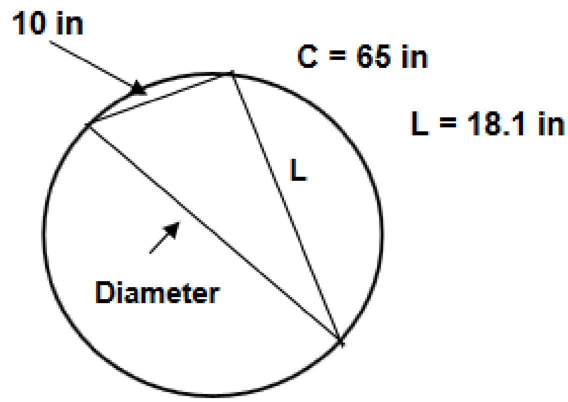
G12EA

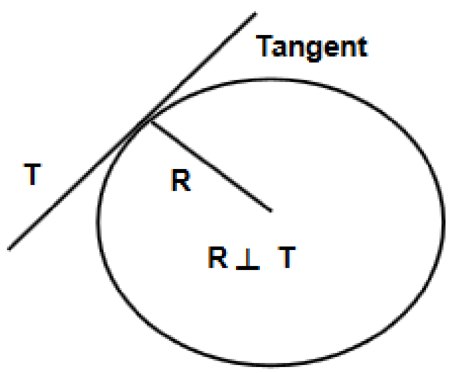
CIRCLES SPECIAL PROPERTIES

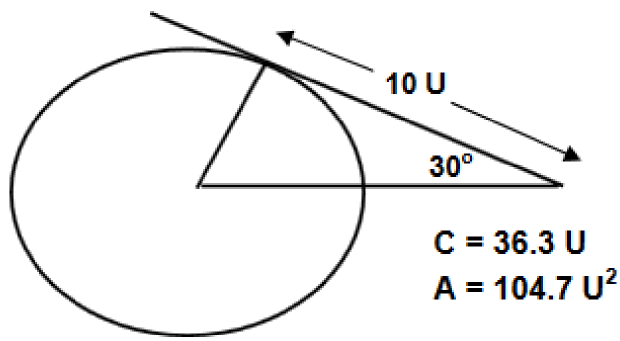
There are three facts about **circles** that I find useful sometimes in a practical problem.

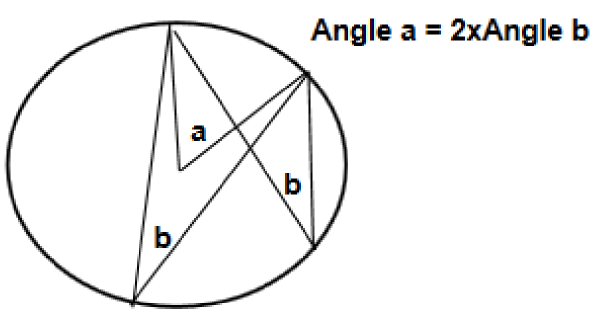
Find the Unknowns.

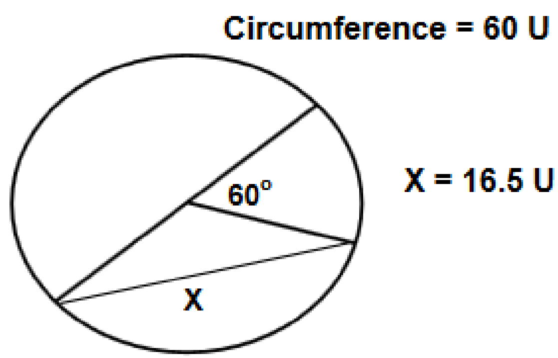










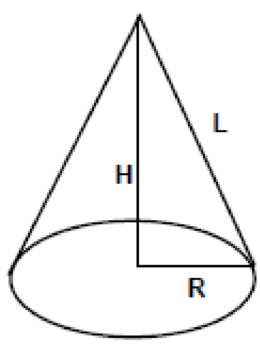


G14 LESSON: SURFACE AREAS CONES

If a **Cone** has **Radius, R**, for its **Base** and has **Height, H,** and **Length, L**, then its **Surface Area** consist of the area of the **Base** plus its **Lateral Area**.

Base Area = πR^2 and **Lateral Area** = $\pi RL = \pi R\sqrt{R^2 + H^2}$

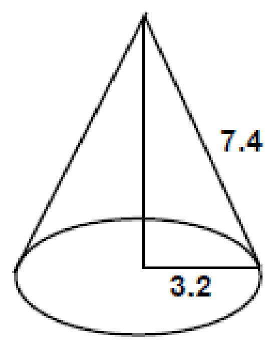
Total Area = $\pi R(R + L)$ measured in U^2



Base Area = πR^2

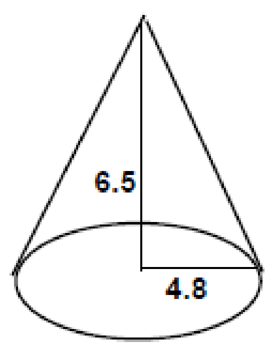
Lateral Area = πRL

Total Area = $\pi R(R + L)$



1

Find Base, Lateral, Total Areas



2

Find Base, Lateral, Total Areas

Answers: 1. 32.2, 74.4, 106.6 U2 **2. 72.4, 121.8, 194.2 U^2**

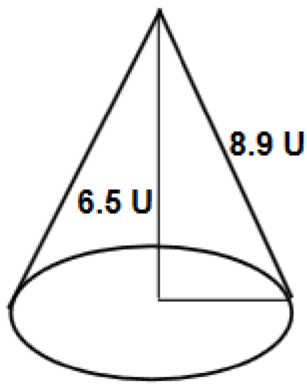
G14E

SURFACE AREAS CONES

Find the **Base Area, BA**

Find the **Lateral Area, LA**

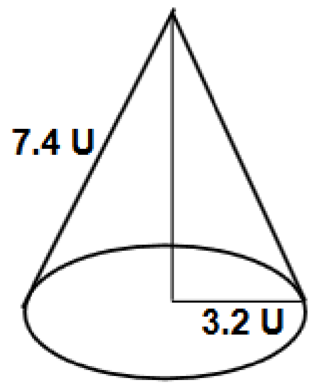
Find the **Total Area, TA**



BA = ? U^2

LA = ? U^2

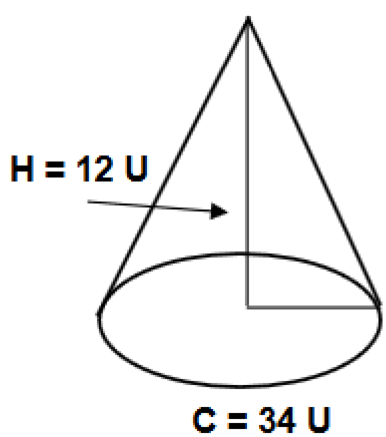
TA = ? U^2



BA = ? U^2

LA = ? U^2

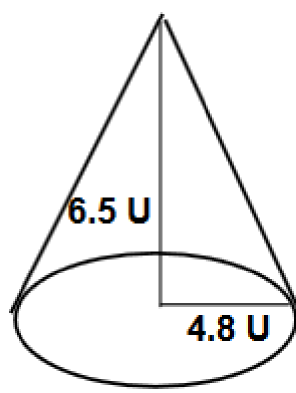
TA = ? U^2



BA = ? U^2

LA = ? U^2

TA = ? U^2



BA = ? U^2

LA = ? U^2

TA = ? U^2

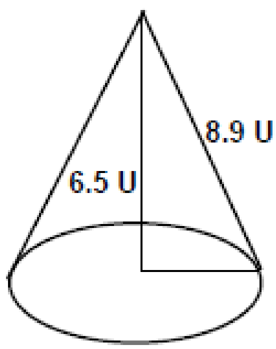
G14EA

SURFACE AREAS   CONES

Find the **Base Area, BA**

Find the **Lateral Area, LA**

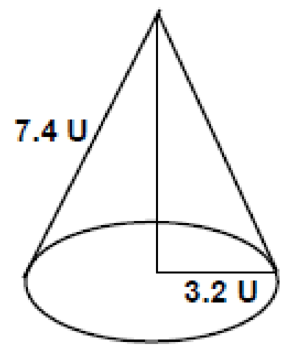
Find the **Total Area, TA**



BA = 116 U^2

LA = 170 U^2

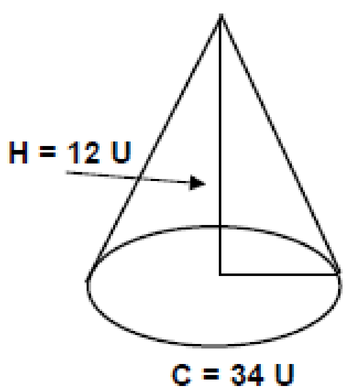
TA = 286 U^2



BA = 32.2 U^2

LA = 74.4 U^2

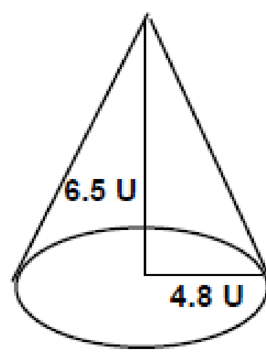
TA = 107 U^2



BA = 92 U^2

LA = 224 U^2

TA = 316 U^2



BA = 72.4 U^2

LA = 121.8 U^2

TA = 194 U^2

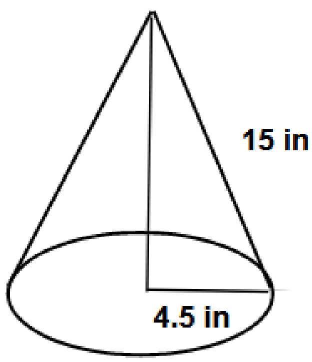
G14ES

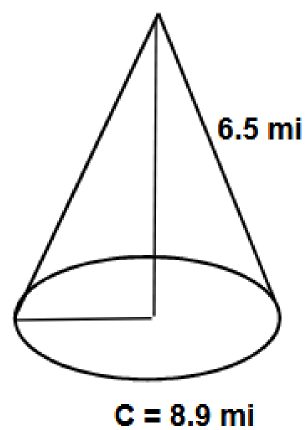
SURFACE AREAS CONES

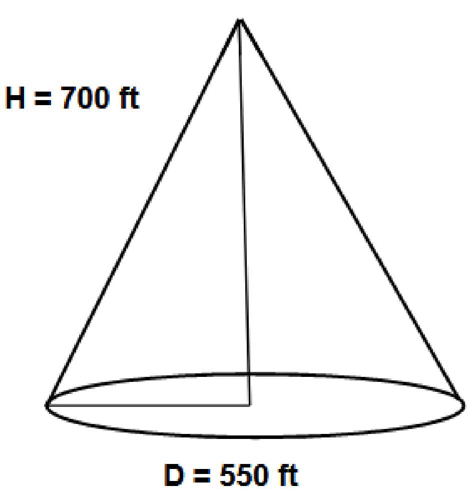
Find the **Base Area, BA**

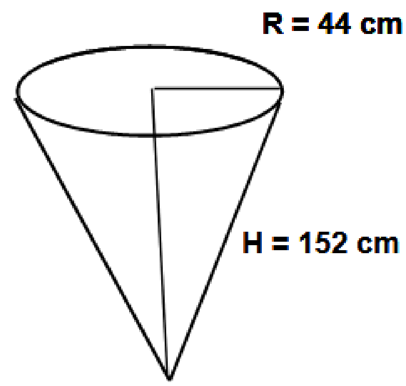
Find the **Lateral Area, LA**

Find the **Total Area, TA**









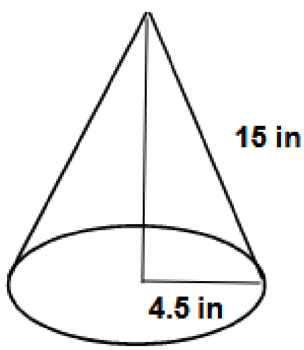
G14ESA

SURFACE AREAS CONES

Find the **Base Area, BA**

Find the **Lateral Area, LA**

Find the **Total Area, TA**



BA = 63.6 in^2

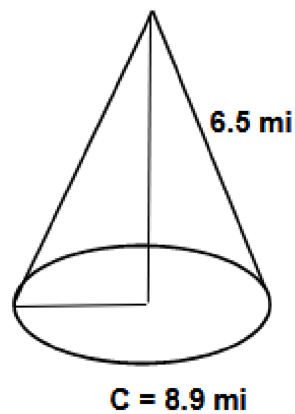
LA= 212.1 in^2

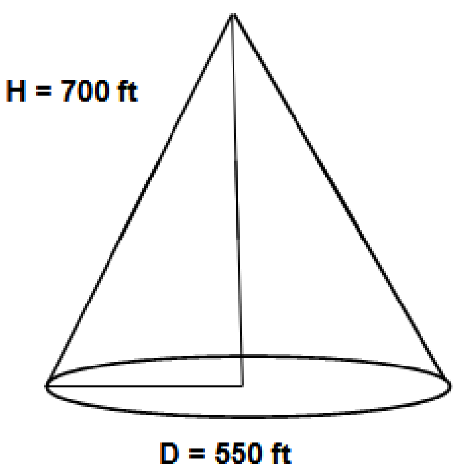
TA = 275.7 in^2

BA = 6.3 mi^2

LA= 28.9 mi^2

TA = 35.2 in^2

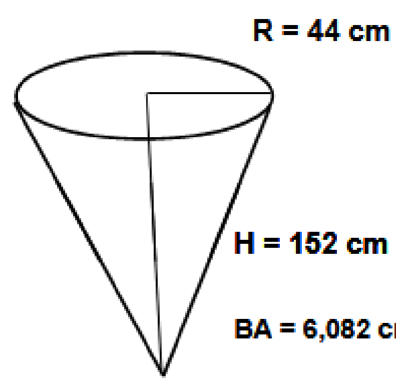




BA = 159,043 ft^2

LA= 635,229 ft^2

TA = 794,272 ft^2



BA = 6,082 cm^2

LA= 21,874 in^2

TA = 27,956 cm^2

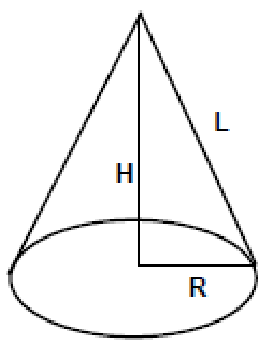
G16 LESSON: VOLUME CONES

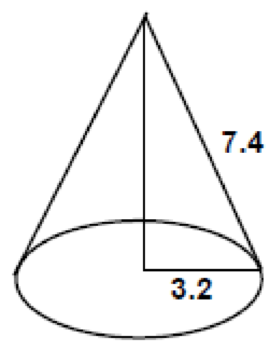
If a **Cone** has **Radius**, **R**, for its **Base** and has **Height**, **H**, and **Length**, **L**, then its **Volume**, **V**, is:

Base Area = πR^2 and **V** = $(1/3)\pi R^2 H = (1/3)\pi R^2\sqrt{L^2 - R^2}$

Volume is measured in **Cubic Units**, U^3, where **U** is a **linear measure.**

For example: cubic inches, in^3



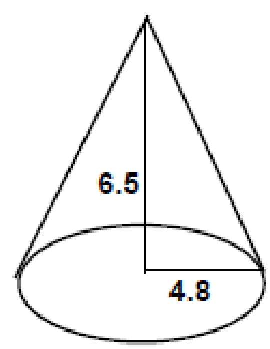


1

Find Volume

Volume = $(1/3)\pi R^2 H$

Volume = $(1/3)\pi R^2\sqrt{L^2 - R^2}$



2

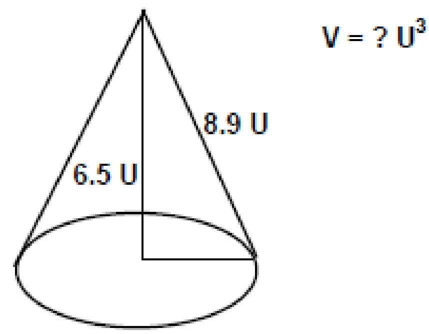
Find Volume

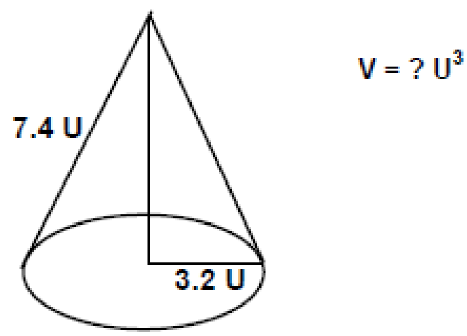
Answers: 1. 71.5 U3 **2. 156.8 cubic units or U^3**

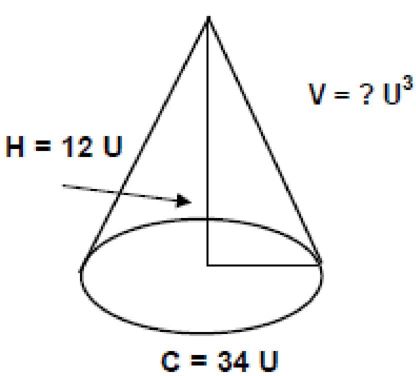
G16E

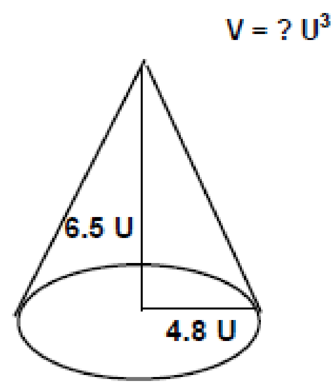
VOLUMES CONES

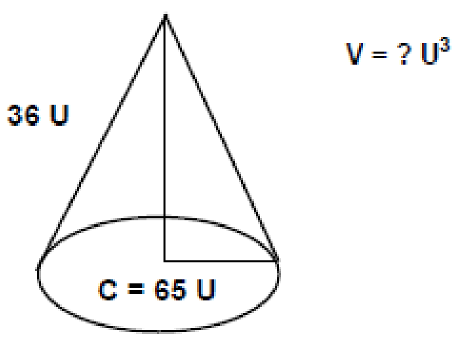
Find the **Volume**, in U^3

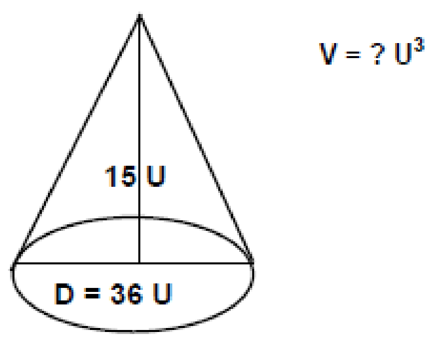








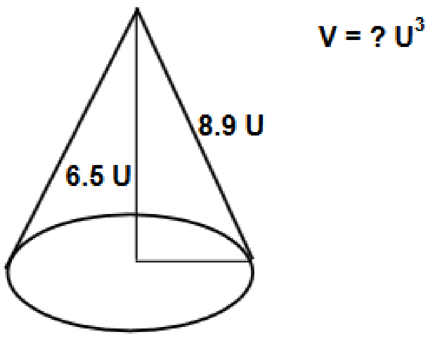


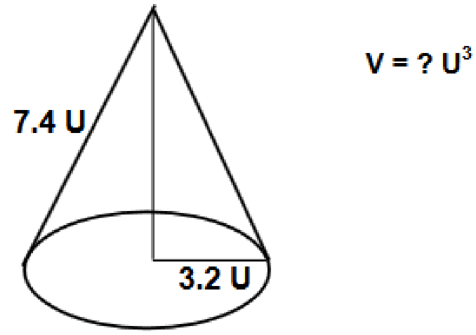


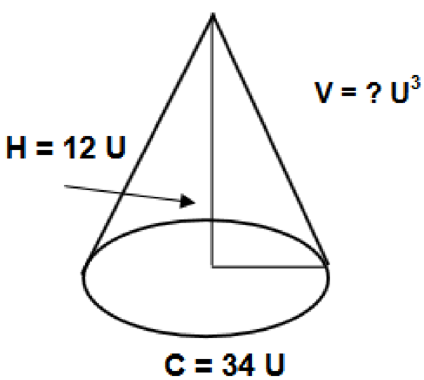
G16EA

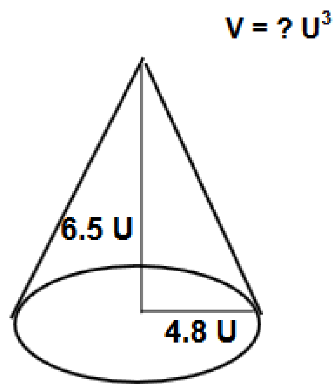
VOLUMES CONES

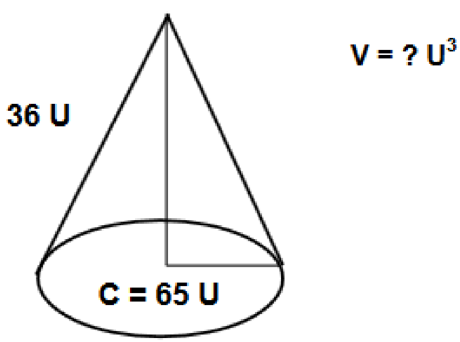
Find the **Volume**, in U^3

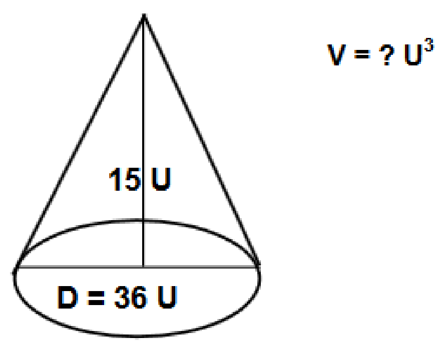






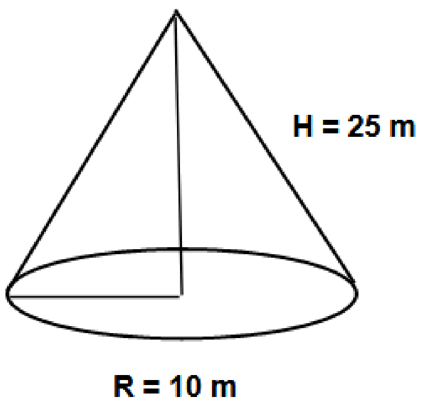


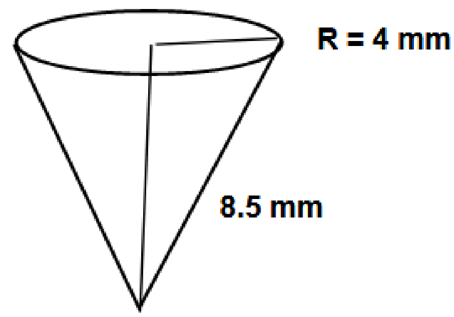


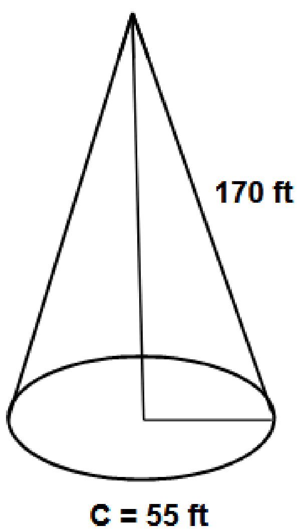


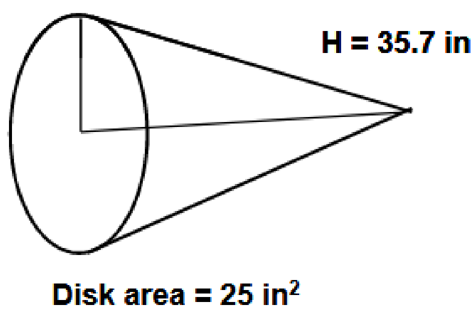
VOLUMES CONES

Find the volume.





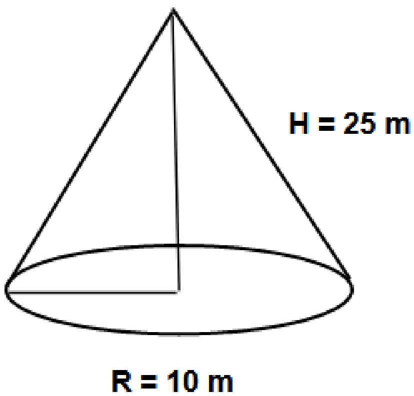




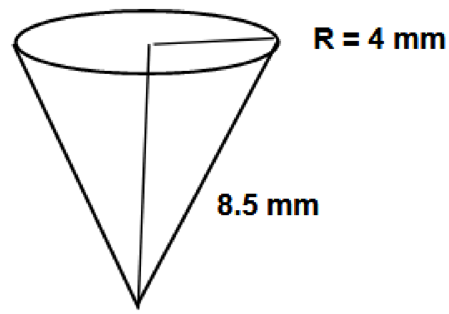
G16ESA

VOLUMES CONES

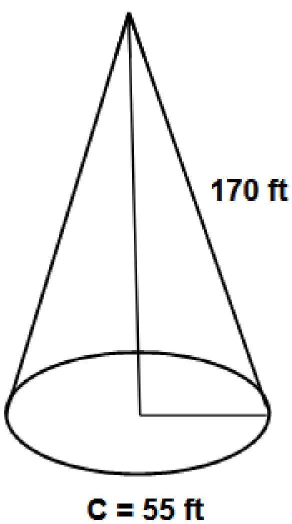
Find the volume.



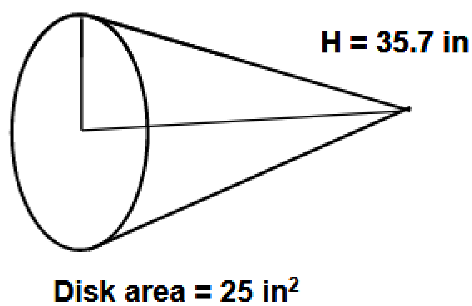
V = 2618 m^3



V = 125.7 mm^3



V = 13,623 ft^3



V = 297.5 in^3

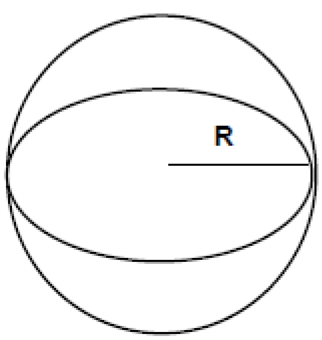
G17 LESSON: SURFACE AREA BALL OR SPHERE

The **Surface Area** of a **Sphere** with **Radius**, **R**, in **Linear Units, U**, is:

$$A = 4\pi R^2 \text{ Square Units, } U^2$$

This is very difficult to prove and we will defer that proof until Tier 4. However, I remember it as follows:

The **Area** of the **circle** of the **cross section** of the **Sphere** through its center is πR^2. I imagine it is rubber and we blow it up like a domed tent. Then its **Area** doubles and that is a **hemisphere** of **Surface Area** $2\pi R^2$. So, the whole **Sphere** is double this, or $4\pi R^2$ U^2.



$$A = 4\pi R^2$$

Problems

R = 5.2 ft	A =	1
R = 150 mi	A =	2
R = .035 cm	A =	3
R = 1 3/4 ft	A =	4

If the Surface Area of a Ball is to be 36 sq. in., what should its Radius be?

$4\pi R^2 = 36$ in^2, then R = $\sqrt{36/4\pi}$ = 1.7 inches

Answers: 1. 340 ft2 2. 282,750 mi^2 3. .0154 cm^2 4. 38.5 ft^2

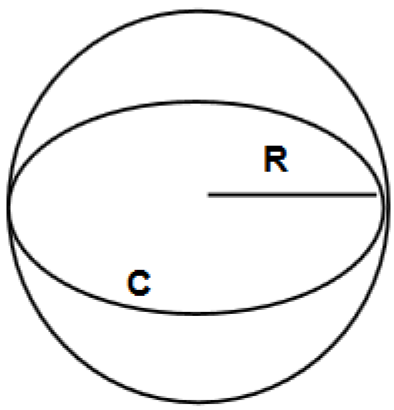
G17E

SURFACE AREA   BALL OR SPHERE

Find the **Surface Area** of the **Spheres** or **Balls**.

What is the formula for the **Surface Area** of a **Sphere** with **Radius** R?

How do you remember it?



R = Radius
D = Diameter
C = Circumference

Exercises

R = 5.2 ft   A = ?

R = 150 mi   A = ?

R = .035 cm   A = ?

R = 1 3/4 ft   A = ?

C = 36 ft   A = ?

C = 120 mi   A = ?

C = 45/8 in   A = ?

D = .025 cm   A = ?

D = 68 in   A = ?

If the Surface Area of a Ball is to be 36 sq. in., what should its Radius be?

G17EA

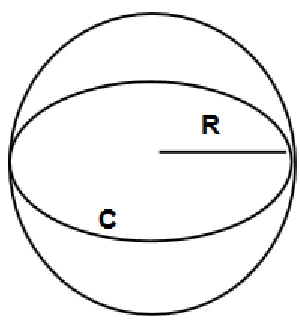
SURFACE AREA BALL OR SPHERE Answers: []

Find the **Surface Area** of the **Spheres** or **Balls**.

What is the formula for the Surface Area of a Sphere with Radius R? [$4\pi R^2$]

What's one way you can remember it? [The Cross Section of the Ball is a circle of Radius R and Area πR^2.]

Now, imagine blowing this up like it's rubber until each point is R from the center. [Turns out the surface area is exactly …thus, Hemisphere area is $2\pi R^2$]



R = Radius
D = Diameter
C = Circumference

Exercises

R = 5.2 ft	A = 340 ft^2
R = 150 mi	A = 282,743 mi^2
R = .035 cm	A = .015 cm^2
R = 1 3/4 ft	A = 38.5 ft^2
C = 36 ft	A = 412.5 ft^2
C = 120 mi	A = 4,584 mi^2
C = 45/8 in	A = 6.8 in^2
D = .025 cm	A =002 cm^2
D = 68 in	A = 14,527 in^2

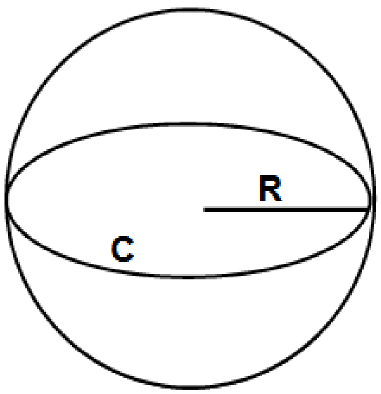
If the Surface Area of a Ball is to be 36 sq. in., what should its Radius be? 1.7 in

G17ES

SURFACE AREA   BALL OR SPHERE

Find the surface area for the spheres with the dimensions below.

1.) Recall the formula for surface area for a sphere, $SA = 4\pi R^2$. If the radious doubled, how much would the SA change? What about if the radius was halved?



2.) R = 35 cm

3.) R = 389 mi

4.) D = 12.6 mm

5.) C = 200,209 km

6.) C = 4π ft

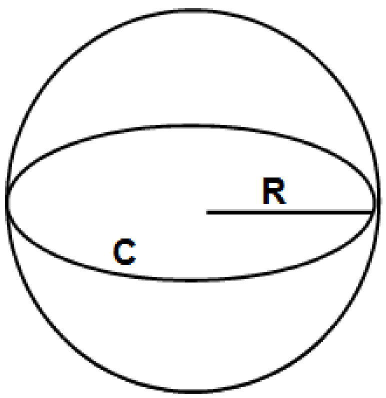
G17ESA

SURFACE AREA BALL OR SPHERE

Find the surface area for the spheres with the dimensions below.

1.) Recall the formula for surface area for a sphere, SA = $4\pi R^2$. If the radious doubled, how much would the SA change? What about if the radius was halved?

Answer: Because the radius is squared, doubling it would cause a 4x increase in surface area. Conversely, halving the radius would result in 4x less surface area.



2.) R = 35 cm
SA = 15,394 cm^2

3.) R = 389 mi
SA = 1,901,556 mi^2

4.) D = 12.6 mm
SA = 498.8 mm^2

5.) C = 200,209 km
SA = 12,759,020,060 km^2

6.) C = 4π ft
SA = 50.3 ft^2

G18 LESSON: VOLUME BALL OR SPHERE ARCHIMEDE TOMBSTONE

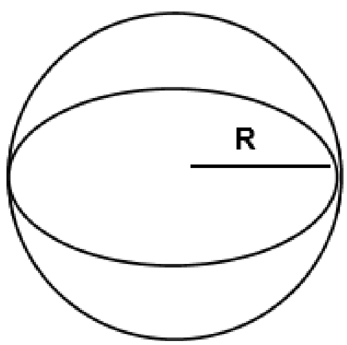
The **Volume** of a **Sphere** with **Radius**, **R**, in **linear units U**, is:

$$V = (4/3)\,\pi R^3 \text{ Cubic Units, } U^3$$

This is very difficult to prove and we will defer that proof until Tier 4. However, I remember it as follows:

> **Archimedes Tombstone:** Imagine a **Sphere** inscribed inside a **Cylinder**. The **Ratio** of the **Volume** or the **Sphere** to the **Volume** of the **Cylinder** is 2:3
>
> The **Cylinder** will have **Base Radius R** and Height **2R**.
> Thus, its **Volume** will be $\pi R^2 x 2R = 2\pi R^3$
> The **Volume** of the **Sphere** is thus, $(2/3)x2\pi R^3 = (4/3)\,\pi R^3$

Note: I say "**triangle**" three times instead of "**tombstone.**"



$A = 4\pi R^2$

$V = (4/3)\pi R^3$

Problems

R = 5.2 ft	V =	1
R = 150 mi	V =	2
R = .035 cm	V =	3
R = 1 3/4 ft	V =	4

If the Volume of a Ball is to be 36 cu. in., what should its Radius be?

$(4/3)\pi R3 = 36$ in3, then $R = \sqrt[3]{36x3/4\pi} = 2.05$ inches

Answers 1. 589 ft^3 2. 14,137,000 mi^3 3. .00018 cm^3 4. 22.4 ft^3

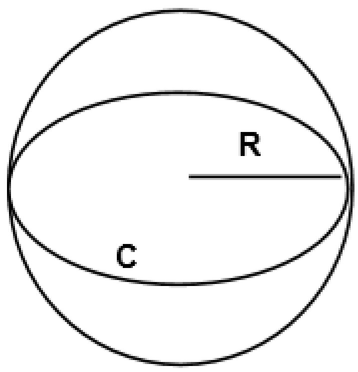
G18E

VOLUME   BALL OR SPHERE

Find the **Volume** of the **Spheres** or **Balls**.

What is the formula for the **Volume** of a **Sphere** with **Radius R**?

What's one way you can remember it?



R = Radius
D = Diameter
C = Circumference

Exercises

R = 5.2 ft   V = ?

R = 150 mi   V = ?

R = .035 cm   V = ?

R = 1 3/4 ft   V = ?

C = 36 ft   V = ?

C = 120 mi   V = ?

C = 4 5/8 in   V = ?

D = .025 cm   V = ?

D = 68 in   V = ?

If the Volume of a Ball is to be 100 cu. in., what should its Radius be?

G18EA

VOLUME BALL OR SPHERE Answers: []

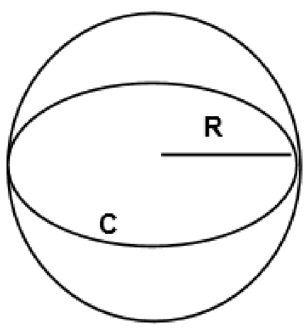
Find the **Volume** of the **Spheres** or **Balls.**

What is the formula for the **Volume** of a **Sphere** with **Radius R**?

[(4/3) πR^3]

What's one way you can remember it?

[Archimedes Tombstone formula whereby the Volume of the Sphere is 2/3 the Volume of a Cylinder the Sphere is inscribed in $(2/3)\times\pi R^2 \times 2R = (4/3)\pi R^3$]



R = Radius
D = Diameter
C = Circumference

Exercises

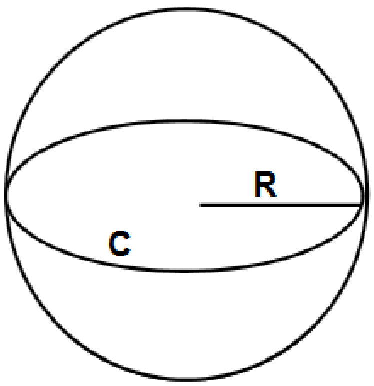
R = 5.2 ft	V = 589 ft^3
R = 150 mi	V = 14,137,167 mi^3
R = .035 cm	V = .00018 cm^3
R = 1 3/4 ft	V = 22.4 ft^3
C = 36 ft	V = 788 ft^3
C = 120 mi	V = 29,181 mi^2
C = 45/8 in	V = 1.67 in^3
D = .025 cm	V = .0000082 cm^3
D = 68 in	V = 164,636 in^3

If the Volume of a Ball is to be 100 cu. in., what should its Radius be? 2.88 in

G18ES

VOLUME BALL OR SPHERE

1.) Recall the formula for volume of a sphere, $V = (4/3)\pi R^3$. If the radious doubled, how much would the V change? What about if the radius was halved?



2.) R = 17 in

3.) R = 2.5 mm

4.) D = 25000 mi

5.) C = 40 km

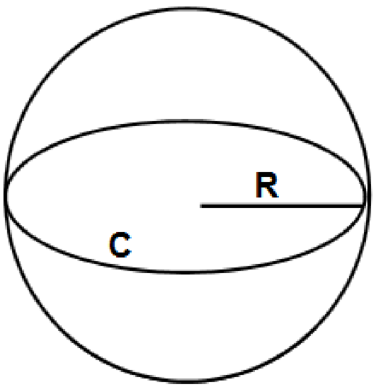
6.) C = 2π

G18ESA

VOLUME BALL OR SPHERE

1.) Recall the formula for volume of a sphere, $V = (4/3)\pi R^3$. If the radious doubled, how much would the V change? What about if the radius was halved?

Answer: Because the radius is cubed, increasing it by a factor of 2 would increase the volume by a factor of 8. Conversely, halving the radius would reduce the volume by a factor of 8.



2.) R = 17 in
$V = 20{,}579.5\ in^3$

3.) R = 2.5 mm
$V = 65.4\ mm^3$

4.) D = 300 mi
$V = 113{,}097{,}336\ mi^3$

5.) C = 40 km
$V = 1{,}039{,}030\ km^3$

6.) $C = 2\pi$ U
$V = (4/3)\pi\ U^3 = 4.19\ U^3$

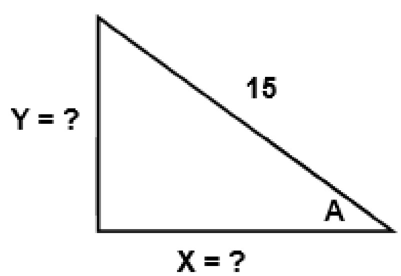
G19 LESSON: WHEN GEOMETRY IS NOT ENOUGH FOR TRIANGLES

We have learned to solve many practical problems using a combination of geometry and algebra. **Triangles** are the most common geometric figure we use in our models.

Yet, there are many practical problems involving **triangles** we still cannot solve with our current knowledge. This Lesson will point out some of these.

That's the "bad news." The "good news" is that we will be able to solve all of these problems using the tools we will learn in the last Section of the Foundation, Trigonometry.

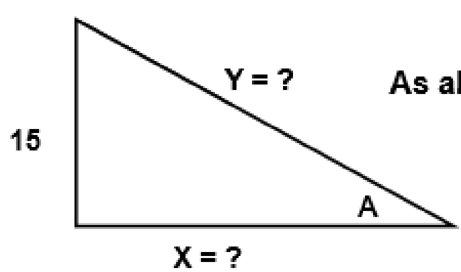
NOTE: See if you can catch the three times I use the word triangle instead of tombstone.



Problem: Find X and Y

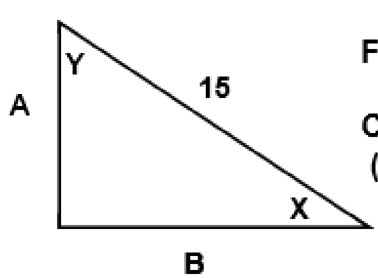
If A = 30^o or 45^o or 60^o we can solve this

With the tool of Trig, we can solve for any angle A.



As above, we can solve for A = 30^o or 45^o or 60^o

Trig will solve for all other angle A's

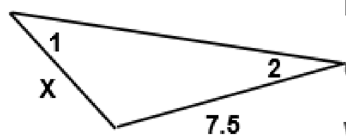


Find Angles X and Y given values of A or B.

Can Solve if A or B equals15 times (1/2) or (√2.2) of (√3/2), not otherwise, so far,

Trig solves for any A or B

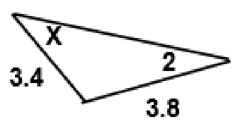
G19 When Geometry is Not Enough Problems



Find X No Similar Triangle available

Currently can only do for special values of 1 and 2

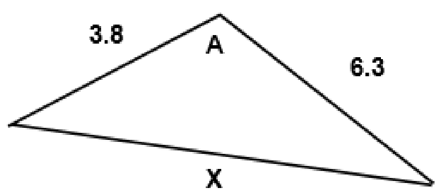
With Trig Tools can do for any angles 1 and 2



Can find X IF we have a similar triangle
with known corresponding sides

With Trig Tools Do Not need the similar triangle

Law of Sines



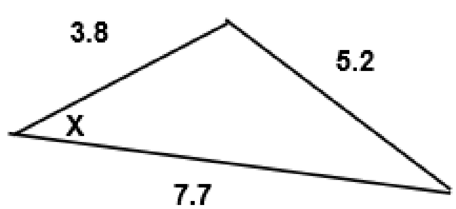
Find X

If A = 90°, OK with Pythagorean Theorem

Trig Tool for any angle A.

Generalized Pythogorean Theorem

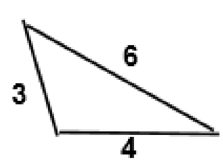
Law of Cosines



Find Angle X

Same Trig Tool as above

Useful in finding area of this triangle



Find the Area of this Triangle

Trigonometry has many profound applications beyond practical math.

G19E

WHEN GEOMETRY IS NOT ENOUGH FOR TRIANGLES

Give four examples of **triangle** "problems" we cannot yet solve with just the geometry and algebra we have learned, but which we will be able to solve with Trig.

This is an Optional Exercise.

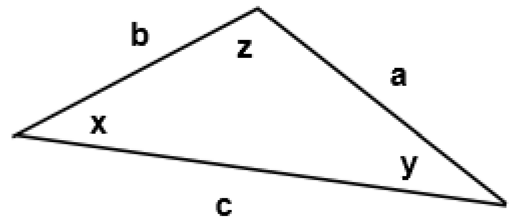
It is designed to help you appreciate the value the powerful Tool of Trigonometry will be for practical problem solving.

Before the scientific calculator was invented, Trig was pretty difficult to learn and apply to practical math.

Now, it is breeze. Aren't Power Tools wonderful?

HINT: Just imagine you know three of the variables below. Then can you find the others? With what you know now?

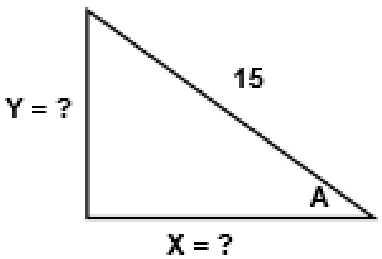
In many cases the answer will be NO. But, with Trig you will be able to solve any solvable triangle problem!



G19EA

WHEN GEOMETRY IS NOT ENOUGH FOR TRIANGLES

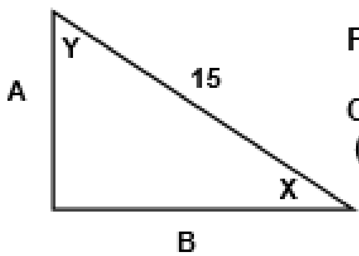
Give four examples of **triangle** "problems" we cannot yet solve with just geometry and algebra we have learned; but, which we will be able to solve with Trig.



Problem: Find X and Y

If A = 30° or 45° or 60° we can solve this now

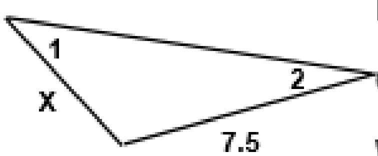
With the tool of Trig, we can solve for any angle A.



Find Angles X and Y given values of A or B.

Can Solve if A or B equals15 times
(1/2) or (√2.2) of (√3/2), not otherwise, so far.

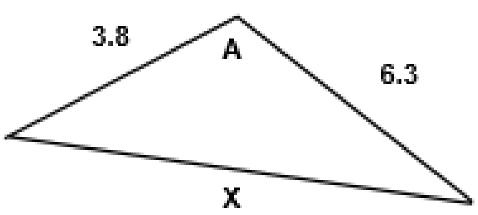
Trig solves for any A or B



Find X No Similar Triangle available

Currently can only do for special values of 1 and 2

With Trig Tools can do for any angles 1 and 2
Law of Sines



Find X or A given the other.

If A = 90°, OK with Pythagorean Theorem

Trig Tool for any angle A.

Generalized Pythogorean Theorem
Law of Cosines

INTRODUCTION TO TRIGONOMETRY

Trigonometry, **Trig**, is the study of triangles.

Trig consists of several powerful tools which will empower you to solve virtually any solvable problem with triangles including the ones discussed in Lesson G19.

It begins with the basic Trig Functions, **SIN**, **COS**, and **TAN**.

These are the "**power tools**" that let us solve problems.

In the old days, there were extensive Trig Tables that were used. It was arduous to learn and apply these tables.

Today, with the power tool of the TI 30XA, we can solve virtually any triangle problem in a matter of minutes or less.

Actually, in some ways Trig is easier than geometry.

We will learn how to use the three Trig Functions, and also, we will learn two very powerful theorems which make these tools even more valuable:

The **Law of Sines** (Lesson T6)

The Generalized **Pythagorean Theorem** commonly called: **The Law of Cosines** (Lesson T7)

Trigonometry then has many extensions into analytical geometry, complex numbers, calculus, and functional analysis which have profound effects in science, engineering and technology.

T1 LESSON: TRIG FUNCTIONS SIN COS TAN

In any **Right Triangle**, there are **Six Ratios** of side lengths. They come in sets of three where one set is just the reciprocal of the other set.

See the triangle below: a/c, b/c, and a/b are one set.

c is called the **Hypotenuse**, or **Hyp**.

b is called the **Adjacent side** (to angle 1), or **Adj**

a is called the **Opposite side** (to angle 1, or **Opp**

So, the Ratios are **Opp**/**Hyp**, **Adj**/**Hyp**, **Opp**/**Adj**

These three ratios are the three **Trig functions of angle 1**.

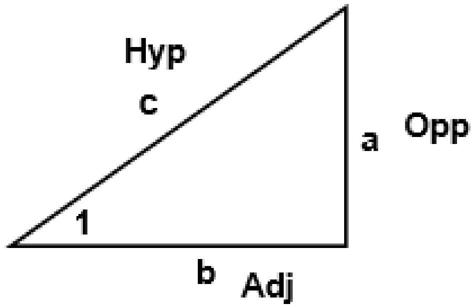
SIN(1) = Opp/Hyp

COS(1) = Adj/Hyp

TAN = Opp/Adj = **SIN**(1)/**COS**(1)

Angle 1 will always be measured in **degrees º** in this Foundation Course.

In advanced applications of Trig, angle 1 is measured in Radians, RAD.



SIN (1) = a/c = Opp/Hyp

COS (1) = b/c = Adj/Hyp

TAN (1) = a/b = Opp/Adj

When turn on the TI 30XA, **DEG** always comes up.

SIN^{-1} COS^{-1} TAN^{-1}

Enter any number between -1 and 1, and find the angle whose **SIN** it is.

Ditto for **COS** and **TAN**. In other words, if

SIN (1) = a, then (1) = **SIN^{-1}**(a)

WARNING: See **T5** for some special information about **SIN^{-1}**

T1 Trig Functions SIN COS TAN Problems

Find **SIN**(1), **COS**(1), **TAN** (1) given **angle** (1) in **degrees** °

Find **Angle** (1), Given **SIN**(1), **COS**(1) using **SIN**$^{-1}$ and **COS**$^{-1}$

Problems:	**Angle 1**	**SIN(1)**	**COS(1)**	**TAN(1)**
in °				
30°	0.5	0.866	0.577	
45°	0.707	0.707	1	
60°	0.866	0.5	1.732	
17°	0.292	0.956	0.306	
38°	0.616	0.788	0.781	
52.7°	0.795	0.606	1.313	
68°	0.927	0.375	2.48	
85°	0.996	0.087	11	
90°	1	0	Error	
100°	0.985	-0.174	-5.68	
115°	0.906	-0.423	-2.15	
135°	0.707	-0.707	-1	
145°	0.574	-0.819	-0.7	
150°	0.5	-0.866	-0.577	
176°	0.07	-0.998	-0.07	

Problems: Find **angle** 1 if

	Angle (1)	
SIN(1) = 0.7865	51.9°	**Note**: **SIN**$^{-1}$(**SIN**(120°) = 60°
SIN(1) = 0.5	30°	
SIN(1) = -0.654	-40.8°	**COS**$^{-1}$(**COS**(120°) = 120°
COS(1) = 0.7865	38.1°	
COS(1) = 0.5	60°	
COS(1) = -0.654	130.8°	
TAN(1) = 0.7865	38.2°	
TAN(1) = 0.5	26.6°	
TAN(1) = -0.654	-33.2	

Note: These problems are repeated on the T1E page as well.

T1E

TRIG FUNCTIONS SIN COS TAN

Find **SIN**(1), **COS**(1), **TAN**(1) given **angle** (1) in degrees ^{o}

Find **Angle** (1), Given **SIN**(1), **COS**(1) using **SIN**$^{-1}$ and **COS**$^{-1}$

Angle 1	**SIN**(1)	**COS**(1)	**TAN**(1)	Evaluate
30^{o}				**SIN**$^{-1}$[**COS**(30^{o})] = ?
45^{o}				**COS**$^{-1}$[**COS**(30^{o})] = ?
60^{o}				**SIN**$^{-1}$[**COS**(120^{o})] = ?
17^{o}				**COS**$^{-1}$[**SIN**(120^{o})] = ?
38^{o}				**COS**$^{-1}$[**SIN**(60^{o})] = ?
52.7^{o}				**COS**$^{-1}$[**SIN**(45^{o})] = ?
68^{o}				**TAN**$^{-1}$[**SIN**(90^{o})] = ?
85^{o}				**SIN**[**COS**$^{-1}$(.5)] = ?
90^{o}				**SIN**[**COS**$^{-1}$(.867)] = ?
100^{o}				**COS**[**SIN**$^{-1}$(.867)] = ?
115^{o}				**SIN**[**COS**$^{-1}$(1)] = ?
135^{o}				**SIN**[**COS**$^{-1}$(0)] = ?
145^{o}				**SIN**[**COS**$^{-1}$.707)] = ?
150^{o}				**TAN**[**SIN**$^{-1}$(.707)] = ?
176^{o}				**TAN**[**COS**$^{-1}$(.707)] = ?

Find **angle** (1) if

Angle (1)

SIN(1) = 0.7865
SIN(1) = 0.5
SIN(1) = -0.654
COS(1) = 0.7865
COS(1) = 0.5
COS(1) = -0.654
TAN(1) = 0.7865
TAN(1) = 0.5
TAN(1) = -0.654

T1EA

TRIG FUNCTIONS SIN COS TAN

Find **SIN**(1), **COS**(1), **TAN**(1) given angle (1) in **degrees** ^{o}

Find **Angle** (1), Given **SIN**(1), **COS**(1) using **SIN**$^{-1}$ and **COS**$^{-1}$

Angle 1	SIN(1)	COS(1)	TAN(1)	Evaluate	
30º	0.5	0.866	0.577	**SIN**$^{-1}$[**COS**(30º)] = ?	60º
45º	0.707	0.707	1	**COS**$^{-1}$[**COS**(30º)]= ?	30º
60º	0.866	0.5	1.732	**SIN**$^{-1}$[**COS**(120º)] = ?	-30º
17º	0.292	0.956	0.306	**COS**$^{-1}$[**SIN**(120º)] = ?	30º
38º	0.616	0.788	0.781	**COS**$^{-1}$[**SIN**(60º)] = ?	30º
52.7º	0.795	0.606	1.313	**COS**$^{-1}$[**SIN**(45º)] = ?	45º
68º	0.927	0.375	2.475	**TAN**$^{-1}$[**SIN**(90º)] = ?	45º
85º	0.996	0.087	11.43	**SIN**[**COS**$^{-1}$(.5)] = ?	0.867
90º	1	0	Error	**SIN**[**COS**$^{-1}$(.867)] = ?	0.5
100º	0.985	-0.174	-5.68	**COS**[**SIN**$^{-1}$(.867)] = ?	0.5
115º	0.906	-0.423	-2.15	**SIN**[**COS**$^{-1}$(1)] = ?	0
135º	0.707	-0.707	-1	**SIN**[**COS**$^{-1}$(0)] = ?	1
145º	0.574	-0.819	-0.7	**SIN**[**COS**$^{-1}$.707)] = ?	0.707
150º	0.5	-0.866	-0.577	**TAN**[**SIN**$^{-1}$(.707)] = ?	1
176º	0.07	-0.998	-0.07	**TAN**[**COS**$^{-1}$(.707)] = ?	1

Find angle (1) if	Angle (1)
SIN(1) = 0.7865	51.9º
SIN(1) = 0.5	30º
SIN(1) = -.654	-40.8º
COS(1) = 0.7865	38.1º
COS(1) = 0.5	60º
COS(1) = -0.654	130.8º
TAN(1) = 0.7865	38.2º
TAN(1) = 0.5	26.6º
TAN(1) = -0.654	-33.2

T1ES

TRIG FUNCTIONS SIN COS TAN

1. $x = 30°$ Find sin(x) , cos(x), and tan(x)
2. $x = 60°$ Find sin(x), cos(x), and tan(x)
3. Find $\cos^{-1}(\sin(60°)) = ?$
4. If $\cos(x) = 0.5$ Find angle x
5. $x = 45°$ Find sin(x) cos(x) and tan(x)
6. $x = 15°$ Find sin(x), cos(x), and tan(x)
7. Find $\sin^{-1}(\cos(30°)) = ?$
8. If $\sin(x) = 0.315$ Find angle x
9. $x = 90°$ Find sin(x), cos(x), and tan(x)
10. $x = 150°$ Find sin(x), cos(x), and tan(x)
11. Find $\sin(\cos^{-1}(0.5)) = ?$
12. If $\tan(x) = 0.425$ Find angle x
13. $x = 117°$ Find sin(x), cos(x), and tan(x)
14. $x = 34.5°$ Find sin(x), cos(x), and tan(x)
15. Find $\sin^{-1}(\tan(17°)) = ?$
16. If $\sin(x) = -0.5$ Find angle x
17. $x = 100°$ Find sin(x), cos(x), and tan(x)
18. $x = 0°$ Find sin(x), cos(x), and tan(x)
19. Find $\tan^{-1}(\cos(70°)) = ?$
20. If $\tan(x) = -0.245$ Find angle x

T1ESA

TRIG FUNCTIONS SIN COS TAN Answers: [ ]

1. x = 30° Find sin(x) , cos(x), and tan(x)

   [**sin(30°) = 0.5, cos(30°) = 0.866, tan(30°) = 0.577**]

2. x = 60° Find sin(x), cos(x), and tan(x)

   [**sin(60°) = 0.866, cos(60°) = 0.5, tan(60°) = 1.732**]

3. Find $\cos^{-1}(\sin(60°))$ = ?

   [**30°**]

4. If cos(x) = 0.5 Find angle x

   [**x = 60°**]

5. x = 45° Find sin(x) cos(x) and tan(x)

   [**sin(45°) = 0.707, cos(45°) = 0.707, tan(45°) = 1**]

6. x = 15° Find sin(x), cos(x), and tan(x)

   [sin(15) = 0.259, cos(15) = 0.966, tan(15) = 0.268]

7. Find $\sin^{-1}(\cos(30°))$ = ?

   [**x = 60°**]

8. If sin(x) = 0.315 Find angle x

   [**x = 18.4°**]

9. x = 90° Find sin(x), cos(x), and tan(x)

   [**sin(90°) = 1, cos(90°) = 0, tan(90°) = undefined**]

10. x = 150° Find sin(x), cos(x), and tan(x)

    [**sin(150°) = 0.5, cos(150°) = -0.866, tan(150°) = -0.577**]

11. Find $\sin(\cos^{-1}(0.5))$ = ?

    [**0.866**]

12. If $\tan(x) = 0.425$ Find angle x

[x = 23.03°]

13. x = 117° Find sin(x), cos(x), and tan(x)

[sin(117°) = 0.891, cos(117°) = -0.454, tan(117°)= -1.96]

14. x = 34.5° Find sin(x), cos(x), and tan(x)

[sin(34.5°) = 0.566, cos(34.5°) = 0.824, tan(34.5°) = 0.687]

15. Find $\sin^{-1}(\tan(17°)) = ?$

[17.8°]

16. If $\sin(x) = -0.5$ Find angle x

[x = 210°, 330° or -30°]

17. x = 100° Find sin(x), cos(x), and tan(x)

[sin(100°) = 0.985, cos(100°) = -0.174, tan(100°) = -5.67]

18. x = 0° Find sin(x), cos(x), and tan(x)

[sin(0°) = 0, cos(0°) = 1, tan(0°) = 0]

19. Find $\tan^{-1}(\cos(70°)) = ?$

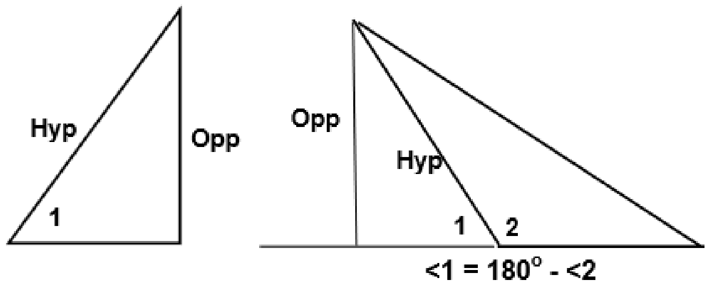
[18.88°]

20. If $\tan(x) = -0.245$ Find angle x

[x = 166.2°, 346.2°, or -13.8°]

T2 LESSON: SIN X SINE OF X X IS AN ANGLE (DEGREES °)

We will extend the definition of **SIN** to include all angles from 0° to 180°. In Tier 3 we will extend the definition to include all angles both positive and negative.



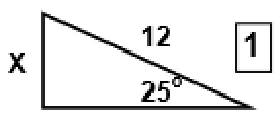
SIN(1) = **Opp/Hyp** **SIN**(2) = **Opp/Hyp** = **SIN**(180° - <2)

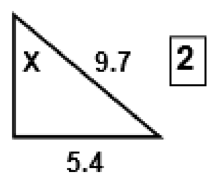
If know two out of three, find the third, **Opp**, **Hyp**, (1)

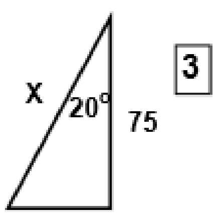
Opp = **SIN**(1)x**Hyp** **Opp** = **SIN**(2)x**Hyp**

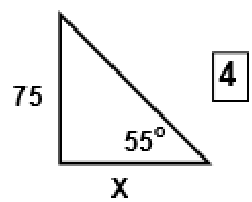
Hyp = **Opp/SIN**(1) **Hyp** = **Opp/SIN**(2)

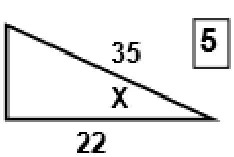
(1) = $\textbf{SIN}^{-1}$(**Opp/Hyp**) (2) = $\textbf{SIN}^{-1}$(**Opp/Hyp**)

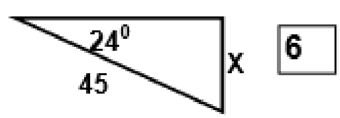












Answers:

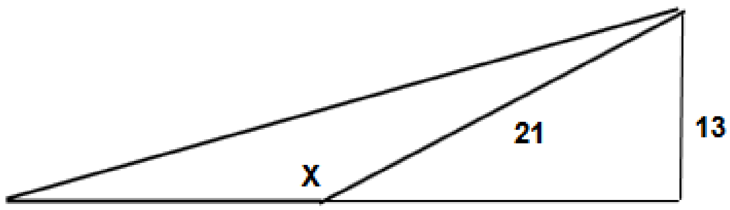
1. 5.1
2. 33.8°
3. 79.8
4. 52.5
5. 51.1°
6. 18.3

T2 SIN Problems

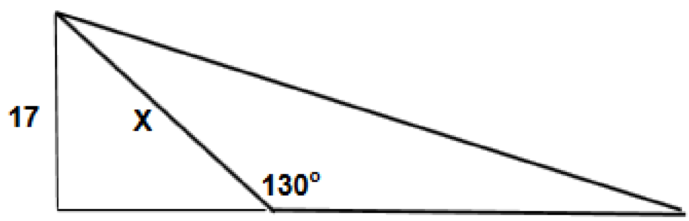
Always set up the Equation first. Then solve.

Use the **Pythagorean Theorem** if necessary.

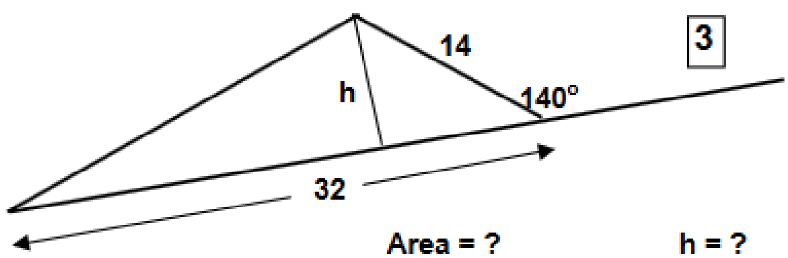
NOTE: Why is **Area** = .5ab**SIN**(<ab) correct?



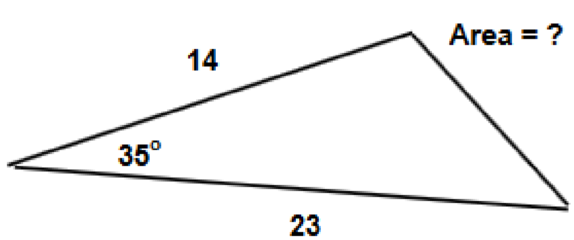
1



2



3



4

Answers:

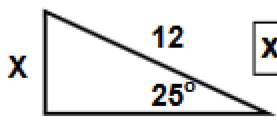
1. 141.8^{o}
2. 22.2
3. A =144, h = 9
4. 92.3

T2E

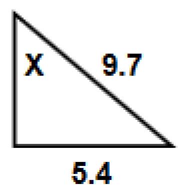
SIN X   SINE OF X

X is an **angle** (**degrees** °)

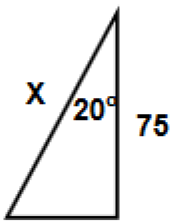
Find **x** in each of the following exercises.



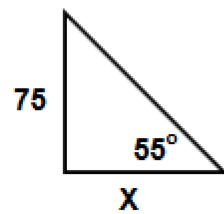
x = ?



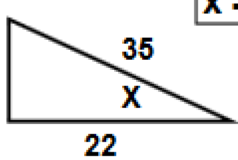
x = ?



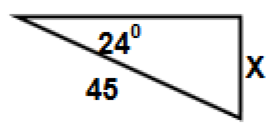
x = ?



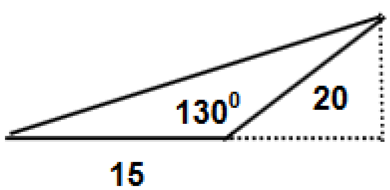
x = ?



x = ?°



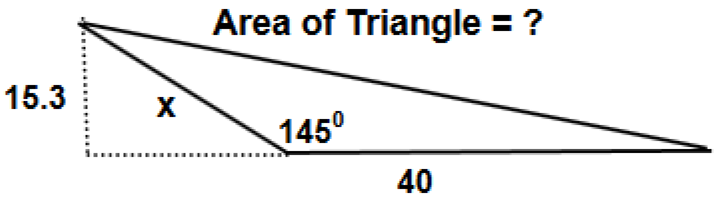
x = ?



X = ?

Area of Triangle = ?

Area of Triangle = ?



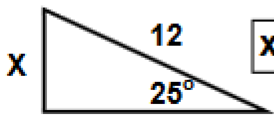
x = ?

T2EA

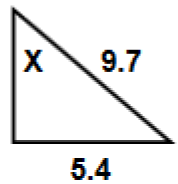
SIN X   SINE OF X

X is an **angle** (**degrees** °)

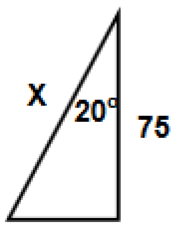
Find **x** in each of the following exercises



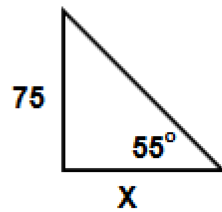
x = 5.1



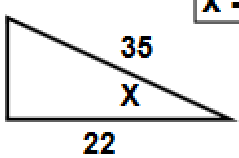
x = 33.8°



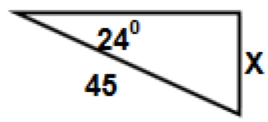
x = 79.8



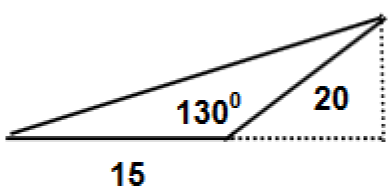
x = 52.5



x = 51°



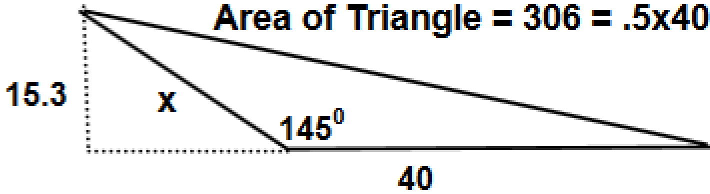
x = 18.3



X = 15.3

Area of Triangle = 115 = .5x15x20xSIN(130°) = .5x15x15.3

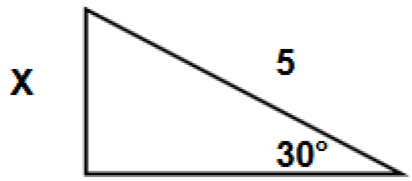
Area of Triangle = 306 = .5x40x15.3 = .5x40x26.7xSIN(145°)

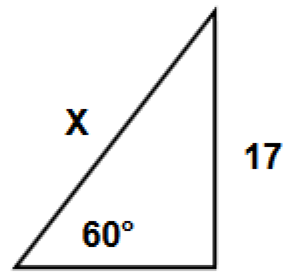


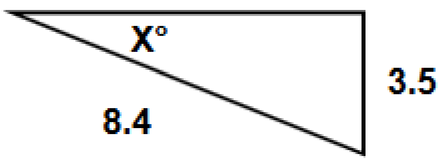
x = 26.7

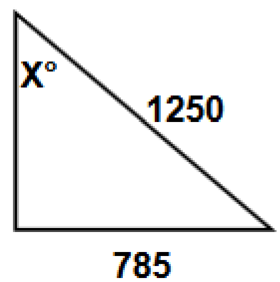
SIN X SINE OF X

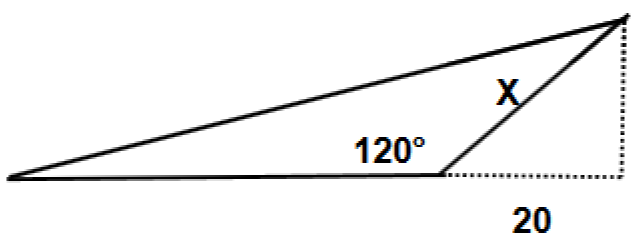
Find X in the following exercises.





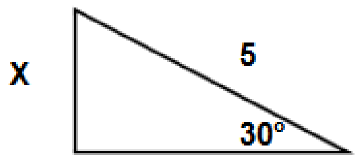




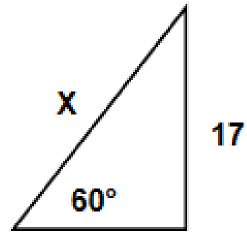


SIN X   SINE OF X

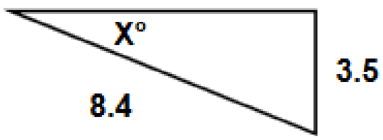
Find X in the following exercises.



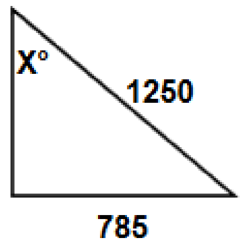
x = 2.5



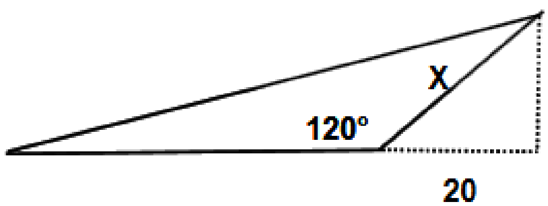
x = 19.6



x = 24.6°



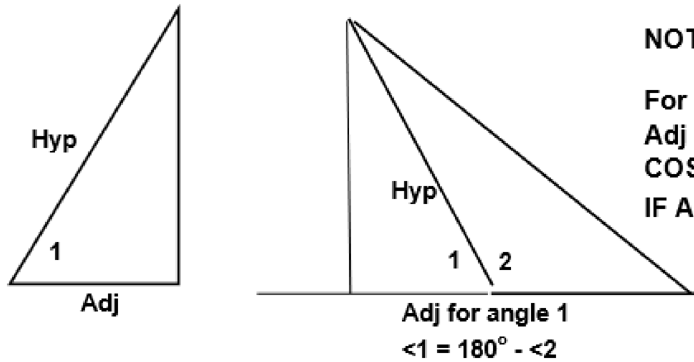
x = 38.9°



x = 40

T3 LESSON: COS X COSINE OF X. X IS AN ANGLE (DEGREES ^{O})

We will extend the definition of **COS** to include all angles from 0^{o} to 180^{o}. In Tier 3 we will extend the definition to include all angles both positive and negative.



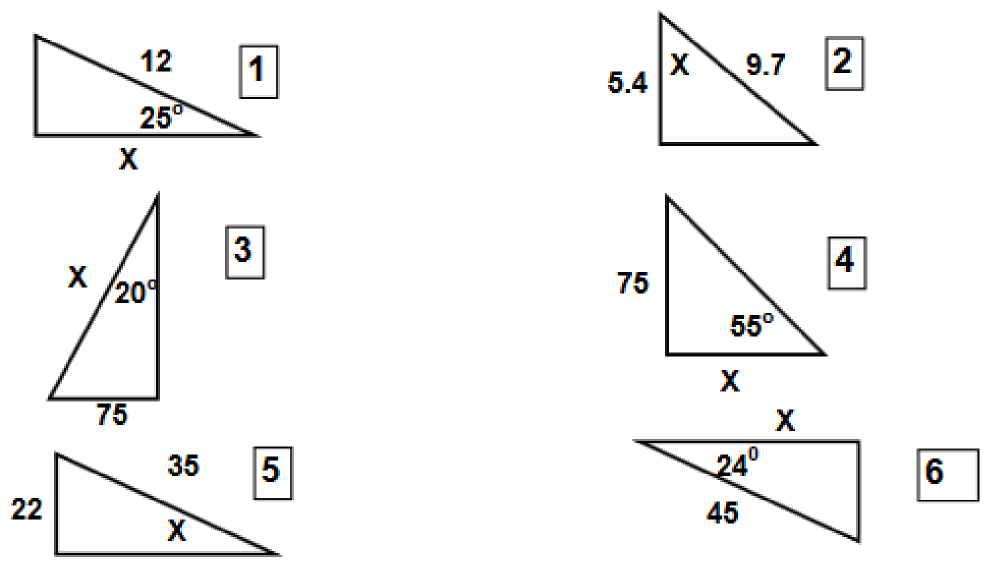
NOTICE

For COS only
Adj is NEGATIVE
COS(2)< 0
IF Angle (2) >90°

COS(1) = **Adj/Hyp** **COS**(2) = **Adj/Hyp** **COS**(2) = -**COS**(180° - <1)

If you know two out of three, find the third, **Opp**, **Hyp**, (**1**)

Adj = **COS**(1)x**Hyp**	**Adj** = **COS**(2)x**Hyp**
Hyp = **Adj/COS**(1)	**Hyp** = **Adj/COS**(2)
(**1**) = **COS**$^{-1}$(**Adj/Hyp**)	(2) = **COS**$^{-1}$(**Adj/Hyp**) **Adj** < 0



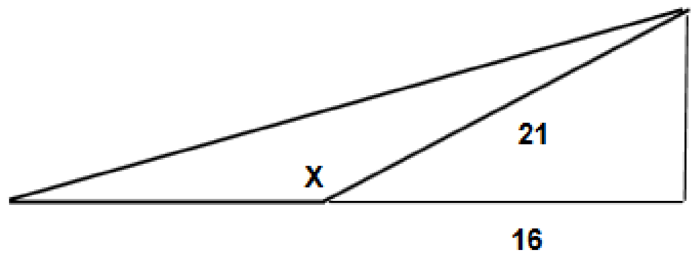
Answers: 1. 10.9 2. 56.2^{o} 3. 219
4. 52.5 5. 38.9^{o} 6. 41.1

T3 COS Problems

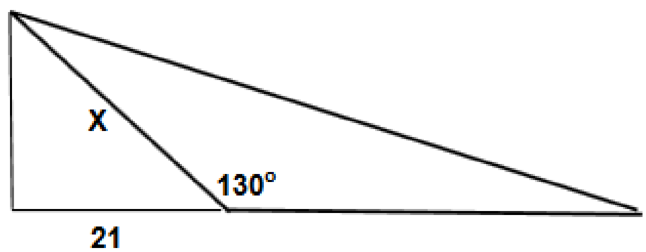
Always set up the Equation first. Then solve.

Use the **Pythagorean Theorem** if necessary.

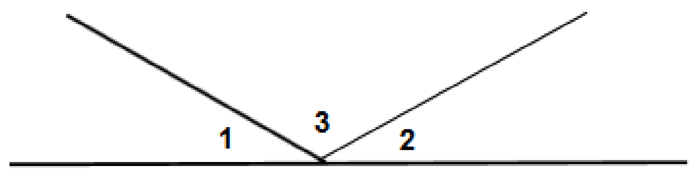
NOTE: **COS**(1) < 0 **IF** $180^\circ > (1) > 90^\circ$



1



2



<1 = <2 and <1 + <3 + <2 =180°

<1 = 35° What does **COS**(<2 + <3) = ? **3**

<3 = 120° What are **COS**(1) and **COS**(2 + 3) ? **4**

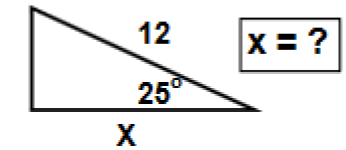
<2 = 30° What are **SIN**(3) and **COS**(3) ? **5**

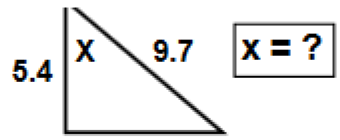
Answers: 1. 137° 2. 32.7 3. -0.819
4. 0.866, -0.866 5. 0.866, -0.5

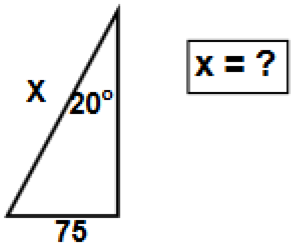
T3E

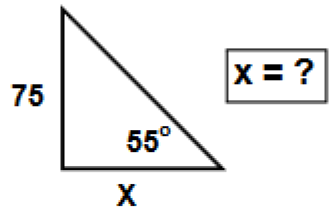
COS X COSINE OF X. X IS AN ANGLE (DEGREES °)

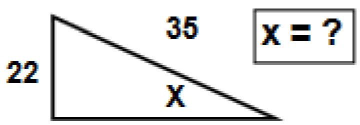
Find **x** in the following exercises.

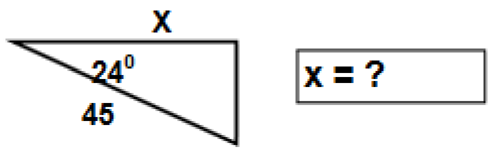


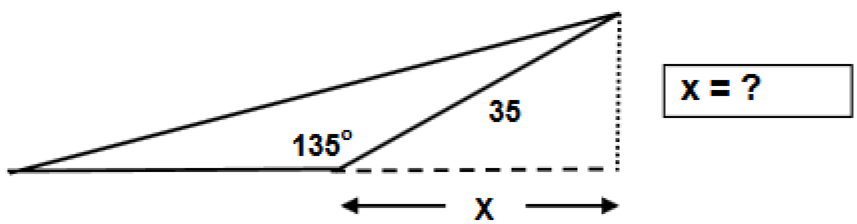








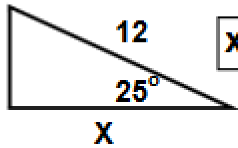




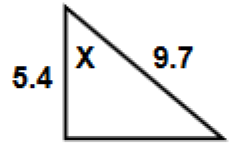
T3EA

COS X COSINE OF X. X IS AN ANGLE (DEGREES ^{O})

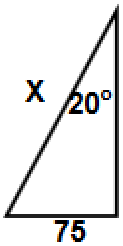
Find **x** in the following exercises.



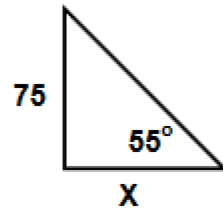
x = 10.9



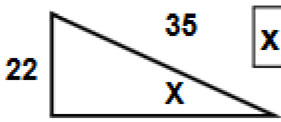
x = 56.2°



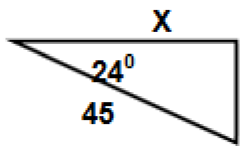
x = 219



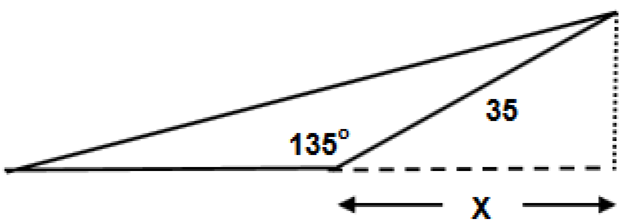
x = 52.5



x = 38.9°



x = 41.1

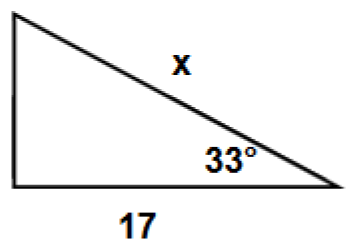


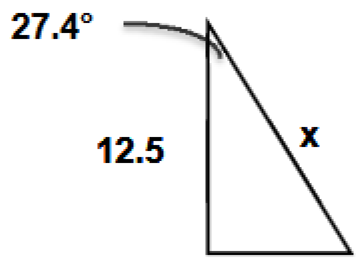
x = 24.7

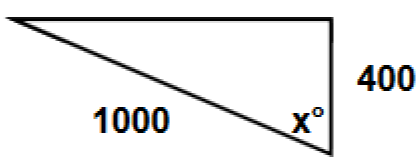
T3ES

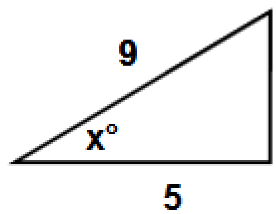
COS X COSINE OF X. X IS AN ANGLE (DEGREES ^{O})

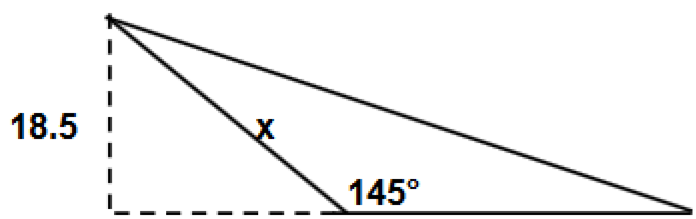
Find X in the following exercises.







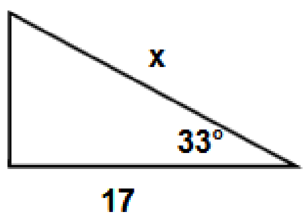




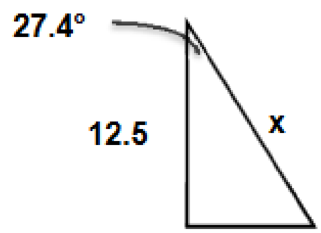
T3ESA

COS X COSINE OF X. X IS AN ANGLE (DEGREES °)

Find X in the following exercises.

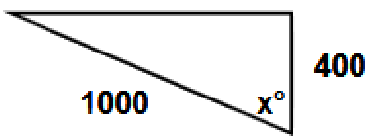


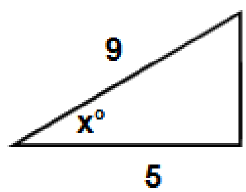
x = 20.3



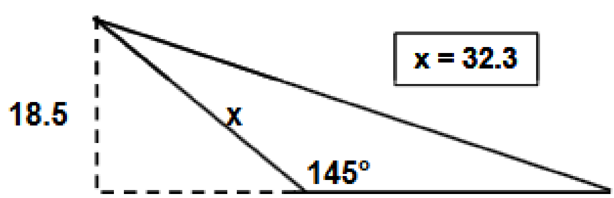
x = 14.1

x = 66.4°





x = 58.3°

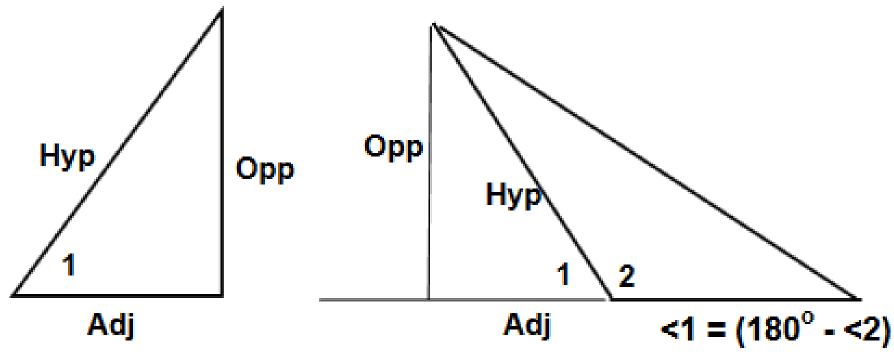


x = 32.3

T4 LESSON: TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{O})

TAN X can take on all values positive and negative

TAN X is **Not** defined at **X** = -90^{o} or 90^{o} (**Error**)



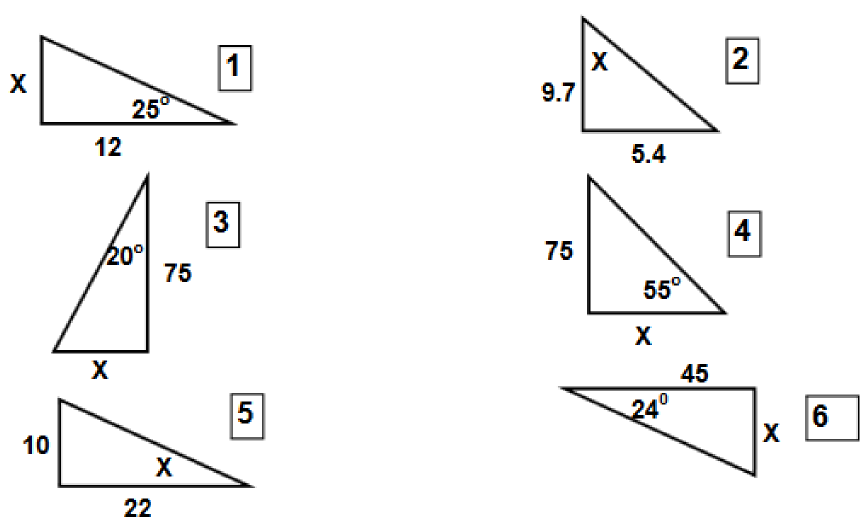
TAN(1) = **Opp/Adj**　　　**TAN**(2) = **Opp/Adj** = -**TAN**(180^{o} - <2)

If know two out of three, find the third, **Opp**, **Adj**, (1)

Opp = **TAN**(1)x**Adj**　　　**Opp** = **TAN**(2)x**Adj**　　　**Adj** < 0

Adj = **Opp/TAN**(1)　　　**Adj** = **Opp/TAN**(2)　　　**Adj** < 0

(1) = **TAN**$^{-1}$(**Opp/Adj**)　(2) = **TAN**$^{-1}$(**Opp/Adj**)

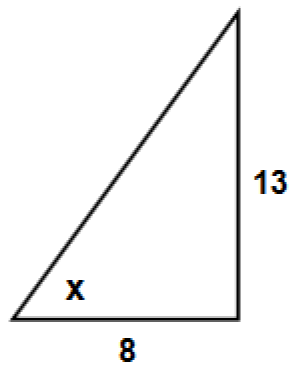


Answers:　1. 5.6　2. 29.1^{o}　3. 27.3
　　　　　4. 52.5　5. 24.4^{o}　6. 20.0

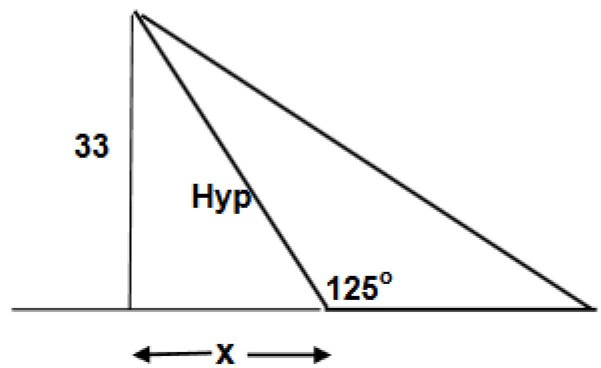
T4E

TAN X TANGENT OF X. X IS AN ANGLE (DEGREES °)

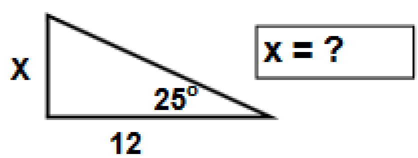
Find X in each of the following exercises.

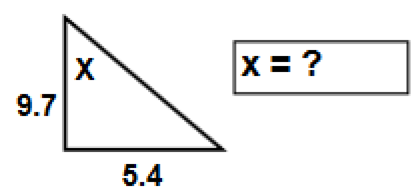


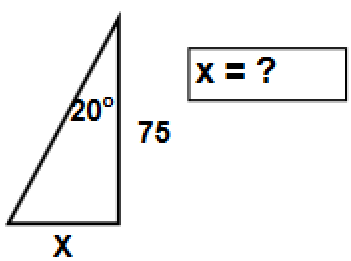
x = ?°

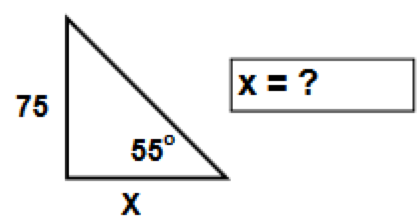


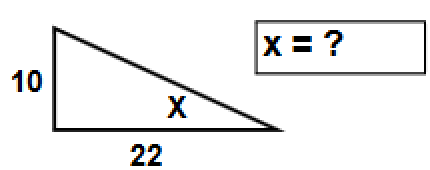
x = ?

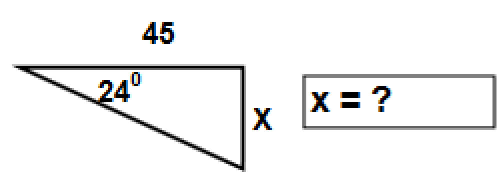








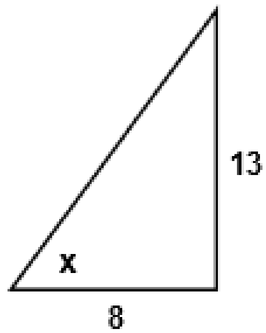




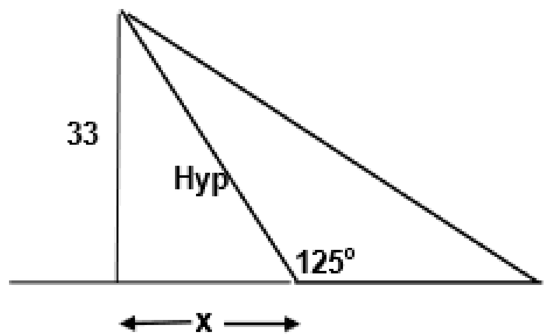
T4EA

TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{o})

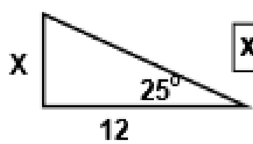
Find X in each of the following exercises.



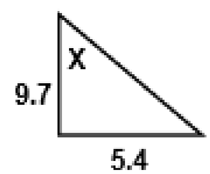
$x = 58.4^{o}$



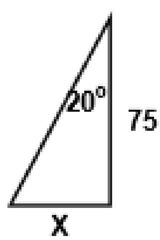
$x = 23.1$



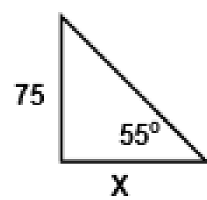
$x = 5.6$



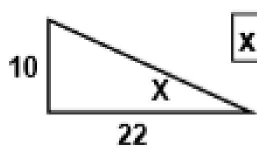
$x = 29.1^{o}$



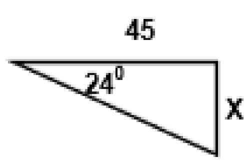
$x = 27.3$



$x = 52.5$



$x = 24.4^{o}$

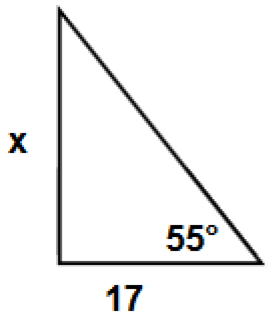


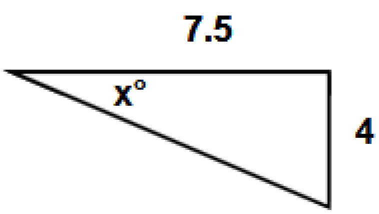
$x = 20.0$

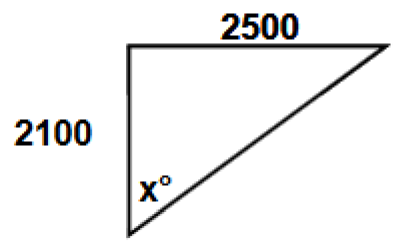
T4ES

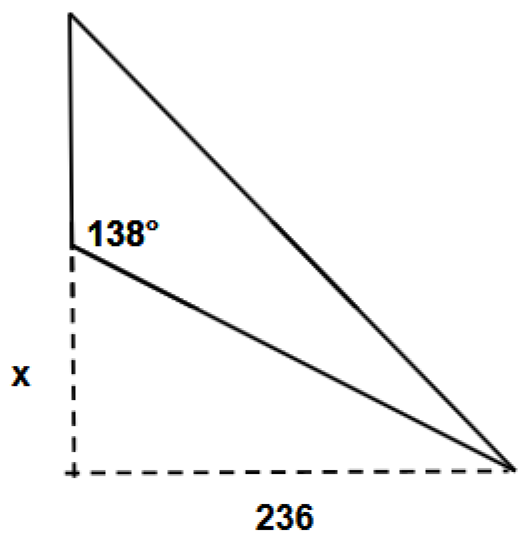
TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{O})

Find X in each of the following exercises.





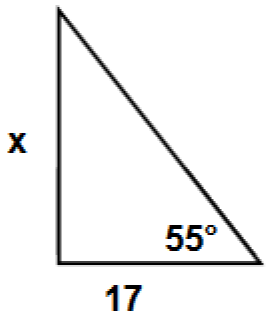




T4ESA

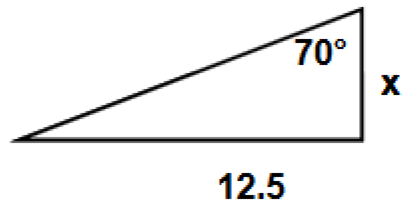
TAN X TANGENT OF X. X IS AN ANGLE (DEGREES °)

Find X in each of the following exercises.

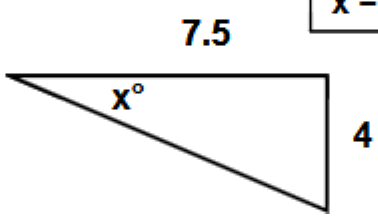


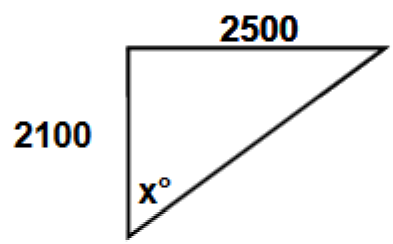
x = 24.3

x = 4.55



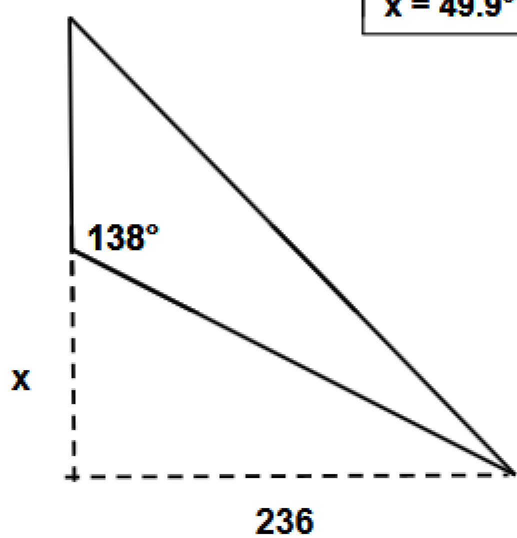
x = 28.1°





x = 49.9°

x = 262.1



T5 LESSON: WARNING ABOUT SIN^{-1}

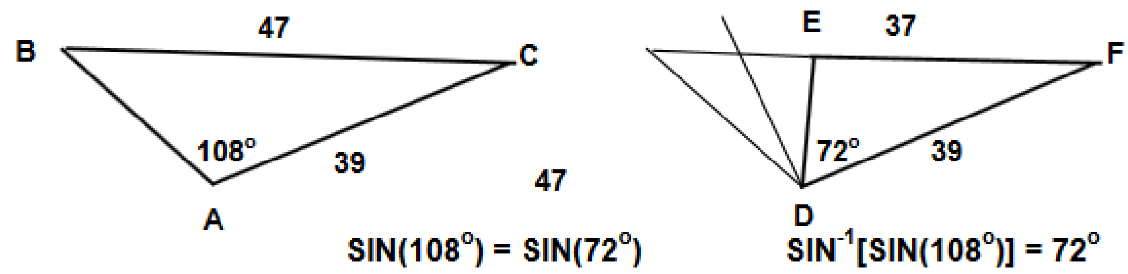
We are interested in **angles**, <A, from 0° to 180°

SIN(<A) = **SIN**(180° - <A)   (see Table below)

So, if we have a **triangle** with an **angle** <A > 90°, with **SIN**(<A), then its **SIN**$^{-1}$ will be wrong.

See below for example:

Suppose we know **SIN**(<A) = .95105, yet **SIN**$^{-1}$(.95105) = 72°



Angle <A	SIN(<A)	SIN^{-1}	Angle <A	COS(<A)	COS^{-1}
0	0.000	0	0	1.000	0
10	0.174	10	10	0.985	10
20	0.342	20	20	0.940	20
30	0.500	30	30	0.866	30
40	0.643	40	40	0.766	40
50	0.766	50	50	0.643	50
60	0.866	60	60	0.500	60
70	0.940	70	70	0.342	70
80	0.985	80	80	0.174	80
90	1.000	90	90	0.000	90
100	0.985	80	100	-0.174	100
110	0.940	70	110	-0.342	110
120	0.866	60	120	-0.500	120
130	0.766	50	130	-0.643	130
140	0.643	40	140	-0.766	140
150	0.500	30	150	-0.866	150
160	0.342	20	160	-0.940	160
170	0.174	10	170	-0.985	170
180	0.000	0	180	-1.000	180

T5E

WARNING ABOUT SIN^{-1}

When dealing with **angles** whose measure is between 90º and 180º, what happens with the **SIN** and **COS** which can lead to confusion?

If **X** is an angle between 90º and 180º how do **SIN** and **COS** behave?

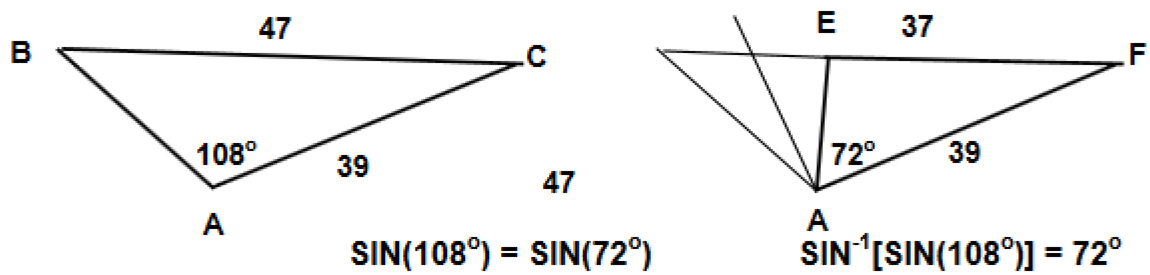
Answer: Give examples:

? For **COS**

? For **SIN**

Suppose we know **SIN**(<A) = .95105, which triangle could this apply to?

Answer: ?



Suppose we know **COS**(<A) = .3090, which triangle could this apply to?

Answer: ?

WHY?

Answer: ?

T5EA

WARNING ABOUT SIN^{-1}

When dealing with angles whose measure is between 90° and 180°, what happens with the **SIN** and **COS** which can lead to confusion?

If **X** is an angle between 90° and 180° how do **SIN** and **COS** behave?

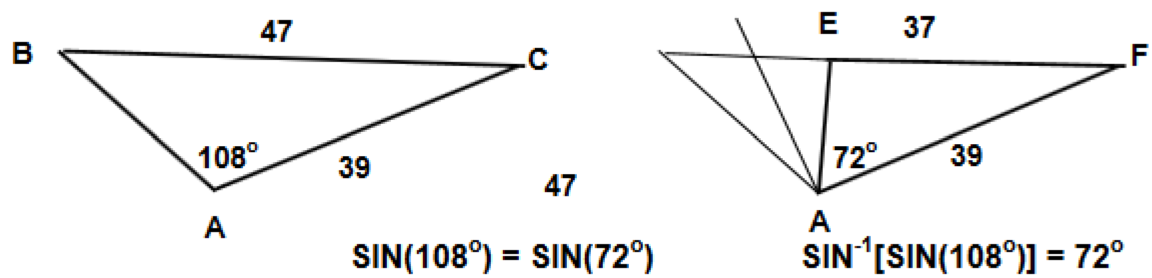
Answer: Give examples:

COS(X°) = - **COS**(180° - X°) **COS**(137°) = -**COS**(43°)

SIN(X°) = **SIN**(180° - X°) **SIN**(137°) = **SIN**(43°)

Suppose we know **SIN**(<A) = .95105, which triangle could this apply to?

Answer: Both



Suppose we know **COS**(<A) = .3090, which triangle could this apply to?

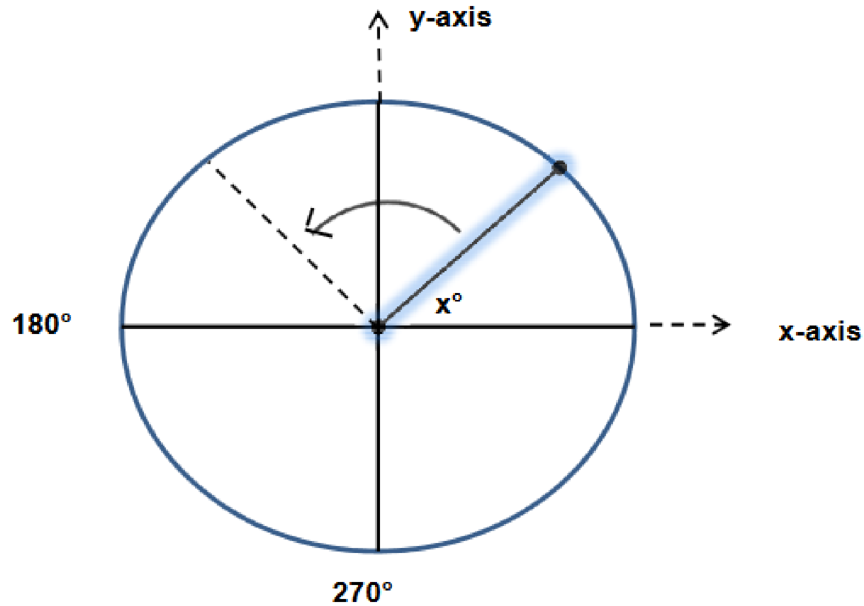
Answer: Only Triangle **AEF**

WHY?

Answer: **COS** (108°) = -.3090

T5ES

WARNING ABOUT SIN^{-1}

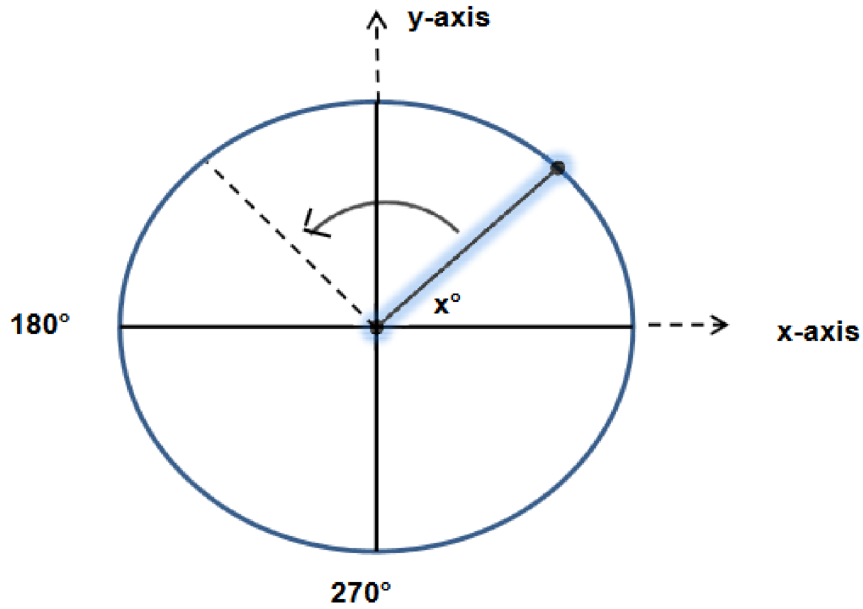


In the diagram above, the highlighted line rotates around in a circle in the xy-plane, starting at 0° and ending at 360°. Fill in the + or -

	0-90°	90-180°	180-270°	270-360°
sin(x)				
cos(x)				
tan(x)				

T5ESA

WARNING ABOUT SIN^{-1}



In the diagram above, the highlighted line rotates around in a circle in the xy-plane, starting at 0° and ending at 360°. Fill in the + or -

	0-90°	90-180°	180-270°	270-360°
sin(x)	positive	positive	negative	negative
cos(x)	positive	negative	negative	positive
tan(x)	positive	negative	positive	negative

T6 LESSON: LAW OF SINES

Problem: Suppose you have a triangle with two angles measuring 40° and 100° and the side opposite the 40° angle is 16 inches.

What is the length, **X**, of the side opposite the 100° angle? Look at the figure below.

Clearly **X** is larger than 16 in. Hmmm…maybe it is just proportional to the angles: How about:

$$X = (100^\circ/40^\circ)x16 = (5/2)x16 = 40 \text{ ?}$$

Construct such a triangle and measure it, and you find it measures about 241/2 inches. SO; no, this doesn't work.

Hmmm…what could we do? How about trying some type of correction factor? How about taking the **SIN** of both angles?

SIN(100°)/**SIN**(40°)x16 = 24.5 **Eureka!** ??

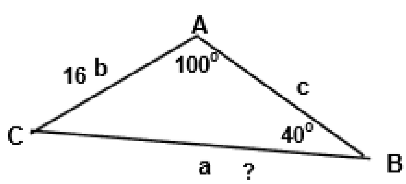
Could this always work? Answer: **YES**.

[**SIN**(<A)/**SIN**(<B)]xb = a, **ALWAYS**, for any angles.

Where **a** is opposite <A and **b** opposite <B

This is called the **Law of Sines**. We prove it in **Tier 3**.

We use it for practical problems. It makes "solving" **triangles** "child's play," especially with a **TI 30XA**.



Law of Sines

a/**SIN**(<A) = b/**SIN**(<B) = c/**SIN**(<C)

[**SIN**<A/**SIN**<B] x b = a

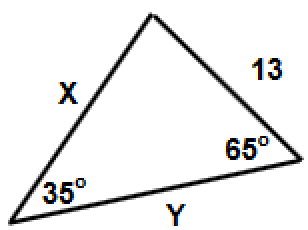
T6 Law of Sines Problems

If you know two angles and an opposite side, you can find them all.

If you know two sides and an opposite angle you can find them all. Sometimes two possibilities.

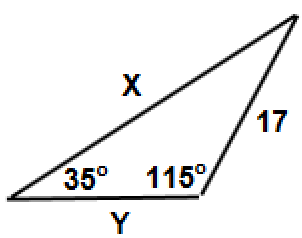
Makes solving problems "child's play."

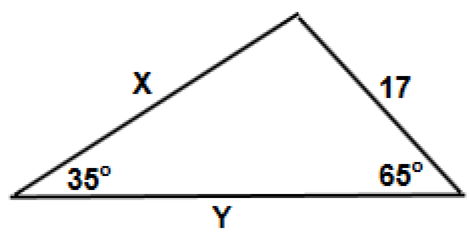
Still, if you know two sides and the included angle, we can't solve for the third side. Need one more tool.



$X/SIN(65^o) = 13/SIN(35^o)$, $X = 20.5$

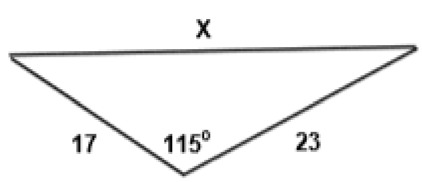
$Y/SIN(80^o) = 13/SIN(35^o)$, $Y = 22.3$





$X/SIN(115^o) = 17/SIN35^o)$ $X = 26.9$ $Y = 14.8$	$X/SIN(65^o) = 17/SIN35^o)$ $X = 26.9$ $Y = 29.2$

Observe: $115^o + 65^o = 180^o$ **Thus:** $SIN(115^o) = SIN(65^o)$



Find X

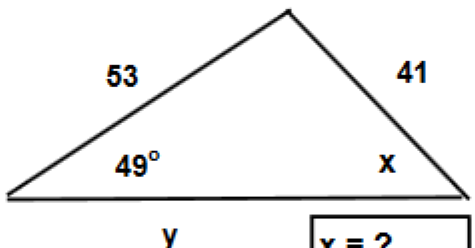
Got to recognize limitations

Need One More Tool

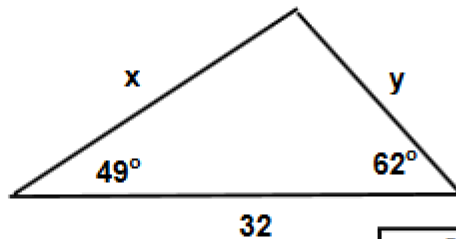
T6E

LAW OF SINES

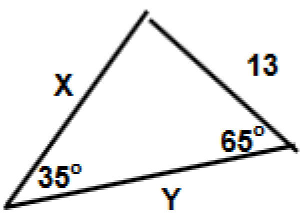
Find the Unknowns and answer questions.



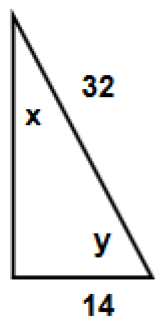
x = ?
y = ?



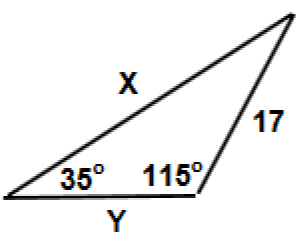
x = ?
y = ?



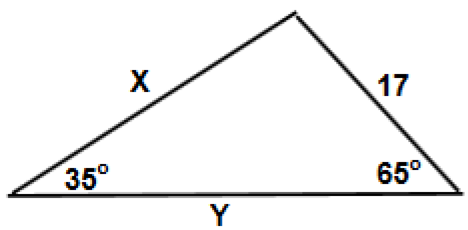
x = ?
y = ?



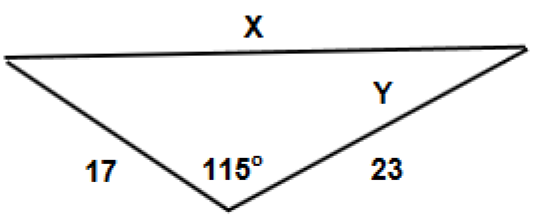
x = ?
y = ?



x = ?
y = ?



x = ?
y = ?

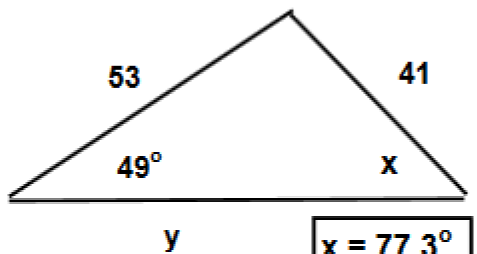


x = ?
y = ?

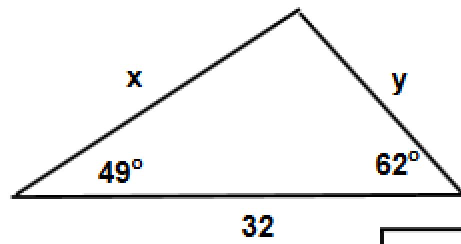
T6EA

LAW OF SINES

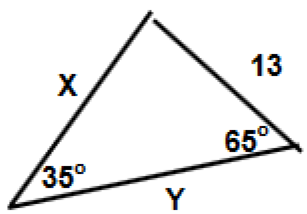
Find the unknowns and answer questions.



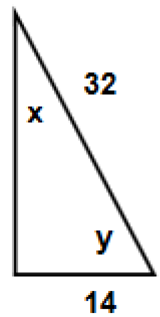
x = 77.3°
y = 43.8



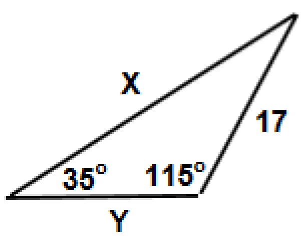
x = 30.3
y = 25.9



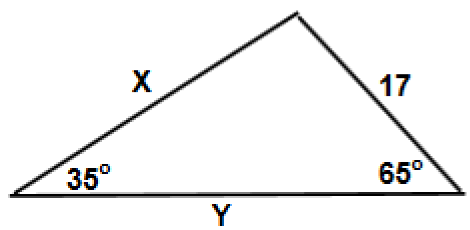
x =20.5
y = 22.3



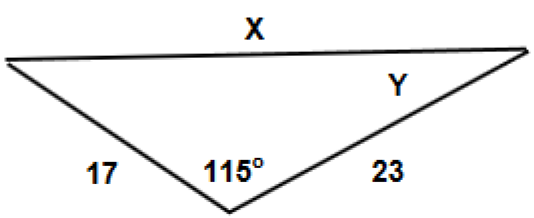
x = 25.9°
y = 64.1°



x = 26.9
y = 14.8



x = 26.9
y = 29.2

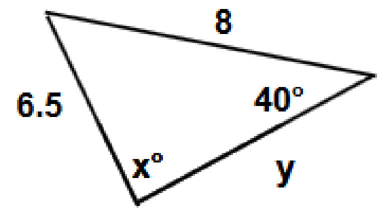


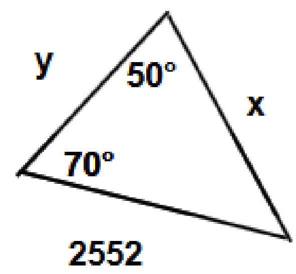
Can not find x and y with tools given so far.
See T7 for solution.

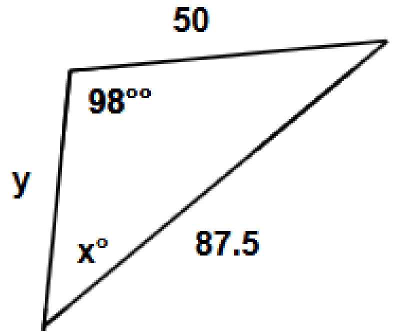
T6ES

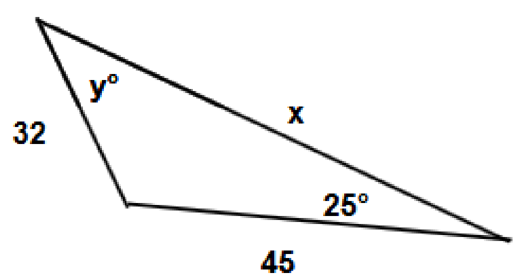
LAW OF SINES

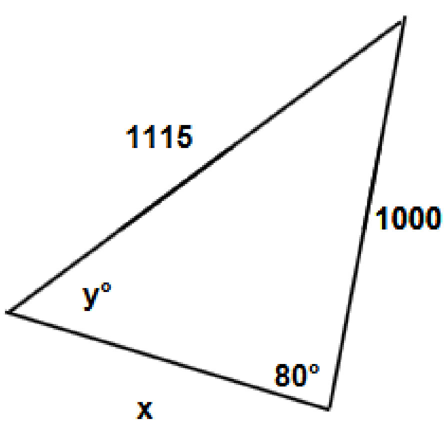
Find X and Y in the following exercises.







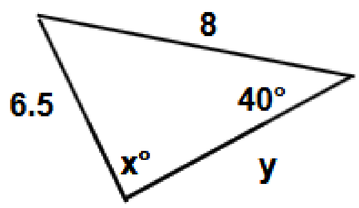




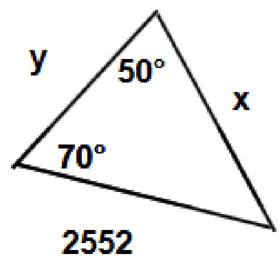
T6ESA

LAW OF SINES

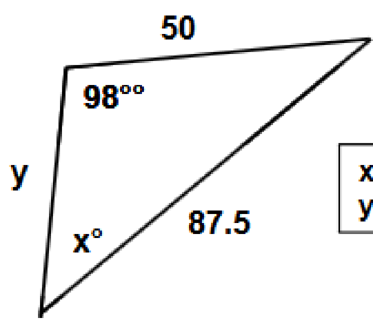
Find X and Y in the following exercises.



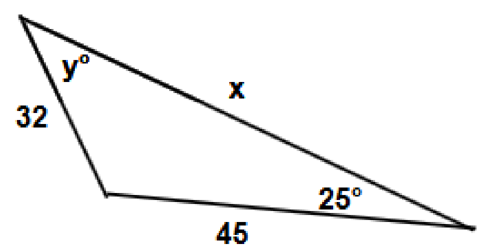
x = 52.3°
y = 6.18



x = 3130.5
y = 2885.1

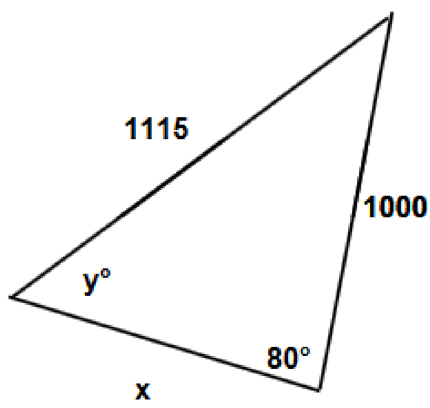


x = 34.5°
y = 72.4



x = 66.5
y = 36.5°

x = 697.1
y = 62°



T7 LESSON: LAW OF COSINES - GENERALIZED PYTHAGOREAN THEOREM

Suppose we know two sides and the included **angle** of a **triangle**. How can we calculate third side's length?

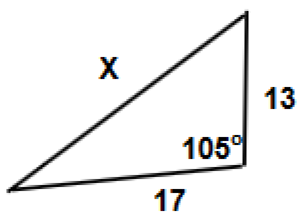
Easy if the **angle** is 90^o. $c^2 = a^2 + b^2$

We need a "**correction factor**" for non-right **angles**,

<a,b $c^2 = a^2 + b^2$ - 2ab**COS**(<a,b), works for all **triangles**.

Also, let us find the **angles** when we only know the three sides of a **triangle**.

<a,b = **COS**$^{-1}$[(a^2 +b^2 - c^2)/(2ab)], where <a,b is included angle.



$X^2 = 17^2 + 13^2$ - 2x13x17x**COS**105^o

Thus, **X** = 23.9 **NOTE: COS**105^o = -.2589, so **CF** is +

$X^2 = 39^2 + 47^2$ - 2x39x47x**COS**20^o

Thus, **X** = 16.9 mi

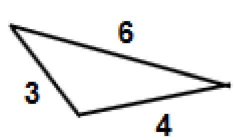
Now we can also calculate y^o Use **Law of Sines**

y^o = 72^o or (180^o - 72^o) = 108^o

Clearly from the diagram 108^o is correct.

Find the Area of the 3, 4, 6 triangle using A = .5ab**SIN**(<a,b)

First, we must calculate <a,b where a = 3, b = 4



<3,4 = **COS**$^{-1}$[(3^2 + 4^2 -6^2)/(2x3x4)] = 117.3^o

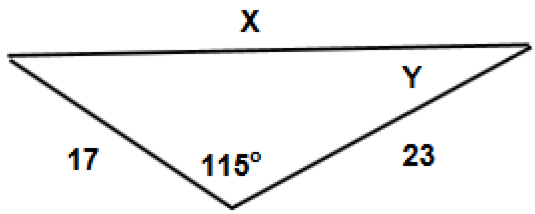
Area = .5x**SIN**(117.3^o)x3x4 = 5.33 U^2

T7E

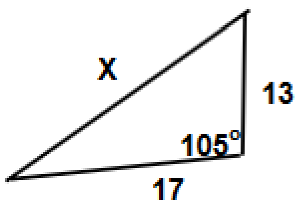
LAW OF COSINES - GENERALIZED PYTHAGOREAN THEOREM

Find the Unknowns

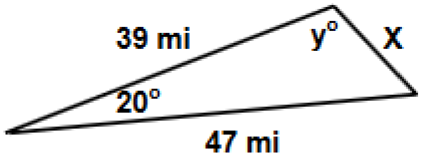
Start with the problem we could not solve in T6



x = ?
y = ?

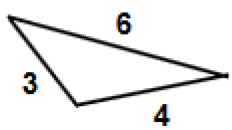


x = ?



x = ?
y = ?

Find the Area of this triangle

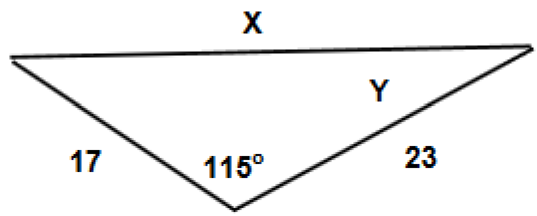


T7EA

LAW OF COSINES - GENERALIZED PYTHAGOREAN THEOREM

Find the Unknowns

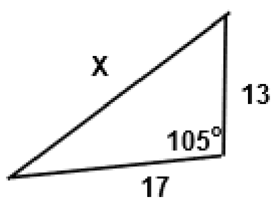
Start with the problem we could not solve in T6



x = 33.9
y = 27°

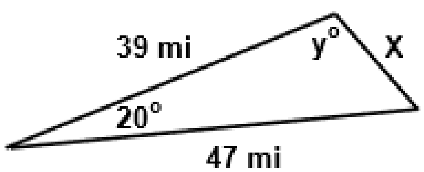
$x^2 = 17^2 + 23^2 - 2x17x23x\mathbf{COS}(115^o)$

$y = \mathbf{SIN}^{-1}[\{\mathbf{SIN}(115^o)/33.9\}x17]$



$X^2 = 17^2 + 13^2 -2x13x17x\mathbf{COS}105^o$

Thus, **X** = 23.9 **NOTE**: **COS**105° = -.2589, so, **CF** is **+**



$X^2 = 39^2 + 47^2 - 2x39x47x\mathbf{COS}20^o$

Thus, **X** = 16.9 mi

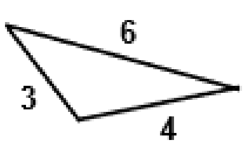
Now we can also calculate y°

Use **Law of Sines**

Clearly from the diagram 108° is correct. y° = 72° or (180° - 72°) = 108°

Find the Area of the 3, 4, 6 triangle using A = .5ab**SIN**(<a,b)

First, we must calculate <a,b where a = 3, b = 4

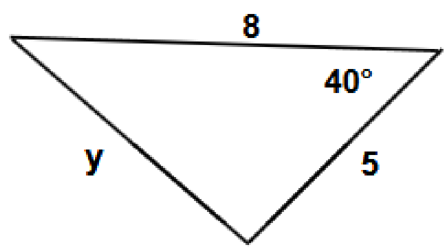


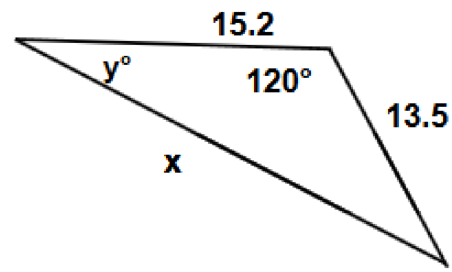
$<3,4 = \mathbf{COS}^{-1}[(3^2 + 4^2 -6^2)/(2x3x4)] = 117.3^o$

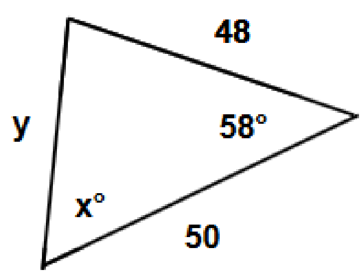
Area = $.5x\mathbf{SIN}(117.3^o)x3x4 = 5.33\ U^2$

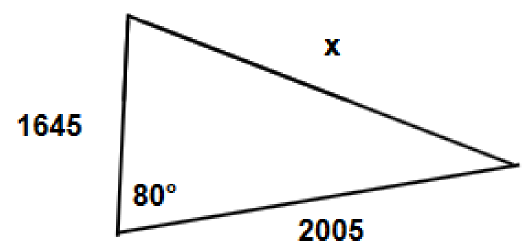
LAW OF COSINES

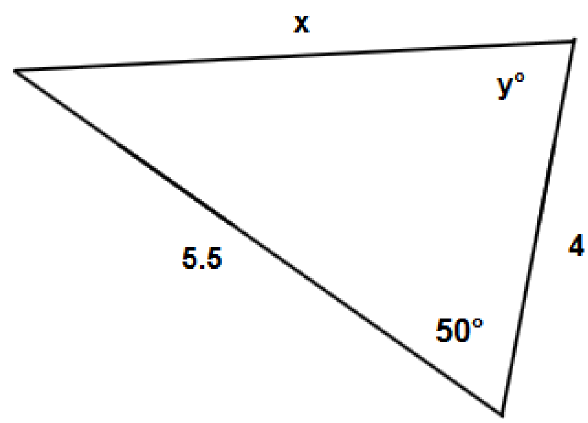
Find X and Y in the following exercises.





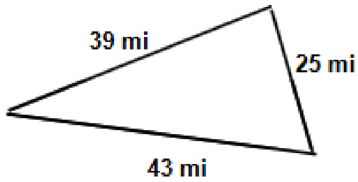


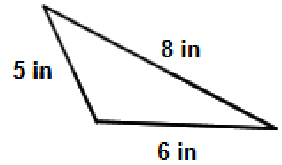


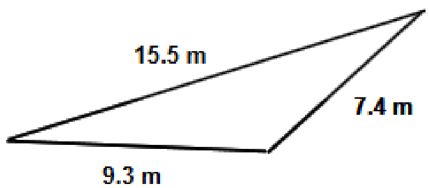


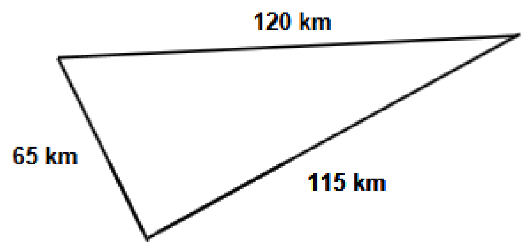
T7ES (cont.)

AREAS OF IRREGULAR TRIANGLES



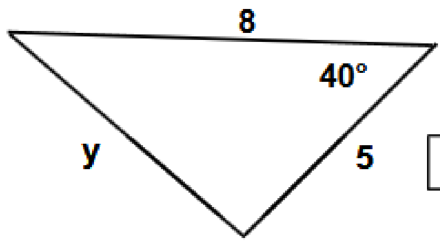




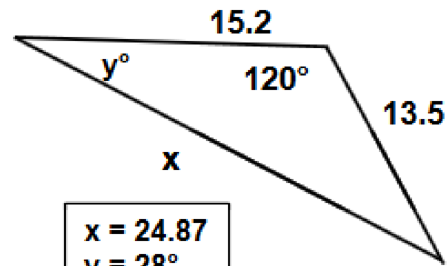


LAW OF COSINES

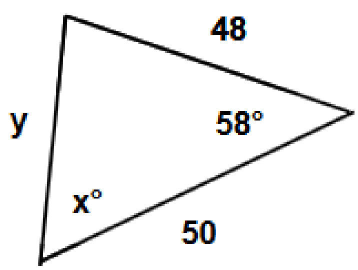
Find X and Y in the following exercises.



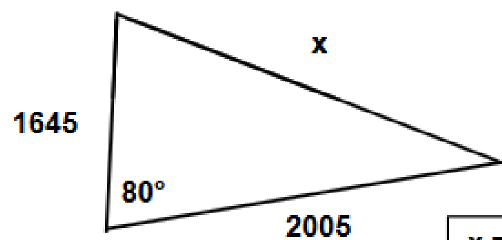
y = 5.26



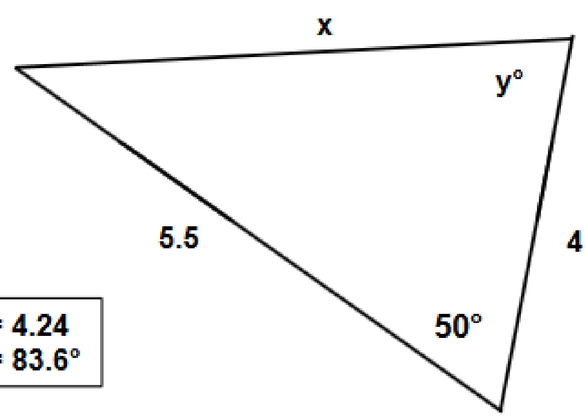
x = 24.87
y = 28°



x = 58.9°
y = 47.5



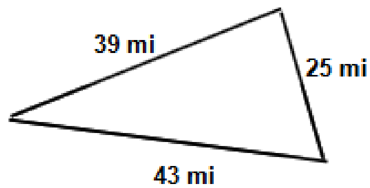
x = 2362.3



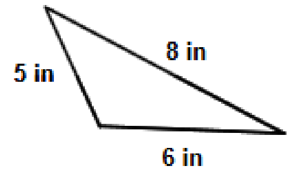
x = 4.24
y = 83.6°

T7ESA (cont.)

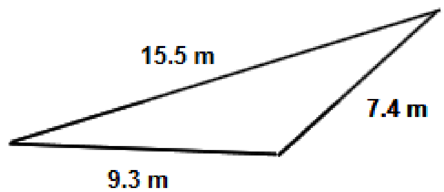
LAW OF COSINES



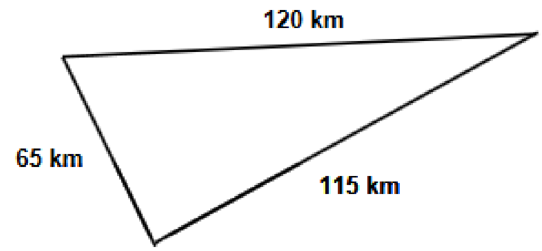
A = 482.1 mi^2



A = 14.98 in^2



A = 23.9 m^2



A = 3,659 km^2

T8 LESSON: TRIGONOMETRY BEYOND PRACTICAL MATH

Trigonometry is a huge extremely important subject with profound applications in science and engineering.

The **Trig Functions** are called the **Circle Functions** and are defined for **ALL** angles, both positive and negative.

Trig Functions are very important in **calculus**.

Trig Functions are probably best understood in the context of the Complex Number System.

Trig Functions are the basis of modern spectrometry via what is called the **Fourier Transform**.

The **Trig Functions** are periodic and that is what makes them so important in any type of **cyclical behavior** such as vibration analysis, and music.

So next, you will need to understand the **Trig functions** via graphs in analytical geometry (Tier 3).

Then one needs to learn about them in the context of the Complex Number System. That is when many of the famous **Trig Identities** will become very natural and understandable. What I consider the most important equation in all of mathematics makes this clear (Tier 4).

Then one needs to learn about their behavior utilizing the **calculus**. It is truly amazing (Tier 5).

Ultimately, they are profound in Functional Analysis and modern physics such as **Quantum Theory** (Tier 9).

T8E

TRIGONOMETRY BEYOND PRACTICAL MATH

Trigonometry is a huge extremely important subject with profound applications in science and engineering and advanced mathematics.

If you are planning to study math beyond Practical Math, then you should be aware of some of the future applications of **Trigonometry**.

List as many things you have heard about where **Trig** will be useful and applicable.

If you study other resources such as Wikipedia you will probably come up with other applications in addition to those I have pointed out.

Please accept my best wishes for your future success.

I hope mathematics will be rewarding to you in your future endeavors, and enjoyable too.

Thank you for studying this **Foundations Course**.

Dr. Del.

T8EA

TRIGONOMETRY BEYOND PRACTICAL MATH

Trigonometry is a huge extremely important subject with profound applications in science and engineering.

The **Trig Functions** are called the **Circle Functions** and are defined for **ALL** angles, both positive and negative.

Trig functions are very important in **calculus**.

Trig Functions are probably best understood in the context of the Complex Number System.

Trig Functions are the basis of modern spectrometry via what is called the **Fourier Transform**.

The **Trig Functions** are periodic and that is what makes them so important in any type of **cyclical behavior** such as vibration analysis, and music.

So next, you will need to understand the **Trig Functions** via graphs in analytical geometry (Tier 3).

Then one needs to learn about them in the context of the Complex Number System. That is when many of the famous **Trig Identities** will become very natural and understandable. What I consider the most important equation in all of mathematics makes this clear (Tier 4).

Then one needs to learn about their behavior utilizing the **calculus**. It is truly amazing (Tier 5).

Ultimately, they are profound in **Functional Analysis** and modern physics such as **Quantum Theory** (Tier 9).

S6 Lesson: Prefixes

In science and engineering Prefixes are used to change the size of units.

For example, Kilometer, km, means 1,000 Meters

So, 1 km = 1,000m = 10^3 m

1 centimeter = .01m = (1/100)m = 10^{-2}m = 1cm

1 decimeter = .1m = (1/10)m = 10^{-1}m = 1dm

1 millimeter = .001m = (1/1000)m = 10^{-3}m = 1mm

The most common Metric Prefixes are listed below along with their exponents of 10.

milli (m)	-3	Kilo (K)	+3	Thousand
micro(μ)	-6	Mega(M)	+6	Million
nano (n)	-9	Giga (G)	+9	Billion
pico (p)	-12	Tera (T)	+12	Trillion

Examples: 27 nS = 27×10^{-9} S = .000000027 S

27 μS = 27×10^{-6} S = .000027 S

45 GH = 45×10^{9} H = 45000000000 H

78KB = 78×10^{3}B = 78000B

3.5KΩ = 3500Ω

Now the laws or rules of exponents are:

$10^n \times 10^m = 10^{n+m}$ for any exponents n and m

Also, $10^0 = 1$ and $10^{-n} = 1/10^n$

So suppose we have, for example:

$7mAx8M\Omega = 7x10^{-3}Ax8x10^6\Omega = 56x10^3V = 56KV$

Since, $1Ax1\Omega = 1V$ [This is Ohm's Law]

Thus, we see mxM = K since $10^{-3}x10^6 = 10^3$

So we multiply, x, **two prefixes to get one prefix by simply adding the exponents**.

mxG = M since -3 + 9 = 6
mxm = μ since -3+ -3 = -6
nxK = μ since -9 + 3 = -6

If you are going to become an electrician or electronics technician you should learn this prefix table, and practice multiplying prefixes.

Then, you will use this along with the Technician's Triangle we will discuss in another lesson.

This will greatly simplify calculations you will be making when you troubleshoot electrical or electronic systems or equipment.

In the **Metric system** we use powers of 10

In the **Digital system** we use powers of 2.

Note: $2^{10} = 1024 \approx 1000 = 10^3$

The most common Digital Prefixes are listed below along with their exponents of 2.

milli	(m)	-10	Kilo	(K)	+10
micro	(μ)	-20	Mega	(M)	+20
nano	(n)	-30	Giga	(G)	+30
pico	(p)	-40	Tera	(T)	+40

If you are going to become a computer or communications technician, you will want to master this system as well. It works just like the metric system.

For example, mSxMH = KC since 1Sx1H = 1C

Because -10 + 20 = +10

The purpose of this Lesson is to make you aware of these Prefixes. You will want to master them IF you decide to learn a technical field where they are used a lot.

Prefix Product Table

0 +3 +6 +9 +12

X		1	K	M	G	T
0	1	1	K	M	G	T
-3	**m**	m	1	K	M	G
-6	**µ**	µ	m	1	K	M
-9	**n**	n	µ	m	1	K
-12	p	p	n	µ	m	1

We will make use of this when we discuss the Technician's Triangle

Of course, this Table can be expanded, but this is what one usually uses.

For example, mxn = p

But, µxn = f

where femto stands for 10^{-15}

Some Musings.

Most of us don't really appreciate the difference between a million and a billion.

How long is one million seconds, 1 MS ?

11.57 days 1,000,000/60/60/24

How long is one billion seconds, 1GS ?

32 years 11,570/365

How long is one trillion seconds, 1 TS ?

32,000 years.

Apply similar questions about our national debt and our money supply.

One million pennies is ten thousand dollars

One billion pennies is ten million dollars.

The DNA in one human cell is about 6 ft long if it unwound. Of course, it is very thin. Similar to extending your little finger from LA to Paris.

There are about one trillion cells in your body. So how long would your DNA be if it was all strung out end to end? How about a billion miles?

S6E

Prefixes

1. Using the generic unit of measure, S, and the **metric** prefixes, calculate the new prefix for the following problems.
   a. mS x nS
   b. mS x MS
   c. KS x MS
   d. µS x µS
   e. nS x GS
   f. TS x µS
   g. GS x KS
   h. mS x µS
   i. GS x pS
   j. TS x µS
2. Using the generic unit of measure, S, and the **metric** prefixes, convert the following to numbers.
   a. 15 nS
   b. 23 KS
   c. 47 TS
   d. 28 µS
   e. 84 GS
   f. 18 MS
   g. 43 pS
   h. 98 mS
   i. 4.2 mS
   j. 3.84 GS
3. Using the generic unit of measure, S, and the **digital** prefixes, calculate the new prefix for the following problems.
   a. mS x nS

b. mS x MS

c. KS x MS

d. µS x µS

e. nS x GS

f. TS x µS

g. GS x KS

h. mS x µS

i. GS x pS

j. TS x µS

Q4. Using the generic unit of measure, S, and the **digital** prefixes, convert the following to numbers.

a. 15 nS

b. 23 KS

c. 47 TS

d. 28 µS

e. 84 GS

f. 18 MS

g. 43 pS

h. 98 mS

i. 4.2 mS

j. 3.84 GS

S6EA

Prefixes

1.

a. $mS \times nS = 10^{-3}S \times 10^{-9}S = 10^{-12}S = pS$
b. $mS \times MS = 10^{-3}S \times 10^{6}S = 10^{3}S = KS$
c. $KS \times MS = 10^{3}S \times 10^{6}S = 10^{9}S = GS$
d. $\mu S \times \mu S = 10^{-6}S \times 10^{-6}S = 10^{-12}S = pS$
e. $nS \times GS = 10^{-9}S \times 10^{9}S = 10^{0}S = S$
f. $TS \times \mu S = 10^{12}S \times 10^{-6}S = 10^{6}S = MS$
g. $GS \times KS = 10^{9}S \times 10^{3}S = 10^{12}S = TS$
h. $mS \times \mu S = 10^{-3}S \times 10^{-6}S = 10^{-9}S = nS$
i. $GS \times pS = 10^{9}S \times 10^{-12}S = 10^{-3}S = mS$
j. $TS \times \mu S = 10^{12}S \times 10^{-6}S = 10^{6}S = MS$

2.

a. 15 nS = $15\text{x}10^{-9}$ S = 0.000000015 S
b. 23 KS = $23\text{x}10^{3}$ S = 23,000 S
c. 47 TS = $47\text{x}10^{12}$ S = 47,000,000,000,000 S
d. 28 μS = $28\text{x}10^{-6}$ S = 0.000028 S
e. 84 GS = $84\text{x}10^{9}$ S = 84,000,000,000 S
f. 18 MS = $18\text{x}10^{6}$ S = 18,000,000 S
g. 43 pS = $43\text{x}10^{-12}$ S = 0.000000000043 S
h. 98 mS = $98\text{x}10^{-3}$ = 0.098 S
i. 4.2 mS = $4.2\text{x}10^{-3}$ = 0.0042 S
j. 3.84 GS = $3.84\text{x}10^{9}$ S = 3,840,000,000 S

3.

a. mS x nS = 2^{-10}S x 2^{-30}S = 2^{-40}S = pS

b. mS x MS = 2^{-10}S x 2^{20}S = 2^{10}S = KS

c. KS x MS = 2^{10}S x 2^{20}S = 2^{30}S = GS

d. µS x µS = 2^{-20}S x 2^{-20}S = 2^{-40}S = pS

e. nS x GS = 2^{-30}S x 2^{30}S = 2^{0}S = S

f. TS x µS = 2^{40}S x 2^{-20}S = 2^{20}S = MS

g. GS x KS = 2^{30}S x 2^{10}S = 2^{40}S = TS

h. mS x µS = 2^{-10}S x 2^{-20}S = 2^{-30}S = nS

i. GS x pS = 2^{30}S x 2^{-40}S = 2^{-10}S = mS

j. TS x µS = 2^{40}S x 2^{-20}S = 2^{20}S = MS

4.

a. 15 nS = $15x2^{-30}$ S = 0.000000014 S

b. 23 KS = $23x2^{10}$ S = 23,552 S

c. 47 TS = $47x2^{40}$ S = $5.167704651x10^{13}$ S

d. 28 µS = $28x2^{-20}$ S = 0.000026703 S

e. 84 GS = $84x2^{30}$ S = 8,589,934,592 S

f. 18 MS = $18x2^{20}$ S = 18,874,368 S

g. 43 pS = $43x2^{-40}$ S = $3.910827218x10^{-11}$ S

h. 98 mS = $98x2^{-10}$ = 0.095703125 S

i. 4.2 mS = $4.2x2^{-10}$ = 0.004101562 S

j. 3.84 GS = $3.84x2^{30}$ S = 34,123,168,604 S

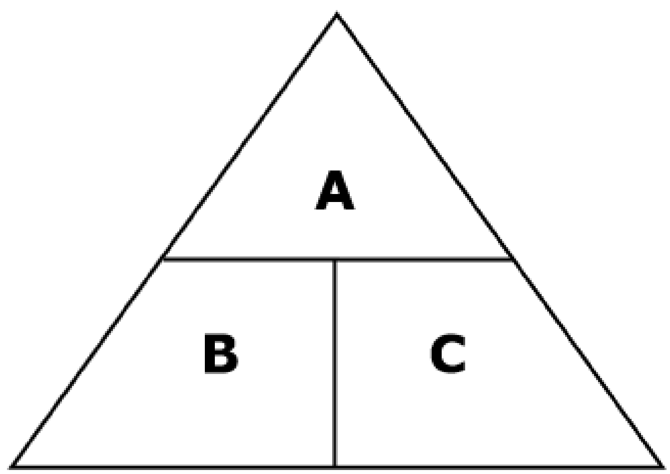
S7 Lesson: Technician's Triangle

Often one is faced with an equation A = BxC, where one must solve for one of these variables when the other two are known.

This yields three equations as you have learned.

A = BxC   B = A/C   C = A/B

Sometimes it is easiest to simply put this into what I call a Technician's Triangle. Then, one can "solve" the equation very easily.



Now to "solve" for any variable, just perform the calculation with the other two variables.

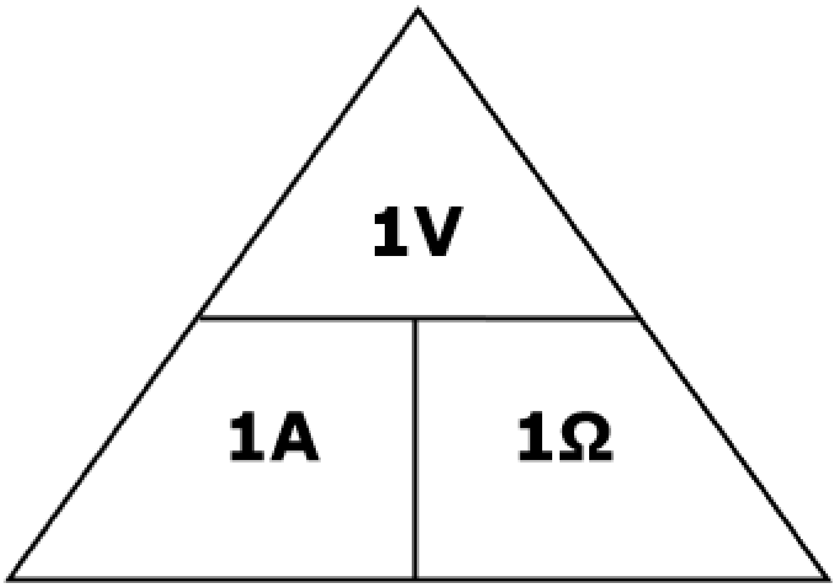
A = BxC   B = A/C   C = A/B

Things get interesting when the units involved have prefixes attached.

Let's look at an example from electronics.

Ohm’s Law is $1V = 1A \times 1\Omega$

Where: V is Volts, A is Amps, Ω is Resistance



But, often one has to deal with prefixes attached to these units. For example, we might have:

$5\mu A \times 7K\Omega = .000005 \times 7000\ V = .035\ V = 35\ mV$

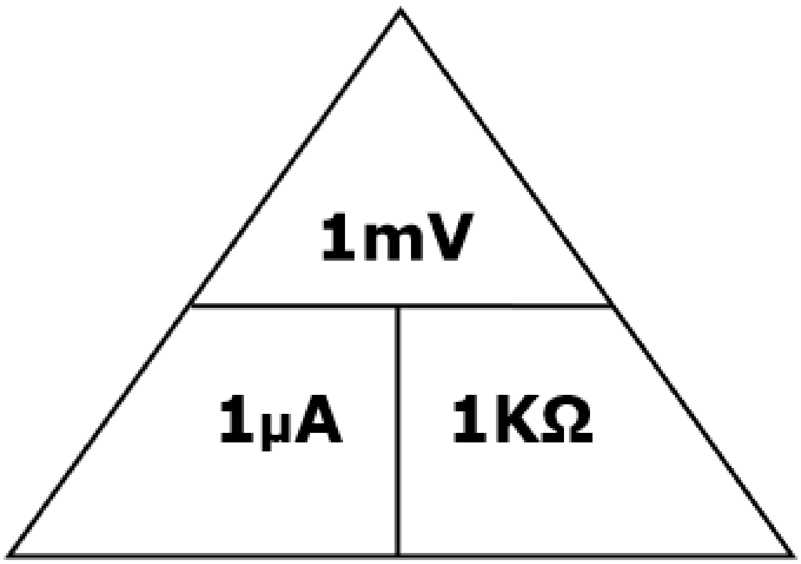
This is the way it has been dealt with classically.

There must be an easier way!

Well, there is.

We learned in the Prefixes lesson that:

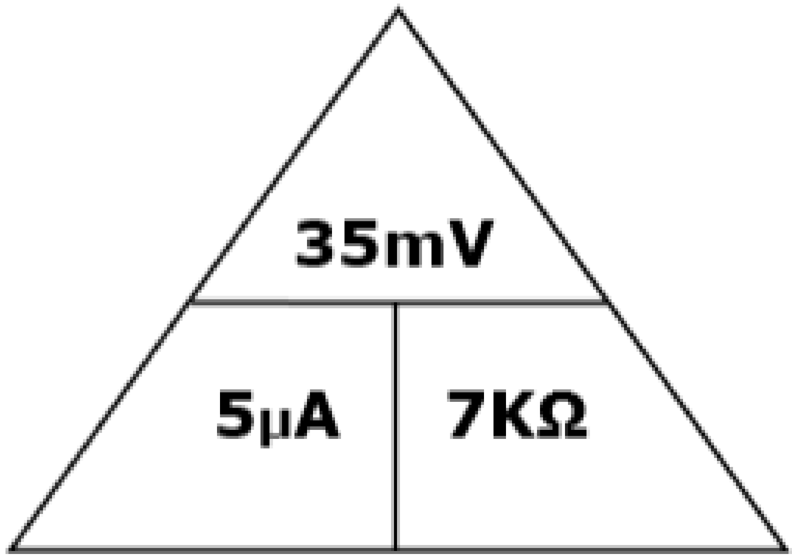
μ x K = m



This then leads us to the following Tech Triangle.

Remember we know μ x K = m

So, this then leads us to the following Tech Triangle



So, all we have to do to solve for any one of these given the other two is simply do the simple arithmetic.
This is much easier than the old-fashioned way.

5µAx7KΩ = .000005x7000V = .035V= 35mV

Or 35mV/7KΩ = .035/7000 A = .000005 A = 5µA

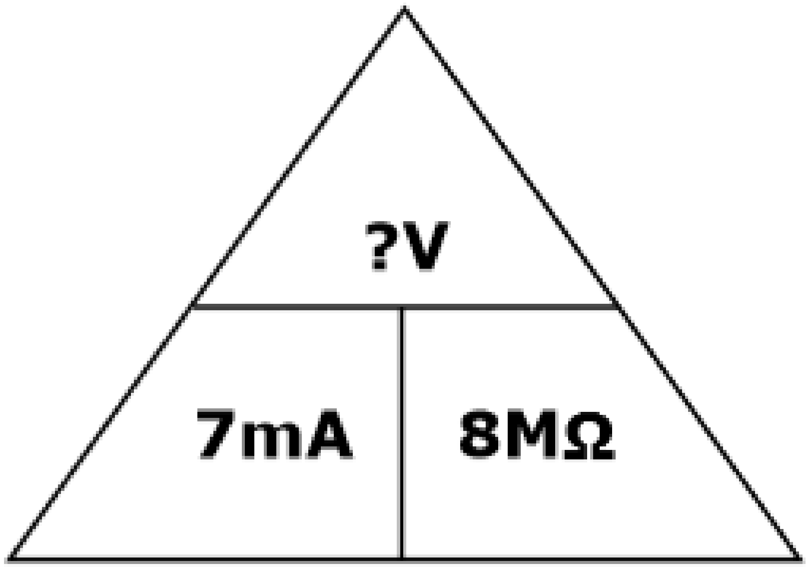
Or 35mV/5µA = .035/.000005 Ω = 7000Ω = 7 KΩ

It was amazing how many times engineers and technicians got the decimal place wrong and were off by an order of magnitude, i.e., 10x.

So, quick now, what is 7 mA times 8 MΩ ?

Remember we know m x M = K

So, this then leads us to the following Tech Triangle



Answer: 56 KV

This is much easier than the old-fashioned way.

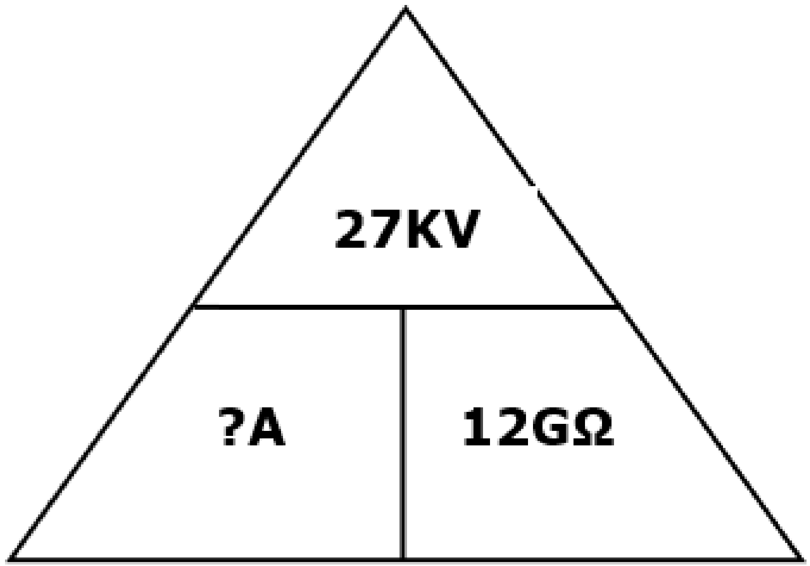
So, quick now, what is 7 mA times 8 MΩ ?

Try it the old-fashioned way if you want to experience what some of our ancestors went through. Even with slide rules and log tables it was more difficult than with a calculator. But, it is even easy to make a mistake with a calculator doing it the old-fashioned way.

Try: 2.4mA x 6.7 MΩ Use the TT, mxM = K

Answer: 2.4x6.7 KV = 16 KV

OK one more, quick. 27KV across a 12GΩ resistor yields how many amps, A? So, this then leads us to the following Tech Triangle



Well look in the Prefix Table.

What times G yields K?   Answer: μ

[G is +9 and K is +3, so we need a -6 since 9+(-6)= +3
  So, we need a μ and Gxμ = K]

So, the answer is 27/12 μA = 2.25 μA

This is much easier than the old-fashioned way.

Try it the old-fashioned way if you want to experience what some of our ancestors went through. They didn't even have calculators. But, our calculator won't even take this many 0's in FLO so you would have to use SCI format.

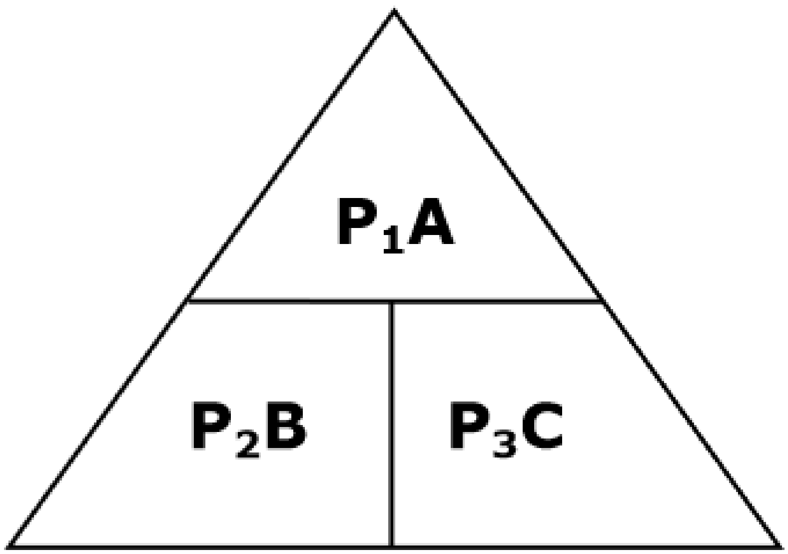
27000/12000000000 = .00000225

There are many fields where you have an equation like

1A = 1Bx1C where A,B,C are some units.

Then, a Technician's Triangle will apply.

You will need to learn the Prefixes and remember to multiply two prefixes you just add their exponents of their power of 10, or of their power of 2 in the digital case.



Where $P_1 = P_2 x P_3$, from the Table of Prefixes

This is much easier than the old-fashioned way.

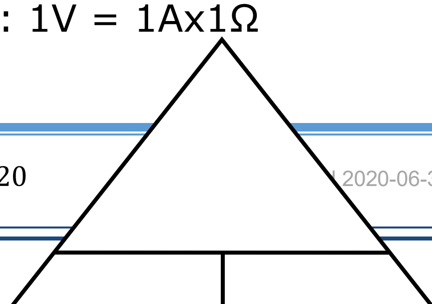
Simply practice in whatever technical field you are in with the relevant equations.

S7E

Technician's Triangle

Solve for the unknown using metric prefixes.

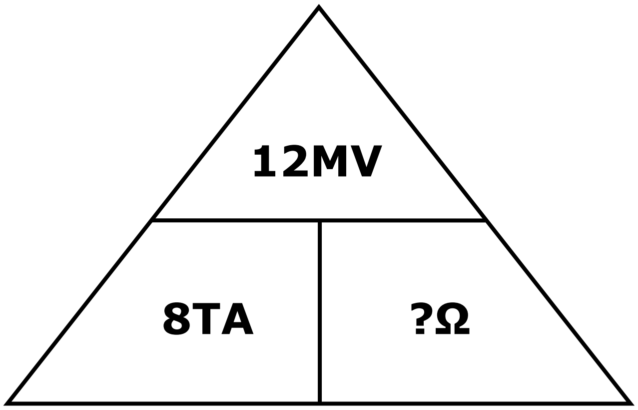
1. Ohm's Law: 1V = 1Ax1Ω



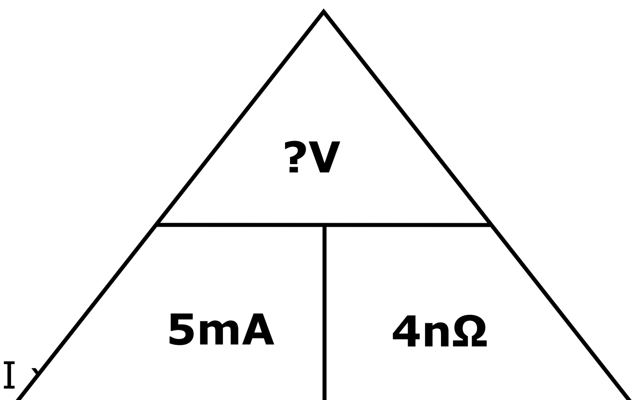
27nV

?A **3pΩ**

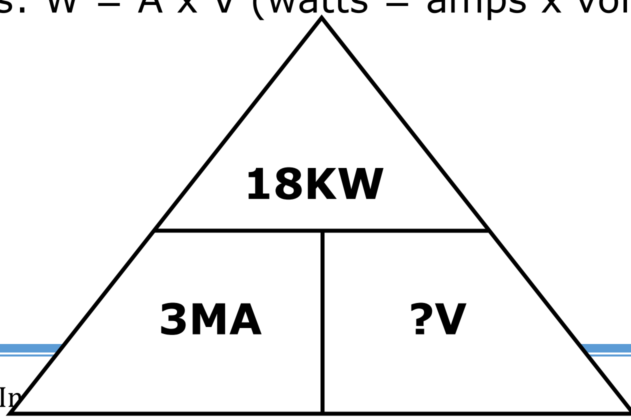
2. Ohm’s Law: 1V = 1Ax1Ω



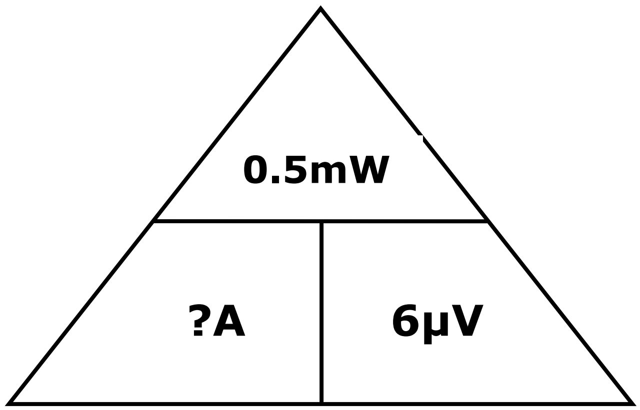
3. Ohm’s Law: 1V = 1Ax1Ω



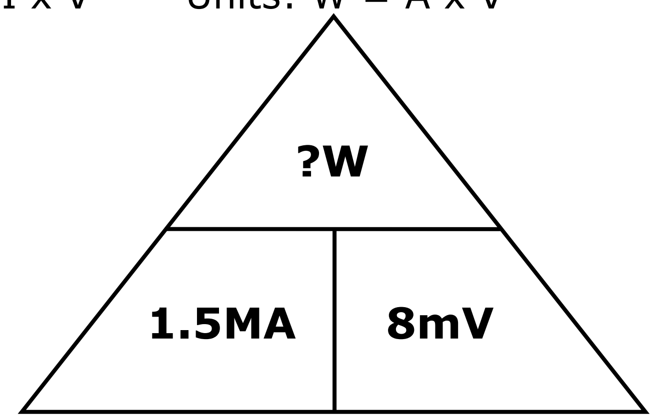
4. P = I x
   Units: W = A x V (watts = amps x volts)



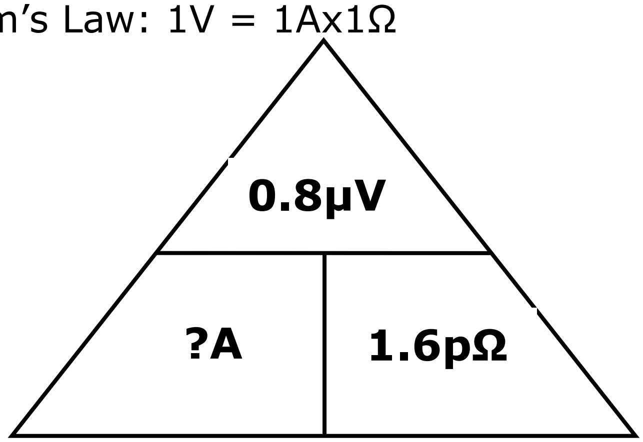
5. P = I x V   Units: W = A x V



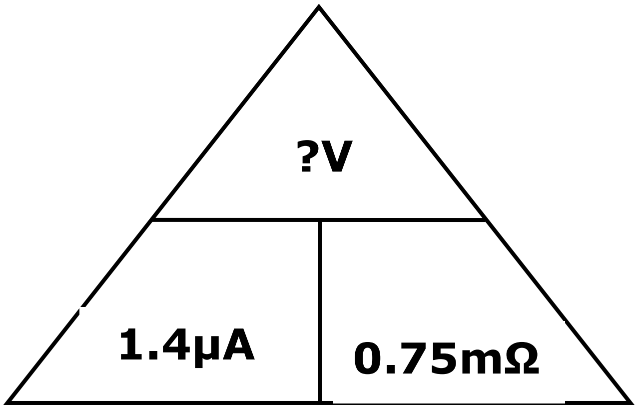
6. P = I x V   Units: W = A x V



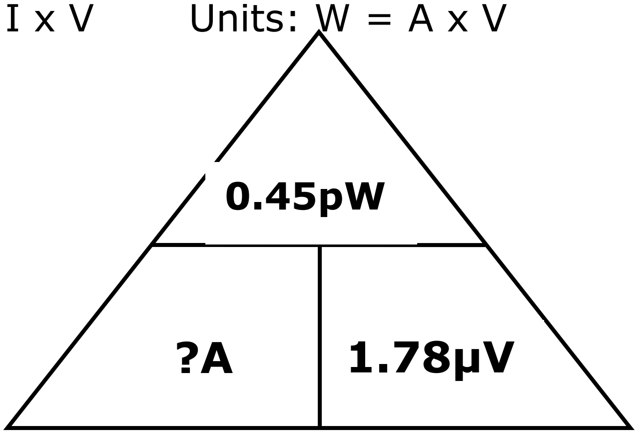
7. Ohm's Law: 1V = 1Ax1Ω



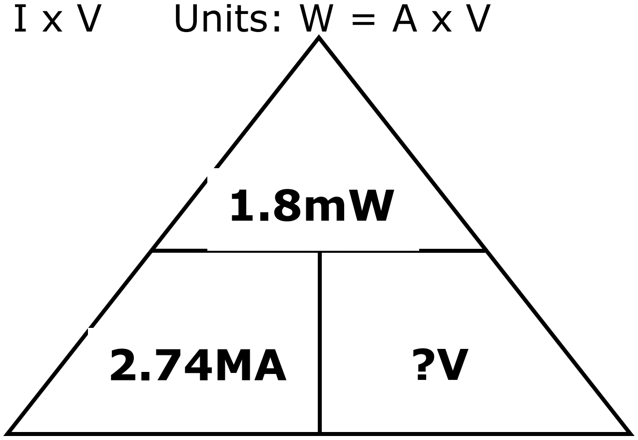
8. Ohm's Law: 1V = 1Ax1Ω



9. P = I x V Units: W = A x V



10. P = I x V Units: W = A x V



S7EA

Technician's Triangle

1. 27nV = ?A x 3pΩ

   n = ? + p --> -9 = ? + -12 --> ? = +3 --> K

   27nV = ?KA x 3pΩ

   27 = ? x 3 -> ? = 9

   Unknown: 9KA

   27nV = 9KA x 3pΩ

2. 12MV = 8TA x ?Ω

   M = T + ? --> +6 = +12 + ? --> ? = -6 --> μ

   12MV = 8TA x ?μΩ

   12 = 8 x ? -> ? = 1.5

   Unknown: 1.5μΩ

   12MV = 8TA x 1.5μΩ

3. ?V = 5mA x 4nΩ

   ? = m + n --> ? = (-3) + (-9) --> ? = -12 --> p

   ?pV = 5mA x 4nΩ

   ? = 5 x 4 -> ? = 20

   Unknown: 20pV

   20pV = 5mA x 4nΩ

4. 18KW = 3MA x ?V

K = M + ? --> +3 = +6 + ? --> ? = -3 --> m

18KW = 3MA x ?mV

18 = 3 x ? -> ? = 6

Unknown: 6mV

18KW = 3MA x 6mV

5. 0.5mW = ?A x 6μV

m = ? + μ --> -3 = ? + (-6) --> = +3 --> K

0.5mW = ?KA x 6μV

0.5 = ? x 6 -> ? = 1/12 or 0.083

Unknown: 0.083KA

0.5mW = 0.083KA x 6μV

6. ?W = 1.5MA x 8mV

? = M + m --> ? = +6 + (-3) --> = +3 --> K

?KW = 1.5MA x 8mV

? = 1.5 x 8 -> ? = 12

Unknown: 12KW

12KW = 1.5MA x 8mV

7. 0.8μV = ?A x 1.6pΩ

μ = ? + p --> -6 = ? + (-9) --> ? = +3 --> K

0.8μV = ?KA x 1.6pΩ

0.8 = ? x 1.6 -> ? = 0.5

Unknown: 0.5KA

0.8μV = 0.5KA x 1.6pΩ

8. ?V = 1.4μA x 0.75mΩ

?= μ + m --> ? = (-6) + (-3) --> ? = -9 --> n

?nV = 1.4μA x 0.75mΩ

? = 1.4 x 0.75 -> ? = 1.05

Unknown: 1.05nV

1.05nV = 1.4μA x 0.75mΩ

9. 0.45pW = ?A x 1.78μV

p = ? + μ --> -12 = -6 + ? --> = -6 --> μ

0.45pW = ?μA x 1.78μV

0.45 = ? x 1.78 -> ? = 0.253

Unknown: 0.253μA

0.45pW = 0.253μA x 1.78μV

10. 1.8mW = 2.74MA x ?V

m = M + ? --> -3 = +6 + ? --> = -9 --> n

1.8mW = 2.74MA x ?nV

1.8 = 2.74 x ? -> ? = 0.657

Unknown: 0.657nV

1.8mW = 2.74MA x 0.657nV

S8 Lesson: Polar Rectangular Coordinates

In the plane, there are two ways to specify a point.

Rectangular Coordinates (**x,y**)

Polar Coordinates (**r**, **θ**) where

r = $(x^2 + y^2)^{1/2}$,

θ = **tan** $^{-1}$(**y/x**) in Quadrants 1 and 4

and θ = **tan**-1(**y/x**) + 180º in Quads 2 and 3

Example 1: (4,3) = (5, 36.87º) since **tan**$^{-1}$(3/4) = 36.87º and 5 = $(4^2 + 3^2)^{1/2}$

Example 2: (-4,3) = (5, 143.13º) since **tan**$^{-1}$(-3/4) = -36.87º + 180º = 143.13º

Fortunately, the **TI30Xa** will do this automatically with the R → P and P → R Keys.

2nd . 2 This fixes the display to two digits past.

FUNCTION	KEY	ENTER	DISPLAY
	4	4	
x <--> y	2nd π		0.00
	3	3	
R< - ->P	2nd -		5.00
x <--> y	2nd π		36.87

FUNCTION	KEY	ENTER	DISPLAY
	4	4	
+< -->-		-4	
x <--> y	2^{nd} π		0.00
	3	3	
R< - ->**P**	2^{nd} -		5.00
x <--> y	2^{nd} π		143.13

You can go from **P** to **R** also.

FUNCTION	KEY	ENTER	DISPLAY
	5	5	
x <--> y	2^{nd} π		0.00
		143.13	143.13
P< - ->R	2^{nd} x		-3.9999
x <--> y	2^{nd} π		3.00

Note: All of this works if you use **RAD** or **GRAD** for the degrees, for those of you who are more advanced in trigonometry.

Now just do some Exercises

(4, 9) = (9.85, 66.03°) **R** to **P**

(7, 197°) = (-6.69, -2.05) **P** to **R**

S8E

Polar Rectangular Coordinates Exercises

For the following exercises, graph the rectangular coordinates to determine quadrant, then solve for the polar coordinates.

1. (5, 12)
2. (8, 15)
3. (-8, -15)
4. (-4.5, 6.3)
5. (3.7, -8.2)
6. (-8.9, -12.5)

For the following exercises, solve for the rectangular coordinates.

7. (9, 45º)
8. (6, 32 º)
9. (12, 127º)
10. (4.7, 118.6º)
11. (5.6, 210º)
12. (7.8, 301.9º)

Using the R → P button on your calculator, convert these rectangular coordinates to polar coordinates.

13. (5, 7)
14. (8, 13)
15. (-7, 16)
16. (6.3, -8.2)

Using the P → R button on your calculator, convert these polar coordinates to rectangular coordinates.

17. (9, 27º)
18. (10, 75º)
19. (4.7, 190.5º)
20. (13.45, 347º)

S8EA

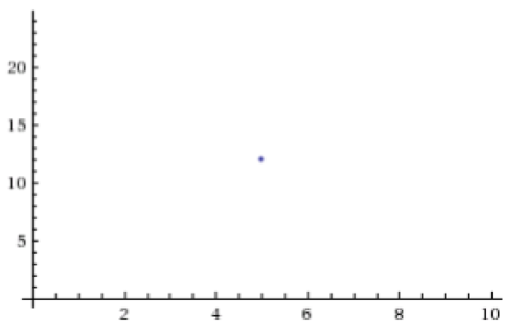
Polar Rectangular Coordinates

1. $r = (x^2 + y^2)^{1/2}$

   $r = (5^2 + 12^2)^{1/2}$

   $r = 13$

   Plot

   Quadrant 1

   $\Theta = \tan^{-1}(y/x)$

   $\Theta = \tan^{-1}(12/5)$

   $\Theta = 67.38^o$

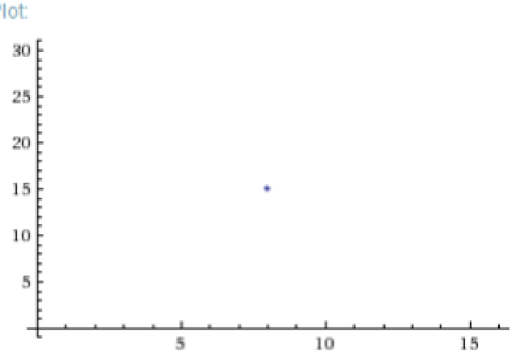
   $(13, 67.38^o)$

2. $r = (x^2 + y^2)^{1/2}$

   $r = (8^2 + 15^2)^{1/2}$

   $r = 17$

   Plot

Quadrant 1

Θ = **tan**$^{-1}$(y/x)

Θ = **tan**$^{-1}$(15/8)

Θ = 61.93º

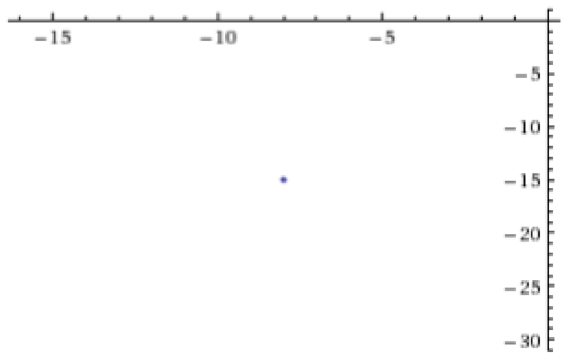
(17, 61.93º)

3. **r** = $(x^2 + y^2)^{1/2}$

r = $((-8)^2 + (-15)^2)^{1/2}$

r = 17

Plot:



Quadrant 3

Θ = **tan**$^{-1}$ (y/x) + 180º

Θ = **tan**-^{1} (-15/-8) + 180º

Θ = 241.93º

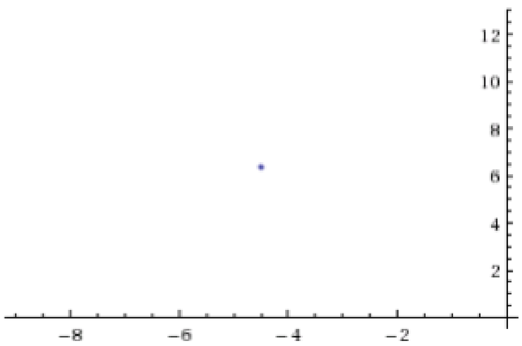
(17, 241.93º)

4. $\mathbf{r} = (x^2 + y^2)^{1/2}$

$\mathbf{r} = ((-4.5)^2 + 6.3^2)^{½}$

$\mathbf{r} = 7.74$

Plot:



Quadrant 2

$\Theta = \mathbf{tan}^{-1}(y/x) + 180^o$

$\Theta = \mathbf{tan}^{-1}(-4.5/6.3) + 180^o$

$\Theta = 144.46^o$

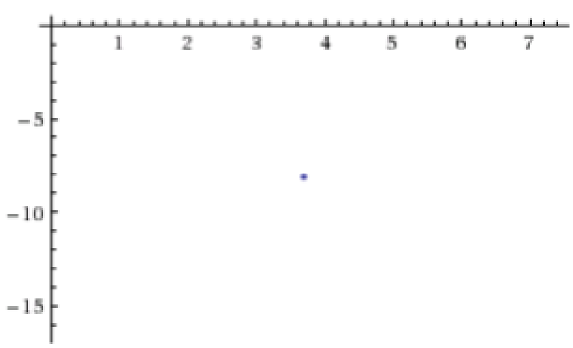
$(7.74, 144.46^o)$

5. $\mathbf{r} = (x^2 + y^2)^{1/2}$

$\mathbf{r} = (3.7^2 + (-8.2)^2)^{1/2}$

$\mathbf{r} = 9.00$

Plot:



Quadrant 4

$\Theta = \tan^{-1}(y/x)$

$\Theta = \tan^{-1}(3.7/-8.2)$

$\Theta = -24.29^o = -24.29^o + 360^o = 335.71^o$

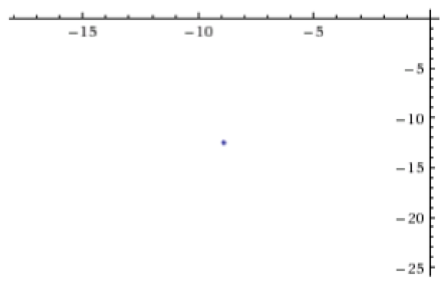
$(9.00, -24.29^o)$ or $(9.00, 335.71^o)$

6. $r = (x^2 + y^2)^{1/2}$

$r = ((-8.9)^2 + (-12.5)^2)^{1/2}$

$r = 15.35$

Plot:



Quadrant 3

$\Theta = \tan^{-1}(y/x) + 180^o$

$\Theta = \tan^{-1}(-8.9/-12.5) + 180^o$

$\Theta = 215.45^o$

$(15.35, 215.45^o)$

7. $x = r\cos(\Theta)$

$x = 9\cos(45^o)$

$x = 6.36$

$y = r\sin(\Theta)$

$y = 9\sin(45^o)$

$y = 6.36$

$(6.36, 6.36)$

8. $x = r\cos(\Theta)$

$x = 6\cos(32°)$

$x = 5.09$

$y = r\sin(\Theta)$

$y = 6\sin(32°)$

$y = 3.18$

(5.09, 3.18)

9. $x = r\cos(\Theta)$

$x = 12\cos(127°)$

$x = -7.22$

$y = r\sin(\Theta)$

$y = 12\sin(127°)$

$y = 9.58$

(-7.22, 9.58)

10. $x = r\cos(\Theta)$

$x = 4.7\cos(118.6°)$

$x = -2.25$

$y = r\sin(\Theta)$

$y = 4.7\sin(118.6°)$

$y = 4.13$

(-2.25, 4.13)

11. $x = r\cos(\Theta)$

$x = 5.6\cos(210°)$

$x = -4.8$

$y = r\sin(\Theta)$

$y = 5.6\sin(210°)$

$y = -2.8$

(-4.8, -2.8)

12. $x = r\cos(\Theta)$

$x = 7.8\cos(301.9^\circ)$

$x = 4.12$

$y = r\sin(\Theta)$

$y = 7.8\sin(301.9^\circ)$

$y = -6.62$

(4.12, -6.62)

13. (8.60, 54.46º)

14. (15.26, 58.39º)

15. (17.46, 113.63º)

16. (10.34, -52.47º) or (10.34, 307.53º) 52.47º + 360º = 307.53º

17. (8.02, 4.09)

18. (2.59, 9.66)

19. (-4.62, -0.86)

20. (13.11, -3.03)

Note: (13.45, -13º) will get you the same answer because 347º-360º=-13º