



Craig Hane, Ph.D., Founder

## Workforce Development: Module 2

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## 1.1 Lessons Abbreviation Key Table

C = Calculator Lesson  
P = Pre-algebra Lesson

The number following the letter is the Lesson Number.

E = Exercises with Answers: Answers are in brackets [ ].  
EA = Exercises Answers: (only used when answers are not on the same page as the exercises.)  
ES = Exercises Supplemental: Complete if you feel you need additional problems to work.

## 1.2 Exercises Introduction

### Why do the Exercises?

Mathematics is like a "game." The more you practice and play the game the better you will understand and play it.

The Foundation's Exercises, which accompany each lesson, are designed to reinforce the ideas presented to you in that lesson's video.

It is unlikely you will learn math very well by simply reading about it or listening to Dr. Del, or anyone else, or watching someone else doing it.

You WILL learn math by "doing math."

It is like learning to play a musical instrument, or write a book, or play a sport, or play chess, or cooking.

You will learn by practice.

Repetition is the key to mastery.

You will make mistakes. You will sometimes struggle to master a concept or technique. You may feel frustration sometimes **"WE ALL DO."**

But, as you learn and do math, you will begin to find pleasure and enjoyment in it as you would in any worthwhile endeavor. Treat it like a sport or game.

**These exercises are the KEY to your SUCCESS!**

**ENJOY!**

## PRE-ALGEBRA INTRODUCTION

In this Foundation course we will be dealing with what are commonly called "Real Numbers" which consist of:

Integers or Whole Numbers, both positive and negative.

Fractions, or quotients, or ratios of integers.

We will usually express numbers in the standard decimal format such as:

$$327.45 = 3 \times 100 + 2 \times 10 + 7 + .4 + .05 \text{ where} \\ .4 = 3/10 \text{ and } .5 = 5/100$$

The Real Numbers correspond to points on a straight line.

There are four basic arithmetic operations:  $+$   $-$   $\times$   $\div$  and a few higher level operations such as:  $x^2$   $1/x$   $\sqrt{x}$

There are several "Rules" or "Laws" of arithmetic.

We assume you already know most of this and will review it briefly in the following Pre-algebra lessons. See the Table of Contents for a listing of the lessons.

We will use the TI 30XA calculator for most of the calculations we perform in this Foundations Course since it accelerates the learning and application of what you will be learning significantly.

The Keys to perform these operations have been discussed in the Lessons on the use of the TI 30XA calculator.

## P1 LESSON: REAL NUMBERS, INTEGERS AND RATIONALS

First, there are the "counting numbers," 1, 2, 3, 4...also called Natural Numbers and Positive Integers.

We count with the usual decimal system which you should know.

Then we have the number Zero (0) which signifies the absence of something.

Then there are the "negative integers." These are just like the integers; but, have a  $-$  sign in front of them, e.g., -5, -6 . . .

Then there are the "fractions" or "rational" numbers which are the ratios or quotients of integers,  $3/4$ ,  $-7/8$ ,  $15/7$ , etc.

We will usually express numbers in the standard decimal format such as:

$$327.45 = 3 \times 100 + 2 \times 10 + 7 + .4 + .05 \text{ where}$$

$$.4 = 4/10 \text{ and } .05 = 5/100$$

It is sometimes easiest to understand these numbers when they are corresponded to points on a straight line, see the next lesson P2.

Later we will review the various operations and "rules" of arithmetic.

Always use the calculator to help yourself understand the various things we are discussing.

We assume this is essentially a review for things you already have learned.

## **P1E**

### **ARITHMETIC REVIEW**

1. What kind of numbers will we deal with in the Foundation Course?
2. What are Integers?
3. What are Rational Numbers?
4. What number is  $3 \times 100 + 2 \times 10 + 7 + 0.4 + 0.05$ ?
5. What do the Real Numbers correspond to?
6. What are the four basic operations?
7. What calculator will we use in the Foundation Course?
8. What other three subjects will we learn about in the Foundations Course after Pre-Algebra?

**Answers on P1EA, page 8.**

**Take the Quiz**

**P1EA**

**ARITHMETIC REVIEW**      Answers: [ ]'s

1. What kind of numbers will we deal with in the Foundation Course?      **[Real Numbers]**
2. What are Integers?      **[Whole or counting numbers both positive and negative]**
3. What are Rational Numbers?      **[Fractions,  $a/b$  where  $a$  and  $b$  are integers,  $b \neq 0$ ]**
4. What number is  $3 \times 100 + 2 \times 10 + 7 + 0.4 + 0.05$ ?      **[327.45]**
5. What do the Real Numbers correspond to?      **[Points on a straight line]**
6. What are the four basic operations?      **[+, -,  $\times$ ,  $\div$ ]**
7. What calculator will we use in the Foundation Course?  
  
**[TI-30XA]**
8. What other three subjects will we learn about in the Foundations Course after Pre-Algebra?  
  
**[Algebra, Geometry, Trigonometry]**

## P2 LESSON: THE NUMBER LINE, NEGATIVE NUMBERS

The Real Numbers we will be using in this Foundation Course can be corresponded to the points on a straight line called the Number Line.

We select a point to call Zero, 0.

We then select a point to the right of 0 and label it 1.

This establishes a "scale" and all numbers now correspond to one unique point on the line. (See below)

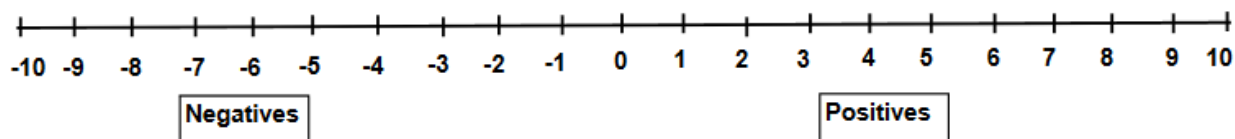
Positive numbers are to the right of 0, and Negative numbers are to the left of 0. The Negatives are a sort of "mirror" image of the Positives.

$a < b$  means a is to the left of b on the number line.

$a > b$  means a is to the right of b on the number line.

$a = b$  means a and b correspond to the same point.

You should be able to find the appropriate point on the line for any number, and vice versa.



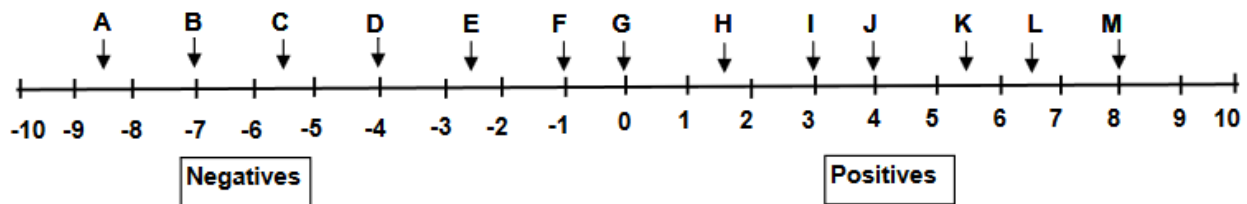


P2E

THE NUMBER LINE, NEGATIVE NUMBERS

Answers: [ ]'s

1. Which letter is above 5.5? [K]
2. Which letter is above 3? [I]
3. Which letter is above -7? [B]
4. Which letter is above -2.5? [E]
5. What number is C above? [-5.5]
6. What number is L above? [6.5]
7. What number is G above? [0]
8. Is  $-3 > -6$ ? [Yes]
9. Is  $-3 < 1$ ? [Yes]
10. Is  $-6 > 0$ ? [No]



**Problem:** Given a number, find its location on the number line.

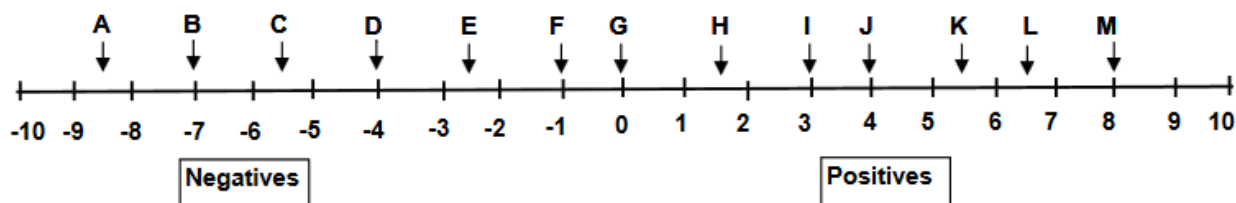
**Problem:** Give a point on the number line, estimate its value.

## P2ES

### THE NUMBER LINE, NEGATIVE NUMBERS

Answers: [ ]'s

1. Which letter is above -4? [D]
2. Which letter is above 1.6? [H]
3. Which letter is above -5.5? [C]
4. Which letter is above 6.5? [L]
5. What number is E above? [-2.5]
6. What number is K above? [5.5]
7. What number is C above? [-5.5]
8. Is  $-1 > -3$ ? [Yes]
9. Is  $-3 < -1$ ? [Yes]
10. Is  $-6 > -7$ ? [Yes]



**Problem:** Given a number, find its location on the number line.

**Problem:** Give a point on the number line, estimate its value.

### P3 LESSON: RULES OF ADDITION + -

Rules of Addition:  $a, b, c$  represent an arbitrary real numbers

1.  $a + 0 = a$

$$7 + 0 = 7$$

2.  $a + b = b + a$

$$15 + 6 = 6 + 15 = 21$$

3.  $(a + b) + c = a + (b + c)$      $(4 + 7) + 5 = 4 + (7 + 5) = 16$

4.  $-(-a) = +a = a$

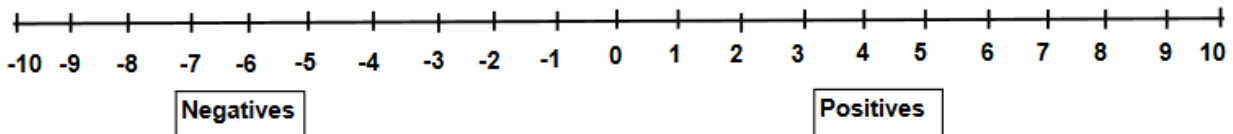
$$-(-8) = 8$$

5.  $b - a = b + (-a)$      $7 - 3 = 7 + (-3) = 4$      $4 - 9 = 4 + (-9) = -5$

6.  $a - a = a + (-a) = 0$

$$8 - 8 = 0 = 8 + (-8)$$

Note how addition works on the Number Line. Watch the video lesson that accompanies this lesson.



**Problem:** Given two numbers, find their sum's location on the number line.

**Problem:** Given two numbers, find their difference's location on the number line.

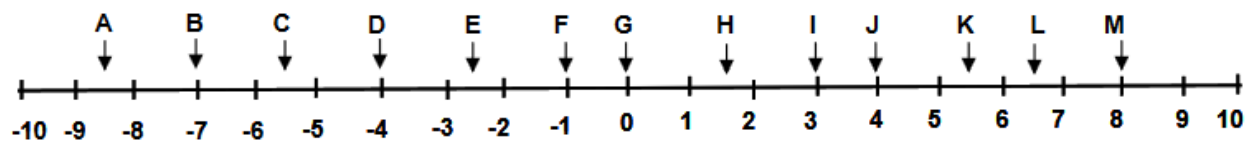


## P3ES

### RULES OF ADDITION +, -

Answers: [ ]'s

1.  $13 + 29 = ?$  [42]
2.  $176 + 839 + 538 = ?$  [1553]
3.  $17.4 + 35.3 + 34.9 = ?$  [87.6]
4.  $57.4 - 89.2 = ?$  [-31.8]
5.  $0.068 + 0.036 = ?$  [0.104]
6.  $83 - 345 = ?$  [-262]
7.  $92 - (-34) = ?$  [126]
8. Where is  $J + F$  on the number line? [I]
9. Where is  $K - F$  on the number line? [L]
10. Where is  $7.7 - 2.2$  on the number line? [K]
11.  $-(-37) + (-23) = ?$  [14]
12.  $-(-37) - (-23) = ?$  [60]



Take Quiz or review

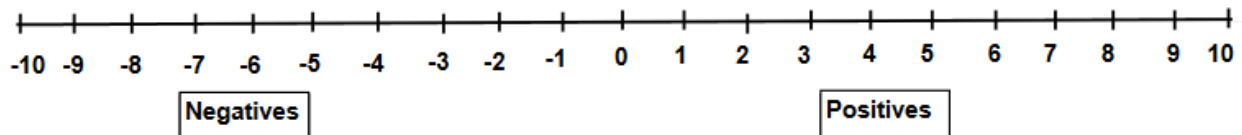
## P4 LESSON: RULES OF MULTIPLICATION X ÷

Multiplication of Real Numbers  $axb$  or  $ab$  or  $a \cdot b$

$a, b, c$  represents arbitrary real numbers

- $a \times 0 = 0$                        $7 \times 0 = 0$
- $a \times 1 = a$                          $13 \times 1 = 13$
- $a \times b = b \times a$  [ $ab = bc$ ]     $15 \times 6 = 6 \times 15 = 90$
- $(ab)c = a(bc)$                      $(4 \times 7) \times 5 = 4 \times (7 \times 5) = 140$
- $(-a) \times b = -(axb)$                $(-13) \times 12 = -156$
- $(-a) \times (-b) = axb$                $-5 \times (-6) = 30$
- $ax(1/a) = 1$  ( $a \neq 0$ )         $7 \times (1/7) = 1$
- $a \div b = ax(1/b)$  ( $b \neq 0$ )     $12 \div 4 = 3 = 12 \times (1/4)$

Note how Multiplication works on the Number Line. Watch the Video lesson that accompanies this lesson.



**Problem:** Given two numbers, find their product's location on the number line.

**Problem:** Given a number, find its reciprocal location on the number line.







## P5 LESSON: DISTRIBUTIVE LAW + AND X COMBINED

### Distributive Law and Factoring Real Numbers

$a, b, c$  represents arbitrary real numbers

1.  $ax(b + c) = axb + axc$  or  $a(b + c) = ab + ac$  Simplifying

2.  $axb + axc = ax(b + c)$  or  $ab + ac = a(b + c)$  Factoring

$x, y, z$  represent arbitrary numbers

3.  $(x + y)z = xz + yz$  Simplifying

4.  $xz + yz = (x + y)z = z(x + y)$  Factoring

Note how the Distributive Law works on the Number Line. Watch the Video lesson that accompanies this lesson.

## P5E

### DISTRIBUTIVE LAW + AND X COMBINED

Answers: [ ]'s

#### Distributive Law and Factoring Real Numbers

- $12x(34 + 23) = ?$  [684]
- $(2.5 - 3.7)x6.9 = ?$  [-8.3]
- $(78.9 + 43.7)x(34.1 + 13.4) = ?$  [5823.5]
- $45x67 + 45x82 = 45x(?)$  [67 + 82 = 149]
- $576x4 - 576x3 = ?$  [576x(4-3) = 576]
- $ab + ad = ax(?)$  [(b + d)]
- $tu + vt = t(?)$  [(u + v)]
- $ab^2 - ac^2 = ?x(b^2 - c^2)$  [a]
- $5.4x2 + 5.4x3 + 5.4x4 + 5.4x5 + 5.4x6 = ?$  [5.4x20 = 108]
- $z^3v + t^2v = (?)v$  [z<sup>3</sup> + t<sup>2</sup>]
- $-3.4x(7.8 - 9.4) = ?$  [5.4]
- $(123 + 876 - 276)x0 = ?$  [0]
- $54.5(21.4 + 87.3 - 17.4)$  [4975.9]
- $0.02x(0.003 + 0.015) = ?$  [0.00036]
- $-17x(-6 - 9) = ?$  [255]

Take Quiz or do more exercises on P5ES

## P5ES

### DISTRIBUTIVE LAW + AND X COMBINED

Answers: [ ]'s

#### Distributive Law and Factoring

1.  $13x(35 + 43) = ?$  [1014]
2.  $(3.5 - 4.9)x6.2 = ?$  [-8.7]
3.  $(7.9 + 43.7)x(4.1 + 13.4) = ?$  [903]
4.  $42x69 + 42x82 = 42x(?)$  [69 + 82 = 151]
5.  $579x7 - 579x6 = ?$  [579]
6.  $as + ad = ax(?)$  [(s + d)]
7.  $ta + bt = t(?)$  [(a + b)]
8.  $ab^3 - abc^2 = ?x(b^2 - c^2)$  [ab]
9.  $abc - ac = acx?$  [(b - 1)]
10.  $z^2v - vt^2 = (?)v$  [ $z^2 - t^2$ ]
11.  $33/4x(7/8 - 9/4) = ?$  [-55/32 = -165/32 = -5.16]
12.  $(12.3 + 886 - 276)x0 = ?$  [0]
13.  $56.5(27.4 + 7.3 - 17.4) = ?$  [977.5]
14.  $0.01x(0.008 + 0.015) = ?$  [0.00023]

**Take Quiz or review**

## P6 LESSON: FRACTIONS, A/B AND C/D, RULES

Rules for adding and multiplying and dividing fractions

a, b, c, d represent arbitrary real numbers with  $b \neq 0$ ,  $d \neq 0$

1.  $a/b + c/d = (ad + bc)/bd$

2.  $a/b - c/d = (ad - bc)/bd$

3.  $(a/b) \times (c/d) = (ac)/(bd)$

4.  $(a/b) \div (c/d) = (a/b) \times (d/c) = (ad)/(bc)$ , now  $c \neq 0$  also

5. Rules regarding  $-$  same as in multiplication.  $- \div - = +$

You may learn to do this manually, or you can learn to use the TI 30XA calculator. It does restrict denominators to be less than 1000.

\*\*\*\*\*

**Review the calculator lessons C10, C11, and C12, if necessary.**

Work problems along with Dr. Del as he does them:

$$2/3 + 3/4 = 17/12 = 1 \frac{5}{12}$$

$$(-1/2) \times (2/3) = -1/3$$

$$(-1/2) \times (-2/3) = 1/3$$

$$(3/4) \times (7/8) = 21/32 = 0.65625$$

**P6E****FRACTIONS, A/B AND C/D, RULES**

Answers: [ ]'s

1.  $2/3 + 3/4 = ?$  [1 5/12 = 17/12 = 1.42]
2.  $5 \frac{6}{7} + 3 \frac{8}{9} = ?$  [9 47/63 = 9.75]
3.  $1 \frac{7}{8} - 1 \frac{1}{2} = ?$  [3/8]
4.  $7/8 - 3/5 = ?$  [11/40]
5.  $6/7 \times 3/8 = ?$  [9/28]
6.  $6/7 \div 3/8 = ?$  [2 2/7 = 16/7 = 2.29]
7. Express 18/5 as a mixed fraction. [3 3/5]
8. Express 18/5 in decimal form. [3.6]
9. Express 0.35 as a fraction. [7/20]
10. Express  $4 \frac{7}{8}$  as an improper fraction. [39/8]
11. Express  $4 \frac{7}{8}$  as a decimal. [4.875]
12.  $3/4 \times (1 \frac{2}{3} + 2 \frac{1}{2}) = ?$  [3 1/8 = 25/8 = 3.125]
13.  $2 \frac{3}{4} - 23/8 = ?$  [3/8]
14.  $3 \frac{5}{8} \times 3 \frac{5}{8} = ?$  [13 9/64 = 13.14]
15. Express 2/3 as a decimal Real Number. [0.6667]
16.  $1/a + 1/b = ?$  [(a + b)/ab]

**Take Quiz or do more exercises on P6ES**

**P6ES****FRACTIONS, A/B AND C/D, RULES**

Answers: [ ]'s

1.  $2/5 + 3/8 = ?$  [31/40]
2.  $2 \frac{6}{7} + 1 \frac{2}{3} = ?$  [ $4 \frac{11}{21} = 95/21 = 4.5$ ]
3.  $1 \frac{5}{6} - 1 \frac{1}{2} = ?$  [1/3]
4.  $5/8 - 4/5 = ?$  [-7/40]
5.  $4/7 \times 5/8 = ?$  [5/14]
6.  $4/7 \div 5/8 = ?$  [32/35]
7. Express  $19/7$  as a mixed fraction. [2 5/7]
8. Express  $18/5$  in decimal form. [3.6]
9. Express 0.22 as a fraction. [11/50]
10. Express  $3 \frac{5}{9}$  as an improper fraction. [32/9]
11. Express  $3 \frac{5}{9}$  as a decimal. [3.56]
12.  $3/4 \times (2 \frac{2}{3} + 3 \frac{1}{2}) = ?$  [ $4 \frac{5}{8} = 37/8 = 4.625$ ]
13.  $2 \frac{3}{5} - 2 \frac{3}{4} = ?$  [-3/20 = -0.15]
14.  $(3 \frac{5}{8})^2 = ?$  [ $13 \frac{9}{64} = 841/64 = 13.1$ ]
15.  $1/ab + 1/cb = ?$  [(c + a)/(abc)]

Take Quiz or review

## P7 LESSON: SQUARES $X^2$ X SQUARED

$A^2 = A \times A$  and we say: A squared

1.  $(AB)^2 = A^2B^2$  Commutative Law yields this.
2.  $(1/A)^2 = 1/A^2$
3.  $(A + B)^2 = A^2 + 2AB + B^2$  Distributive Law yields this.
4.  $(A - B)^2 = A^2 - 2AB + B^2$  Distributive Law again.

The  $x^2$  Key will automatically square any number.

Work problems along with Dr. Del as you watch the video:

$$(3 \times 4)^2 = 144 = 3^2 \times 4^2 \text{ or } (3 \times 4)^2 = 144 = (3^2) \times (4^2)$$

$$(1/7)^2 = 1/7^2$$

$$(25.3)^2 = (25.3)^2 = 640.09$$

$$(-8)^2 = (-8)^2 = 64$$

$A^2 > 0$   $A^2$  is positive, if A is non zero

**P7E****SQUARES  $X^2$  X SQUARED****Answers: [ ]'s**

1.  $(34.5)^2 = ?$  [1190.25]

2.  $(87)^2 = ?$  [7569]

3.  $(-23)^2 = ?$  [529]

4.  $(2.4^2 + 3.5^2)^2 = ?$  [324.4]

5.  $(65.9)^2 = ?$  [4343]

6.  $(89 + 57 - 32)^2 = ?$  [12996]

7.  $(12.3)^2/7.6$  [19.9]

8.  $(15.4 \div 0.35)^2 = ?$  [1936]

9.  $(1 + 0.08)^2 = ?$  [1.167]

10.  $(X + Y)^2 - X^2 - Y^2 = ?$  [2XY]

11.  $(A - B)^2 - A^2 - B^2 = ?$  [-2AB]

12.  $(3/4)^2 = ?$  [9/16 = 0.5625]

13.  $3^2 + 4^2 = ?$  [25 = 5<sup>2</sup>]

14.  $(0.25)^2 = ?$  [0.0625]

**Take Quiz or do more exercises on P7ES.**



## P7ES

SQUARES  $X^2$  X SQUARED

Answers: [ ]'s

1.  $(3 \frac{4}{5})^2 = ?$  [14.44 = 14  $\frac{11}{25}$ ]

2.  $(8.7)^2 = ?$  [75.7]

3.  $(-2/3)^2 = ?$  [0.444 = 4/9]

4.  $(1.4^2 + 2.5^2)^2 = ?$  [67.4]

5.  $(1 \frac{2}{3} - 2 \frac{3}{4})^2 = ?$  [1.17 = 1  $\frac{25}{144}$ ]

6.  $(8.9 + 5.7 - 3.2)^2 = ?$  [130.0]

7.  $(3.3)^2 / (2.6)^2 = ?$  [1.6]

8.  $(12.4 \div 0.85)^2 = ?$  [212.8]

9.  $[(1 + 0.05)^2]^2 = ?$  [1.22]

10.  $X^2 + Y^2 + 2XY = ?$   $[(X + Y)^2]$

11.  $(0.01)^2 = ?$  [0.0001]

12.  $(2/3)^2 = ?$  [4/9 = 0.444]

13.  $1^2 + 2^2 + 3^2 + 4^2 + 5^2 = ?$  [55]

14.  $(1.25)^2 = ?$  [1.56]

Take Quiz or review

## P8 LESSON: SQUARE ROOTS $\sqrt{x}$

$\sqrt{A}$  is a number whose square will equal A.

$(\sqrt{A})^2 = A$ ,  $\sqrt{A}$  can be positive or negative

A must be positive or  $\sqrt{A}$  will not be a real number.

The  $\sqrt{x}$  Key will calculate the square root of any positive number and give you the positive square root.

$\sqrt{x}$  will return an Error message on the TI 30XA if  $x < 0$ .

$\sqrt{a^2} = a$ ,  $\sqrt{x}$  and  $x^2$  are inverses, i.e., undo each other.

Note:  $\sqrt{(a + b)} \neq \sqrt{a} + \sqrt{b}$

\*\*\*\*\*

$$\sqrt{9} = 3 \quad (-3)^2 = 3^2 = 9$$

$$\sqrt{16} = 4 \quad 4^2 = 16$$

$$\sqrt{89} = 9.4 \quad \text{Note: } (9.4)^2 = 88.36$$

$\sqrt{2} = 1.414213562\dots$  "Irrational" Number Infinite non-repeating decimal

Irrational means NO fraction will equal  $\sqrt{2}$

Fractions  $a/b$ , where a, b are integers, are called "Rational numbers"

$\sqrt{-6}$  Error "Complex Number"

$$\sqrt{(12 + 54)} = 8.12 \quad \text{where } \sqrt{12} + \sqrt{54} = 3.46 + 7.39 = 10.8$$

You DO IT! Then "play with" the  $\sqrt{\phantom{x}}$  function, key.

**P8E****SQUARE ROOTS  $\sqrt{x}$** 

Answers: [ ]'s

1.  $\sqrt{81} = ?$  [9]
2.  $\sqrt{56.9} = ?$  [7.5]
3.  $\sqrt{745365} = ?$  [863]
4.  $\sqrt{(87)^2} = ?$  [87]
5.  $(\sqrt{95})^2 = ?$  [95]
6.  $\sqrt{(9 + 16)} = ?$  [5]
7.  $(1 + \sqrt{32})^2 = ?$  [44.3]
8.  $\sqrt{0.25} = ?$  [0.5]
9.  $\sqrt{0.0001} = ?$  [0.01]
10.  $(\sqrt{16} + \sqrt{9})^2 = ?$  [49]
11.  $\sqrt{1/4} = ?$  [1/2]
12.  $\sqrt{1/2} = ?$  [0.707]
13.  $\sqrt{9/16} = ?$  [3/4]
14.  $\sqrt{-9} = ?$  [Error]

Take Quiz or do more exercises on P8ES.

**P8ES****SQUARE ROOTS  $\sqrt{x}$** 

Answers: [ ]'s

1.  $\sqrt{144} = ?$  [12]
2.  $\sqrt{256} = ?$  [16]
3.  $\sqrt{123456} = ?$  [351.4]
4.  $\sqrt{(67)^2} = ?$  [67]
5.  $(\sqrt{67})^2 = ?$  [67]
6.  $\sqrt{(3^2 + 4^2)} = ?$  [ $\sqrt{5^2} = 5$ ]
7.  $(\sqrt{23} + \sqrt{32})^2 = ?$  [109.3]
8.  $\sqrt{0.1111} = ?$  [0.3333]
9.  $\sqrt{0.000001} = ?$  [0.001]
10.  $\sqrt{0.00001} = ?$  [0.0032]
11.  $\sqrt{(1/25)} = ?$  [0.2 = 1/5]
12.  $\sqrt{(1+ 3 +5 + 7 +9)} = ?$  [5]
13.  $\sqrt{(1+3+5+7+9+11+13+15)} = ?$  [8]
14. Do you see a pattern in the last two problems?

**Take Quiz or review**

**P9 LESSON: RECIPROCAL     1/X     X ≠ 0**

- 1.  $1/x = 1 \div x$                        $1/4 = 1 \div 4 = .25$
- 2.  $1/(1/x) = x$                        $1/x$  is its own inverse
- 3.  $1/a + 1/b = (a + b)/ab$      see fractions
- 4.  $(1/x)^2 = 1/x^2$                       see rules of exponents (P10)
- 5.  $1/\sqrt{x} = \sqrt{(1/x)}$                       see rules of exponents (P10)

\*\*\*\*\*

$1/0$  is undefined                       $1/0$  Error     Never divide by 0

$1/1/4 = 4$                                    $1/x$  Key is its own inverse

$1/9 = .111111111111...$

$(1/3)^2 = 1/3^2 = 1/9 = .111111111111...$

$1/\sqrt{16} = 1/4 = \sqrt{(1/16)} = .25$

$\sqrt{.5} = .707$      and      $.5 < .707$

**P9E****RECIPROCAL  $1/X$ ,  $X \neq 0$       Answers: [ ]'s**

1.  $1/7 = ?$       [0.1429]
2.  $1/25 = ?$       [0.04]
3.  $1/0.05 = ?$       [20]
4.  $1/(0.1 + 0.2) = ?$       [3.33]
5.  $(1/3.3)^2 = ?$       [0.0918]
6.  $1/(3.3)^2 = ?$       [0.0918]
7.  $\sqrt{(1/9)} = ?$       [1/3]
8.  $1/\sqrt{(3^2 + 4^2)} = ?$       [0.2]
9.  $1/1/7$       [7]
10.  $1/0$       [Error]
11.  $1/(a + b) = ?$       [1/(a + b)]
12.  $1/\sqrt{9} = ?$       [1/3]
13.  $1/(\sqrt{16} + \sqrt{25})$       [0.1111111111]
14.  $(1 + 1/10)^2 = ?$       [1.21]
15. What operation is its own inverse?      [1/x]

**Take Quiz or do more exercises on P9ES**



# P10 LESSON: EXPONENTS $Y^X$ $Y > 0$ , $X$ CAN BE ANY NUMBER

Definitions  $A^0 = 1$   $y^x$  is sometimes used for  $y^x$

1.  $A^n = AxAx \dots xA$ ,  $n$  times when  $n$  positive integer
2.  $A^{1/n}$  is number such that  $(A^{1/n})^n = A$
3.  $A^{-n} = 1/A^n$  Negative exponents.
4.  $A^{n/m} = (A^{1/m})^n$  Exponents defined for any rational number.
5.  $A^x$  can be defined for any real number.  $A > 0$ .

## Rules of Exponents

6.  $A^n \times A^m = A^{n+m}$
7.  $(A^n)^m = A^{nm}$

\*\*\*\*\*

$y^x$   $y$  times itself  $x$  times,  $y$  is base,  $x$  is exponent or power

$$3^4 = 81 : 4^3 = 64 : 2^3 = 8$$

<u>Name</u>	<u>Digital Base 2</u>	<u>Metric Base 10</u>	
Kilo K	$2^{10} = 1024$	$10^3 = 1000$	Thousand
Mega M	$2^{20} = 1048576$	$10^6 = 1000000$	Million
Giga G	$2^{30} = 11073741824$	$10^9 = 1000000000$	Billion
Tera T	$2^{40} =$ You do it.	$10^{12} = 12$ Zeros	Trillion

$$8^{1/3} = 2 \qquad (987)^{1/3} = 9.956$$

$$9^{-2} = .0123 = 1/9^2 \qquad (16)^{-1/2} = .25 = 1/4$$

$$(81)^{-1/4} = .3333... = 1/81^{1/4}$$

$$177,147 = 3^{11} = 3^{(4+7)} = 3^4 \times 3^7 = 81 \times 2187$$

$$9^3 = (3^2)^3 = 3^6 = 729 \qquad 5^{2.6} = 65.66$$



## P10E

### EXPONENTS $Y^X$ ; $Y > 0$ , $X$ ANY NUMBER

Answers: [ ]'s

1.  $2^8 = ?$  [256]
2.  $12^3 = ?$  [1728]
3.  $(17.1)^4 = ?$  [85504]
4.  $10^9 = ?$  [1,000,000,000]
5.  $(1 + 0.06)^{20} = ?$  [3.2]
6.  $15^{2.7} = ?$  [1498]
7.  $1/(0.5)^4 = ?$  [16]
8.  $25^{1/2} = ?$  [5]
9.  $81^{1/4} = ?$  [3]
10.  $5^{-2} = ?$  [0.04 = 1/25]
11.  $2^{30} = ?$  [1,073,741,824 1 GIG]
12.  $1000 \times (1.06)^{100} = ?$  [339,302]
13.  $1000 \times (1.07)^{100} = ?$  [867,716]
14.  $26 \times (1 + 0.06)^{400} = ?$  [3.446x10<sup>11</sup> = 344,600,000,000]

Take Quiz or do more exercises on P10ES.

## P10ES

### EXPONENTS $Y^X$ ; $Y > 0$ , $X$ ANY NUMBER

Answers: [ ]'s

1.  $2^{10} = ?$  [1,024 K]
2.  $2^{20} = ?$  [1,048,576 M]
3.  $2^{30} = ?$  [1,073,741,824 G]
4.  $10^3 = ?$  [1,000 K]
5.  $10^6 = ?$  [1,000,000 M]
6.  $10^9 = ?$  [1,000,000,000 G]
7.  $1/5^2 = ?$  [0.04]
8.  $5^{-2} = ?$  [0.04]
9.  $1281^{1/4} = ?$  [5.98]
10.  $(5.98)^4 = ?$  [1279]
11.  $2^{64} = ?$  [1.845x10<sup>19</sup>]
12.  $(1.02)^{2000} = ?$  [158,000,000,000,000,000]

[\$1 invested at time of Christ's birth earning 2% per year compounded would be more money than in the world today. 1% would yield only 440 million.]

Take Quiz or review