

Craig Hane, Ph.D., Founder

# Workforce Development: Module 4

1.1	Lessons Abbreviation Key Table
1.2	Exercises Introduction
INTROD	JCTION TO GEOMETRY
G1 LESSO	DN: WHAT IS GEOMETRY?
G1E	
G1EA.	9
G2 LESS	ON: STRAIGHT LINES AND ANGLES
G2E	
G2EA.	
G3 LESSO	DN: PARALLEL LINES
G3 Pro	oblems for Parallel Lines
G3E	
G3EA.	
G3 EA	(cont'd)
G3ES.	
G3ESA	A19
G4 LESSO	ON: TRIANGLES, DEFINITION, SUM OF ANGLES

G4 Triangle Problems	21
G4E	22
G4EA	23
G4ES	24
G4ESA	25
5 LESSON: RIGHT TRIANGLES - PYTHAGOREAN THEOREM	
G5 Right Triangle Problems	27
G5E	
G5EA	29
G5ES	
G5ESA	
5 LESSON: SIMILAR TRIANGLES	
G6 Similar Triangles Problems	
G6E	
G6EA	
G6ES	
G6ESA	
LESSON: QUADRILATERALS, POLYGONS, PERIMETERS (P)	
G7 Quadrilaterals, Polygons, Perimeters (P) Problems	
G7E	40
G7EA	41
G7ES	

### 1.1 Lessons Abbreviation Key Table

- C = Calculator Lesson
- P = Pre-algebra Lesson
- A = Algebra Lesson
- G = Geometry Lesson
- T = Trigonometry Lesson
- S = Special Topics

#### The number following the letter is the Lesson Number.

- E = Exercises with Answers: Answers are in brackets [].
- EA = Exercises Answers: (only used when answers are not on the same page as the exercises.)
- ES = Exercises Supplemental: Complete if you feel you need additional problems to work.

#### 1.2 Exercises Introduction

#### Why do the Exercises?

Mathematics is like a "game." The more you practice and play the game the better you will understand and play it.

The Foundation's Exercises, which accompany each lesson, are designed to reinforce the ideas presented to you in that lesson's video.

It is unlikely you will learn math very well by simply reading about it or listening to Dr. Del, or anyone else, or watching someone else doing it.

You WILL learn math by "doing math."

It is like learning to play a musical instrument, or write a book, or play a sport, or play chess, or cooking.

You will learn by practice.

Repetition is the key to mastery.

You will make mistakes. You will sometimes struggle to master a concept or technique. You may feel frustration sometimes "WE ALL DO."

But, as you learn and do math, you will begin to find pleasure and enjoyment in it as you would in any worthwhile endeavor. Treat it like a sport or game.

### These exercises are the KEY to your SUCCESS!

## **ENJOY!**

#### INTRODUCTION TO GEOMETRY

The Foundation Course is dedicated to your learning how to solve practical math problems that arise in a wide variety of industrial and "real world" situations.

In addition to learning how to use the power tool called a scientific calculator, you need to learn material from three fields, Algebra, Geometry and Trigonometry.

Geometry is the "Centerpiece" of math that you will use in most problems. It is all about physical space in one, two, and three dimensions: Lines, Flat Surfaces and 3-D objects.

Algebra is a tool that is often used along with Geometry to solve problems.

You use Geometry to set up an equation which you then solve for the unknown. The unknown might be a length, or some dimension you need to know, or area, or volume.

Trigonometry is a special subject used for triangles. There are occasions where you cannot solve a problem with just algebra and geometry alone and where you need trigonometry. It deals with triangles.

Geometry is one of humankind's oldest mathematical subjects along with numbers and algebra.

Geometry is the foundation of modern science and technology and much modern mathematics.

Mathematics is like a "contact" sport, or a game.

You learn by practicing and "doing."

Each Lesson will include a video discussion of the topic just as we did in Algebra.

Then you will be given Homework Problems to work.

You are encouraged to make up your own problems.

The more you "play" and the more questions you ask, the better you will learn.

When you think you are ready, take the Online Quiz.

This will give you an indicator if you have mastered the material. If not, go back and "play" some more.

Learning math is like climbing a ladder. If you do it one small step at a time, it is pretty easy. But, it is difficult to go from rung 4 to rung 9 directly.

This Foundation course has been designed to let you climb the ladder of math understanding in small steps.

But, **YOU** must do the climbing. Watching someone else climb isn't enough. Play the game.

### G1 LESSON: WHAT IS GEOMETRY?

Mathematics is based on two fundamental concepts:

#### Numbers and Geometry

Numbers are used to count and measure things.

Geometry is used to model physical things.

There are actually several different kinds of geometry.

We will study the oldest of all geometries, Euclidean.

Euclidean Geometry is used in most practical situations.

We will study:

	Points: 0 dimensional
Lines:	1 dimensional
Surface Objects:	2 dimensions
And:	3-D objects

We will learn how to analyze many geometric situations and then set up **Equations** to find the value of various unknowns. This could be how long something is, or how much area something is, or the volume of something.

Many of the practical problems one comes across in many walks of life involve some type of geometric object.

Historically, in our schools, emphasis has been placed on proving theorems (statements about geometric objects) with rigorous logic and step by step deductions.

This can be difficult and tedious, and sometimes seemingly meaningless. We will emphasis sound reasoning in the Foundation Course, but not formal "proofs."

### G1E

### WHAT IS GEOMETRY?

- 1. Math is based on what two fundamental concepts?
- 2. Numbers are used to?
- 3. Geometry is used to?
- 4. The oldest kind of Geometry is?
- 5. In Geometry we will study what four things?
- 6. What will we use to find unknowns in **Geometry**?
- 7. What kind of **Unknowns** might we wish to find?

G1EA

## WHAT IS GEOMETRY? Answers: []

1. Math is based on what two fundamental concepts?

[Numbers and Geometry]

- 2. Numbers are used to? [Count and measure things]
- 3. Geometry is used to? [Model physical things]

4. The oldest kind of geometry is? [Euclidean]

5. In Geometry we will study what four things?

[Points: 0 dimensional] [Lines: 1 dimension] [Surface Objects: 2 dimensions] [And: 3-D objects]

- 6. What will we use to find Unknowns in Geometry? [Equations and Algebra]
- 7. What kind of Unknowns might we wish to find?

[This could be how long something is, or how much area something is, or the volume of something.]

Many of the practical problems one comes across in many walks of life involve some type of geometric object.

### G2 LESSON: STRAIGHT LINES AND ANGLES

A **Point** is ideally a location in space with no length or width. It has zero area.

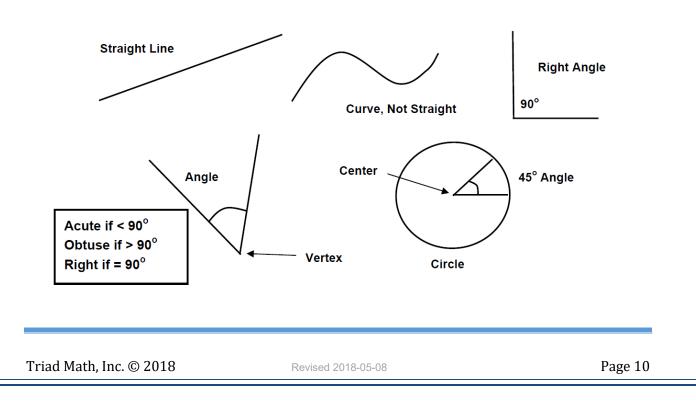
A **Plane** is a flat surface consisting of points. Think of a wall or blackboard as a plane. It is a surface with zero curvature.

A **Straight Line** (Segment) is the collection of points between two points that represents the shortest distance between them. It too has zero curvature. A **Straight Line** can be extended indefinitely.

The intersection of two lines (**straight**, unless I otherwise state), forms an **Angle** and their point of intersection is called the **Vertex**.

Angles are measured in **Degrees** (°) where there are 360° in a complete circle, a set of points equidistant from a point, center.

A **Right Angle** measures 90° and the two sides are **Perpendicular**.



### G2E

### STRAIGHT LINES AND ANGLES

- 1. What are: Point, Plane, and Straight Line?
- 2. What are an Angle and a Vertex?
- 3. How are Angles measured?
- 4. What is a Right Angle?
- 5. What are Acute and Obtuse Angles?

G2EA

### STRAIGHT LINES AND ANGLES Answers: []

1. What are: Point, Plane, and Straight Line?

[A Point is ideally a location in space with no length or width. It has zero area.

A Plane is a flat surface consisting of points. Think of a wall or blackboard as a plane. It is a surface with zero curvature.

A Straight Line (Segment) is the collection of points between two points that represents the shortest distance between them.]

2. What are an Angle and a Vertex?

[The intersection of two lines (straight, unless I otherwise state), forms an Angle and their point of intersection is called the Vertex.]

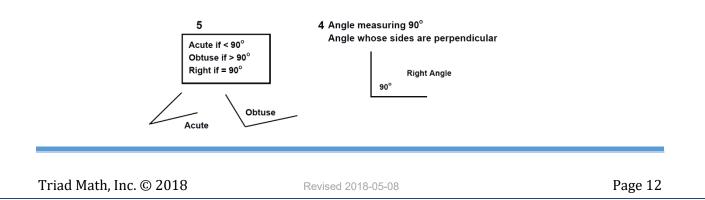
3. How are Angles measured?

[Angles are measured in Degrees (°) where there are 360° in a complete circle, a set of points equidistant from a point, center.]

4. What is a Right Angle?

[See Below Right]

5. What are Acute and Obtuse Angles? [See Below Left]



#### G3 LESSON: PARALLEL LINES

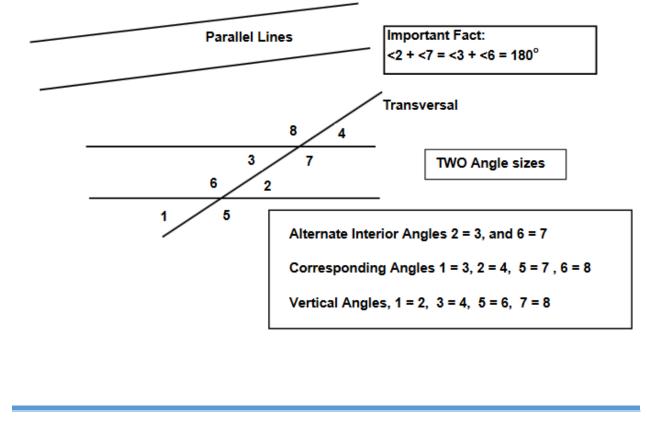
Two straight lines are **parallel** if they never intersect no matter how far they are extended in either direction.

The Fundamental Property in Euclidean Geometry is:

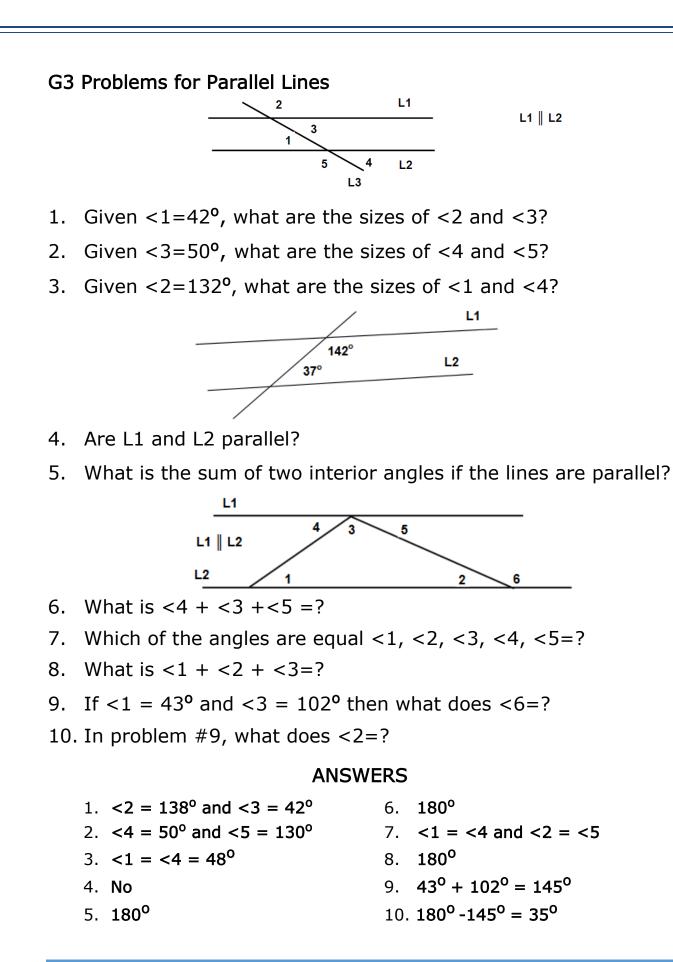
Given a straight line and an external point, there is exactly one straight line through this point parallel to the given line.

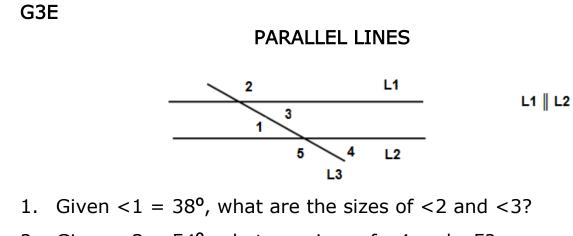
This is called the **Parallel Postulate** and it is not true for other **non-Euclidean** geometries.

When two parallel lines are crossed by another straight line, called a **transversal**, eight angles are created in two sets of four equal-sized angles. This is a critical property.

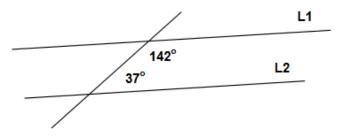


Triad Math, Inc. © 2018



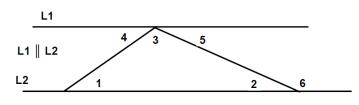


- 2. Given  $<3 = 54^{\circ}$ , what are sizes of <4 and <5?
- 3. Given  $<2 = 138^{\circ}$ , what are sizes of <1 and <4?
- 4. Given:

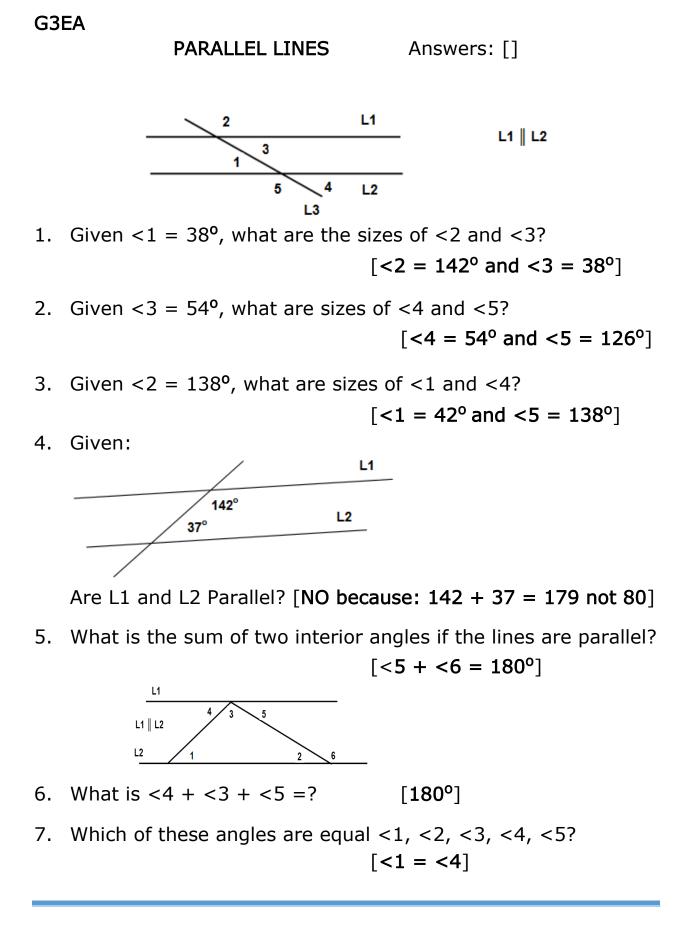


Are L1 and L2 Parallel?

5. What is the sum of two opposite interior angles if the lines are parallel?



- 6. What is <4 + <3 + <5 =?
- 7. Which of these angles are equal <1, <2, <3, <4, <5?
- 8. What is <1 + <2 + <3 =?
- 9. If  $<1 = 42^{\circ}$  and  $<3 = 105^{\circ}$ , what does <6 =?
- 10. In problem #9, what does <2 =?
- 11. The sum of the three angles of a triangle equal?



Triad Math, Inc. © 2018

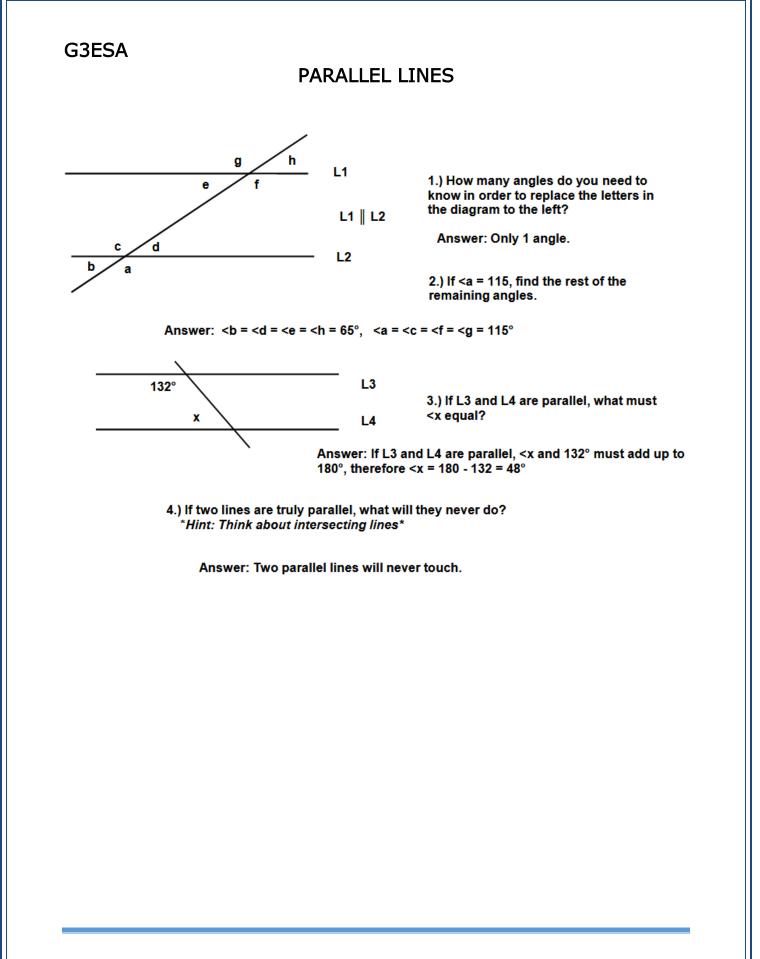
G3 EA (cont'd) PARALLEL LINES (cont'd) Answers: []

8. What is <1 + <2 + <3 =?	[180 <sup>o</sup> ]	
9. If $<1 = 42^{\circ}$ and $<3 = 105^{\circ}$ , what	does <6 =?	[ <b>147</b> <sup>o</sup> ]
10. In problem #9, what does $<2 =?$		[ <b>33</b> º]

11. The sum of the three angles of a triangle equal? [180°]

G3ES **PARALLEL LINES** h g L1 1.) How many angles do you need to f е know in order to replace the letters in the diagram to the left? L1 || L2 С d L2 a 2.) If <a = 115, find the rest of the remaining angles. L3 132° 3.) If L3 and L4 are parallel, what must <x equal? Х L4

> 4.) If two lines are truly parallel, what will they never do? \*Hint: Think about intersecting lines\*



Triad Math, Inc. © 2018

#### G4 LESSON: TRIANGLES, DEFINITION, SUM OF ANGLES

A Triangle is a three-sided **polygon**, i.e., a geometric figure created by three intersecting straight lines. Thus, a triangle has three sides and three vertices.

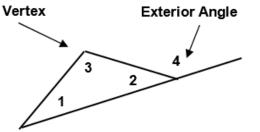
The sum of the three interior angles of a triangle is always 180°. Exterior Angle = Sum of opposite Interiors

 $1 + 2 + 3 = 180^{\circ}$  and 4 = 1 + 3

Triangles are often used to model a physical situation.

There are several types of triangles:

Right, Acute, Obtuse, Isosceles, and Equilateral. See below.



All angles < 90°

One angle = 90°

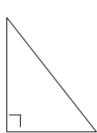
<1 + <2 + <3 = 180°

Obtuse One angle > 90°

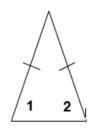
<4 = <1 + <3

Acute

Right

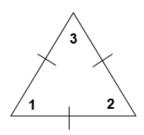


Right Triangle



Isosceles Triangle <1 = <2

Two Equal Sides



Equilateral Triangle <1 = <2 = <3 = 60°

**Three Equal Sides** 

#### Triad Math, Inc. © 2018

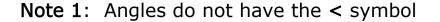
#### G4 Triangle Problems

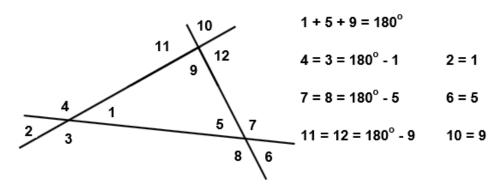
Finding unknown angles from known angles.

Each **vertex** of a triangle has four angles associated with it for a total of twelve angles for a triangle. There will be six values.

If you know any two angles from two different vertices, then you can calculate all the other angles.

This is demonstrated below.



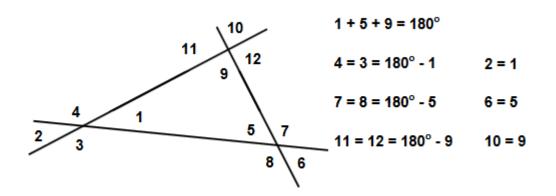


Given any two angles from two vertices, we can calculate all the other angles.

Example 1		1 = 40° and 7 = 120°	Find the other angles
Answers		$5 = 6 = 180^{\circ} - 120^{\circ} = 60^{\circ}$	8 = 120°
		$4 = 3 = 180^{\circ} - 40^{\circ}$	2 = 40°
	**	9 = 10 = 180° - 1 - 5 = 180° - 40°	$-60^{\circ} = 80^{\circ}$
		$11 = 12 = 180^{\circ} - 80^{\circ} = 100^{\circ}$	
Example 2		9 = $75^{\circ}$ and 8 = $110^{\circ}$ Find oth	ner angles
Answers		$5 = 6 = 180^{\circ} - 110^{\circ} = 70^{\circ}$ and $7 = 10^{\circ}$	= 110 <sup>°</sup>
		11 = 12 = 180° - 75° = 105° and	10 = 75°
	**	2 = 1 = 180° - 75° - 70° = 35° an	d 4 = 3 = 180° - 35° = 145°

#### TRIANGLES

Find the unknown angles from known angles below.



Given any two angles from two vertices, we can calculate all the other angles.

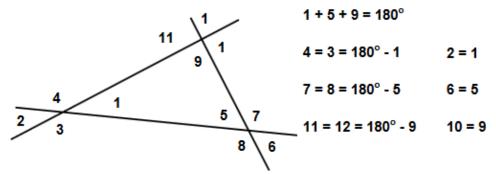
Exercise 1:	$1 = 40^{\circ}$ and $7 = 120^{\circ}$	Find the other angles.
Exercise 2:	$9 = 75^{\circ}$ and $8 = 110^{\circ}$	Find the other angles.
Exercise 3:	$2 = 38^{\circ}$ and $10 = 70^{\circ}$	Find the other angles.
Exercise 4:	9 = 72° and 6 =68°	Find the other angles.
Exercise 5:	4 = 135° and 7 =118°	Find the other angles.
Exercise 6:	10 = 85° and 12 = 95°	Find the other angles.

G4E

### G4EA

#### TRIANGLES

Find the unknown angles from known angles below.



Given any two angles from two vertices, we can calculate all the other angles.

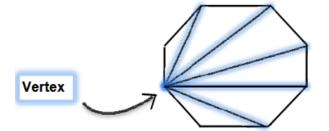
Exercise 1 Answers	1 = 40° and 7 = 120° 5 = 6 = 180° - 120° = 60° 4 = 3 = 180° - 40° = 140° 11 = 12 = 180° - 80° = 100°	Find the other angles 8 = 120° 2 = 40° 9 = 10 = 80°
Exercise 2 Answers	9 = 75° and 8 = 110° 5 = 6 = 180° -110° = 70° and 7 11 = 12 = 180° - 75° = 105° and 2 = 1 = 180° - 75° - 70° = 35° an	10 = 75°
Exercise 3 Answers	2 = 38° and 10 = 70° 1 = 38° and 3 = 4 = 142° 9 = 70° and 11 = 12= 110° 5 = 6 = 72° and 7 = 8 = 108°	Find the other angles
Exercise 4 Answers	9 = 72° and 6 =68° 5 = 6 = 68° and 7 = 8 = 112° 11 = 12 = 108° and 10 = 72° 2 = 1 = 40° and 4 = 3 = 140°	Find the other angles
Exercise 5 Answers	4 = 135° and 7 =118° 5 = 6 = 62° and 7 = 8 = 118° 11 = 12 = 107° and 9 =10 = 73° 2 = 1 = 45° and 4 = 3 = 135°	Find the other angles
Exercise 6 Answers	10 = 85° and 12 = 95° 9 = 85° and 11 = 95° Not enough information for the	Find the other angles e other angles.

#### G4ES

#### TRIANGLES

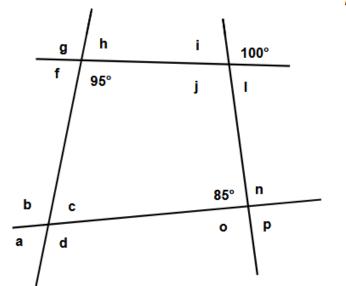
Note: The interior angles of any polygon add up to the number of sides the shape has - 2 and then multiplied by 180.

Ex. Triangles have 3 sides --> (3 - 2) times  $180 = 180^{\circ}$ Ex. Rectangles have 4 sides --> (4 - 2) times  $180 = 360^{\circ}$ 



1.) The reasoning behind this trick all comes back to triangles. How many degrees does a triangle's interior angles add up to?

2.) Now how many triangles can we break up this octagon into from a single vertex?



\*Note: A vertex is just a corner made by two lines!\*

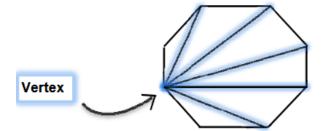
3.) With the help of this trick, find the remaining angles in the diagram to the left.

#### G4ESA

#### TRIANGLES

Note: The interior angles of any polygon add up to the number of sides the shape has - 2 and then multiplied by 180.

Ex. Triangles have 3 sides  $\rightarrow$  (3 - 2) times 180 = 180° Ex. Rectangles have 4 sides  $\rightarrow$  (4 - 2) times 180 = 360°



1.) The reasoning behind this trick all comes back to triangles. How many degrees does a triangle's interior angles add up to?

Answer: 180°

2.) Now how many triangles can we break up this octagon into from a single vertex?

\*Note: A vertex is just a corner made by two lines!\*

Answer: 6 triangles

3.) With the help of this trick, find the remaining angles in the diagram to the left.

Answer:  $a = c = 80^{\circ}$   $b = d = 100^{\circ}$   $f = h = 85^{\circ}$   $g = 95^{\circ}$   $i = I = 80^{\circ}$   $j = 100^{\circ}$   $n = o = 95^{\circ}$  $p = 85^{\circ}$ 

h i g 100° f 95° i I n 85° b С р ο d а

#### G5 LESSON: RIGHT TRIANGLES - PYTHAGOREAN THEOREM

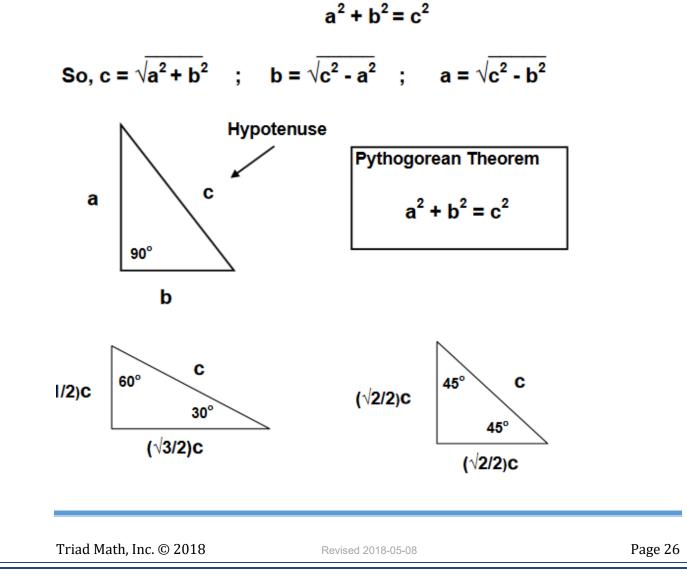
A **Right Triangle** has one of its angles = 90°

The side opposite the **right angle** is called the **Hypotenuse**.

The sum of the other two angles will sum to 90°

The Lengths of the three sides of a **Right Triangle** are related by the **Pythagorean Theorem**.

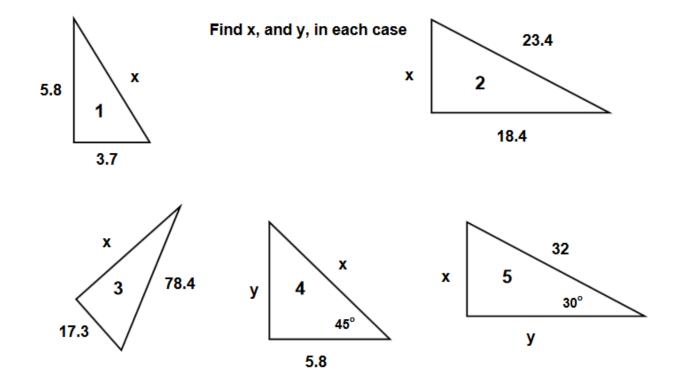
If, they are **a**, **b**, and **C** where "**C**" is the **hypotenuse**, then:



#### G5 Right Triangle Problems

Typically, you are given one or two sides or angles and want to figure out the other sides or angles.

Here are a few examples (You will typically use the **Pythagorean Theorem** and a calculator):



Answers 1. x = 6.9 2. x = 14.5 3. 76.5 4. y = 5.8, x = 8.2 5. x = 16, y = 27.7

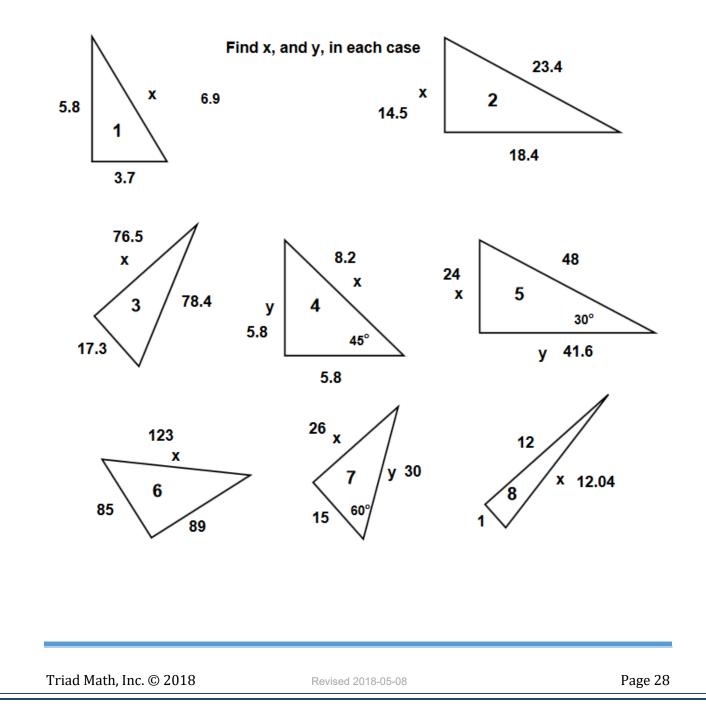
G5E

### **RIGHT TRIANGLES**

Find x and y in each of the Exercises below.

You will typically use the Pythagorean Theorem and a calculator.

All triangles below are **right triangles**.

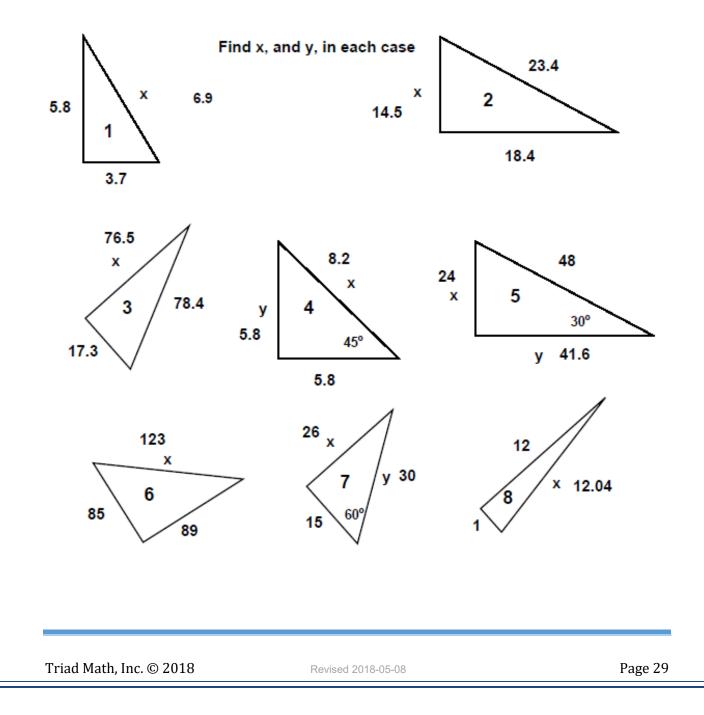


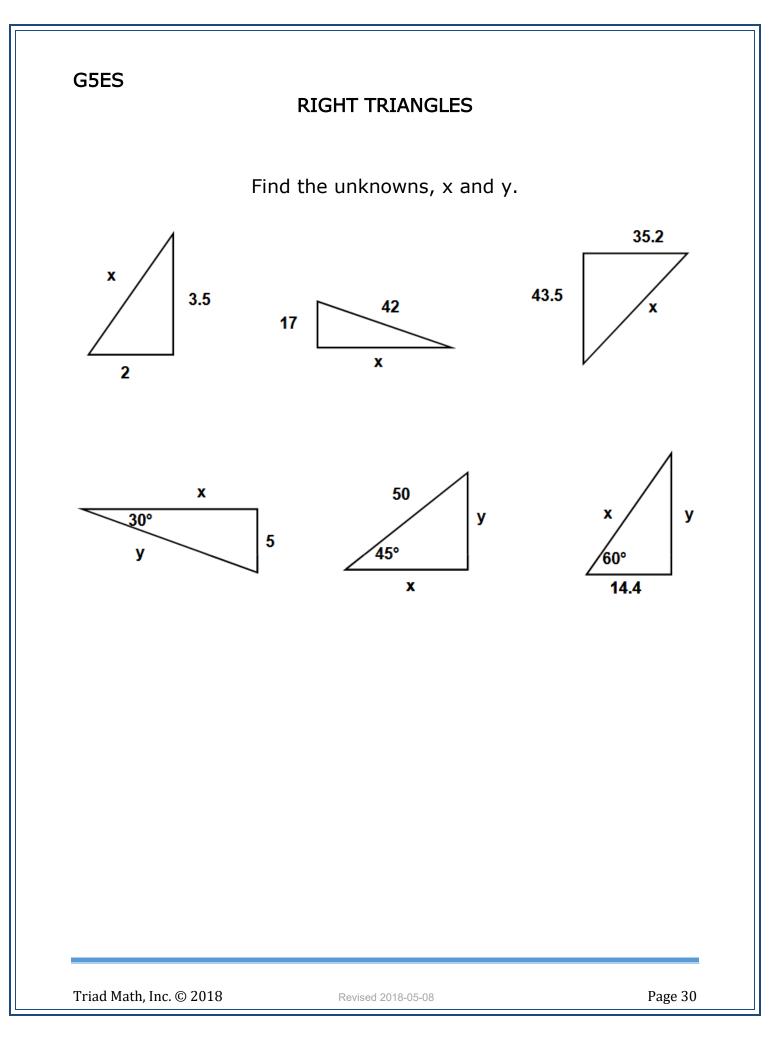
### **RIGHT TRIANGLES**

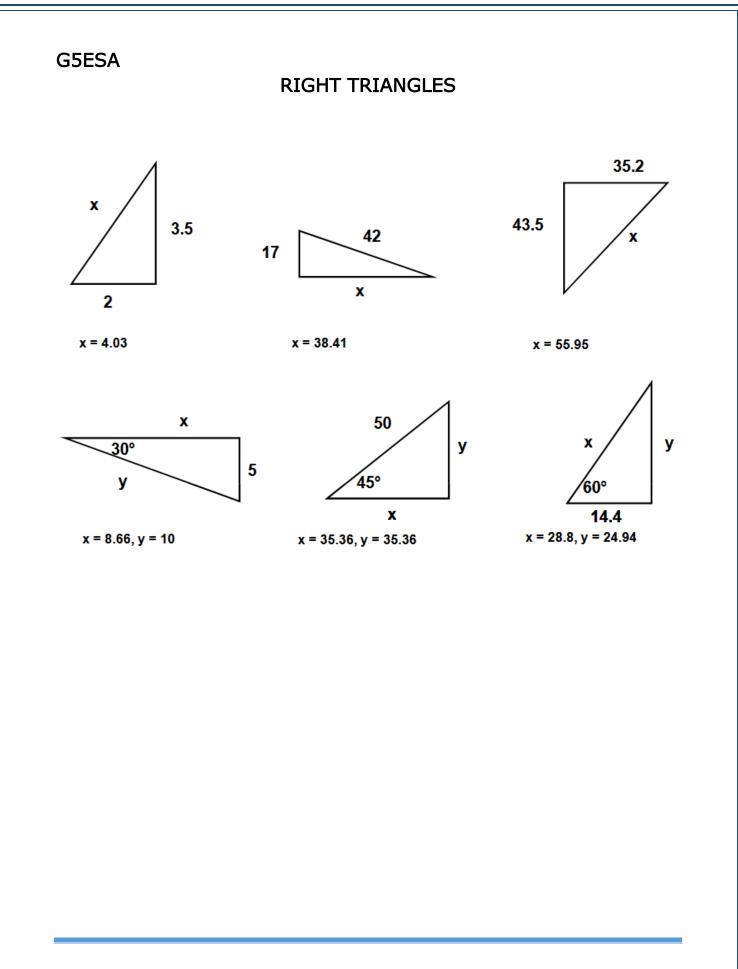
Find **x** and **y** in each of the Exercises below.

You will typically use the Pythagorean Theorem and a calculator.

All triangles below are **right triangles**.







Triad Math, Inc. © 2018

#### G6 LESSON: SIMILAR TRIANGLES

Two Triangles are similar if they have equal angles.

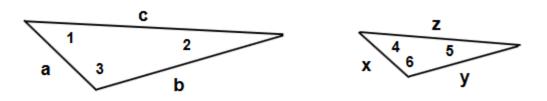
This means they have the same "**shape**" but may be of different sizes. If they also are the same size they are **congruent**.

Similar triangles appear frequently in practical problems.

Their corresponding ratios are equal, and that is what makes them so important and useful.

This is often the way you set up an **Equation** to find an **Unknown**.

**Note:** If **two** sets of angles are equal, the **third** must be equal also, and the triangles are similar.



Given: 1 = 4; 2 = 5; 3 = 6, Called corresponding angles

Corresponding sides are:  $a \leftrightarrow x$ ;  $b \leftrightarrow y$ ;  $c \leftrightarrow z$ 

The Following Ratios are Equal

a/x = b/y = c/z and $x/a = y/b = z$	/c
-------------------------------------	----

a/b = x/y a/c = x/z b/c = y/z

b/a = y/x c/a = z/x c/b = z/y

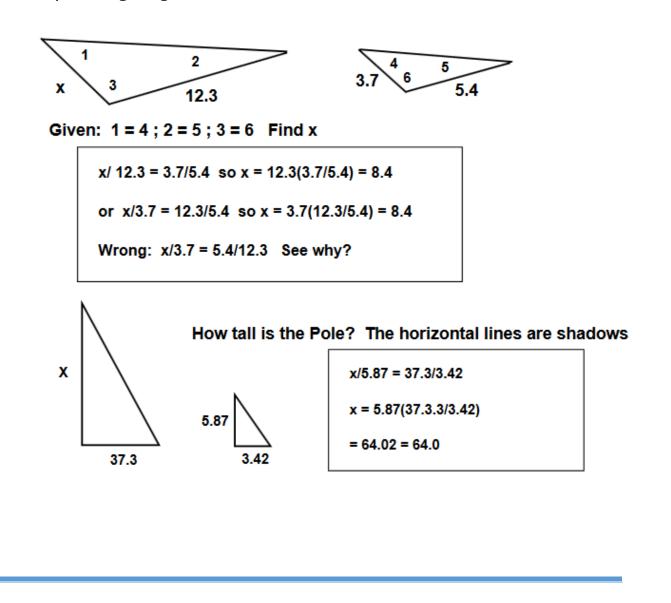
Triad Math, Inc. © 2018

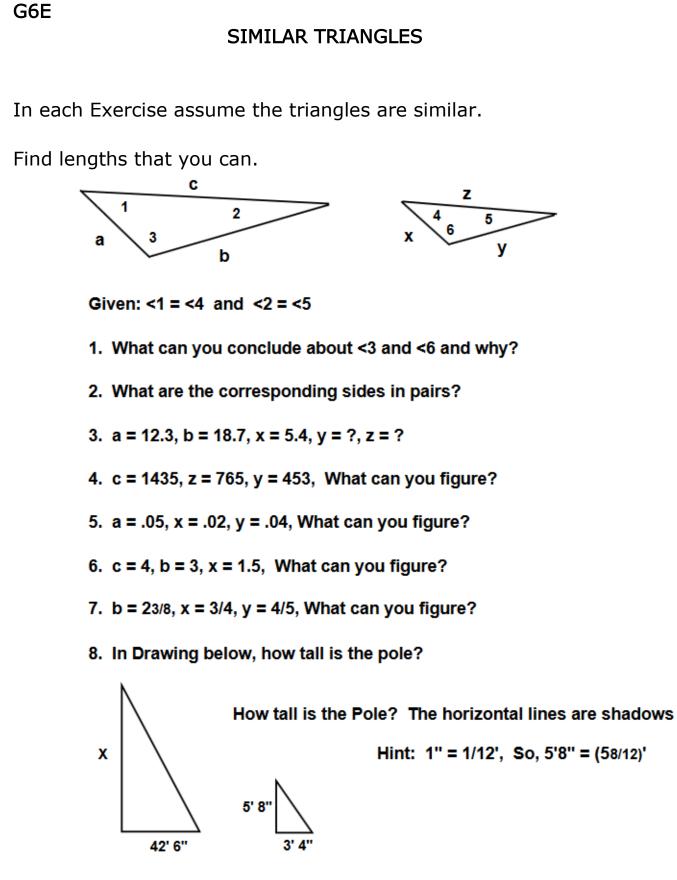
#### G6 Similar Triangles Problems

When you have **two equal ratios** with one **unknown** it is a simple algebra problem to solve for the unknown **X**.

X/a = b/c and X = a(b/c) X/3 = 7/12 and X = 3x(7/12) = 1.75a/X = b/c and X = a(c/b) 3/X = 7/12 and X = 3x(12/7) = 5.15

Find two similar triangles where the **unknown** is one side and you know three more sides, one of which is opposite the corresponding angle of the unknown.





G6EA

#### **SIMILAR TRIANGLES** Answers: []

In each Exercise assume the triangles are similar. Find lengths that you can.



Given: <1 = <4 and <2 = <5

- What can you conclude about <3 and <6 and why?</li>
  [They are equal due to sum of angles of triangle equals 180°]
- 2. What are the corresponding sides in pairs?

$$[a \leftrightarrow x, b \leftrightarrow y, c \leftrightarrow z]$$

3. a = 12.3, b = 18.7, x = 5.4, y = ?, z = ?

[y = 8.2 Have not yet learned how to calculate z] 4. c = 1435, z = 765, y = 453 What can you figure? [b = 850]

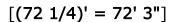
5. a = 0.05, x = 0.02, y = 0.04 What can you figure? [b = 0.1]

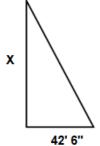
6. c = 4, b = 3, x = 1.5 What can you figure? [Nothing with just similar triangles]

7. b = 2 3/8, x = 3/4 y = 4/5, What can you figure? [a = 2 29/128 = 2.23]

8. In drawing below, how tall is the pole?

5' 8'





How tall is the Pole? The horizontal lines are shadows

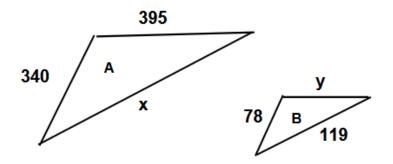
Hint: 1" = 1/12', So, 5'8" = (58/12)'

Triad Math, Inc. © 2018

G6ES

### SIMILAR TRIANGLES

Find the unknowns, x and y.

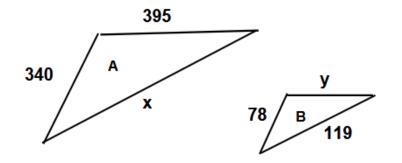


Assume that the two triangles to the left are similar. Using this knowledge, find the unknown lengths.

Triad Math, Inc. © 2018

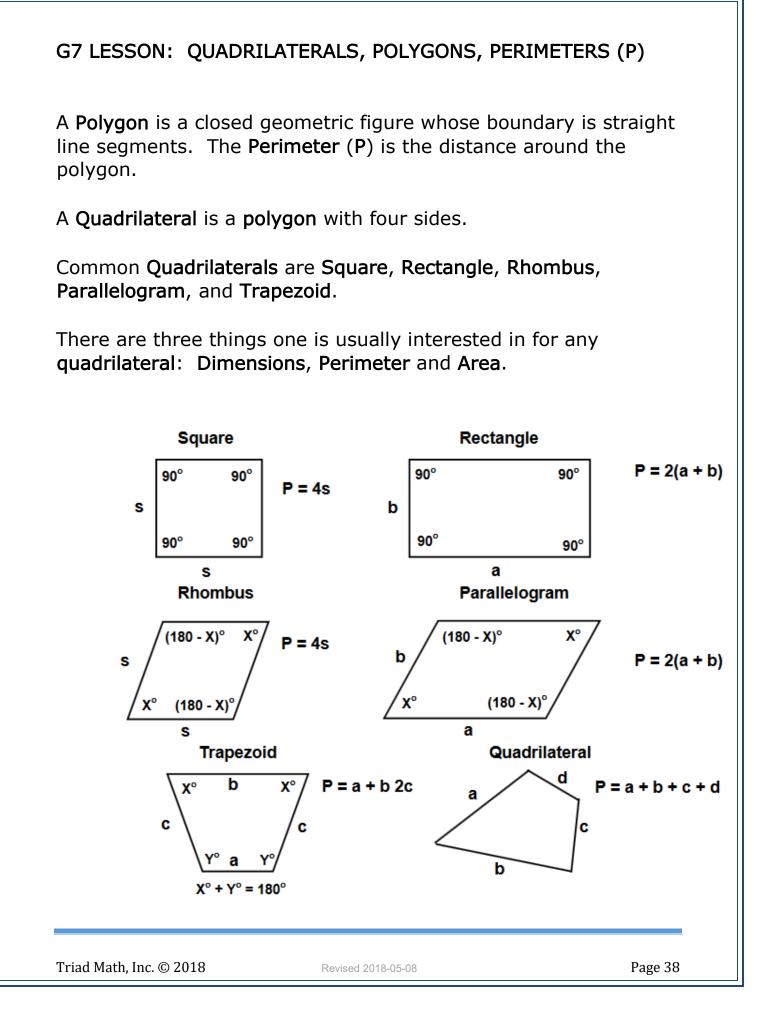


### SIMILAR TRIANGLES



Assume that the two triangles to the left are similar. Using this knowledge, find the unknown lengths.

x = 518.72, y = 90.62



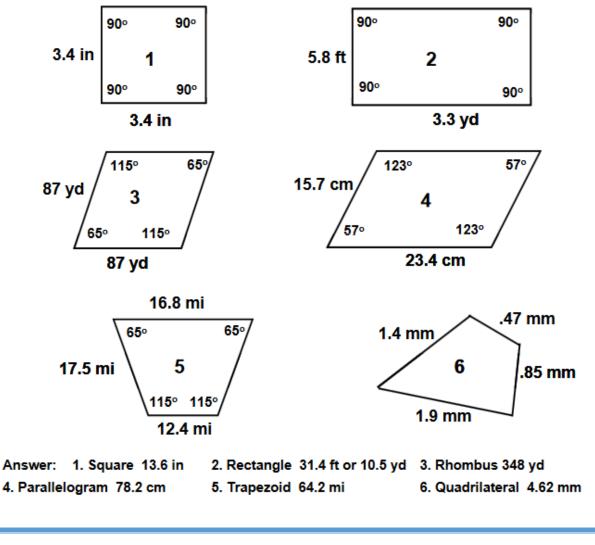
G7 Quadrilaterals, Polygons, Perimeters (P) Problems

Identify the figures below and compute their Perimeters.

Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.

Suppose a **rectangle** has one side 11/2 feet, and the other side 8 inches. Then, convert feet to inches or inches to feet.

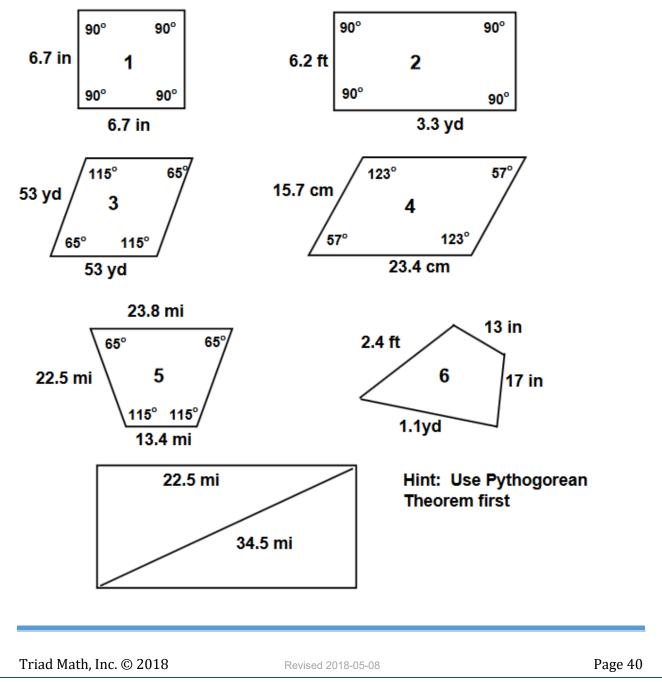
Answers are at bottom of page - Number, name, Perimeter.



## QUADRILATERALS, POLYGONS, PERIMETERS (P)

Identify the figures below and compute their Perimeters

Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.



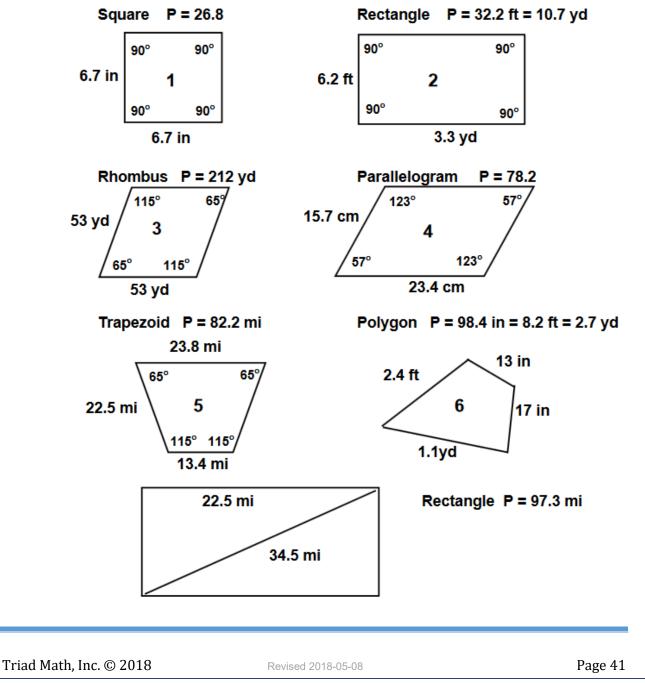
G7E

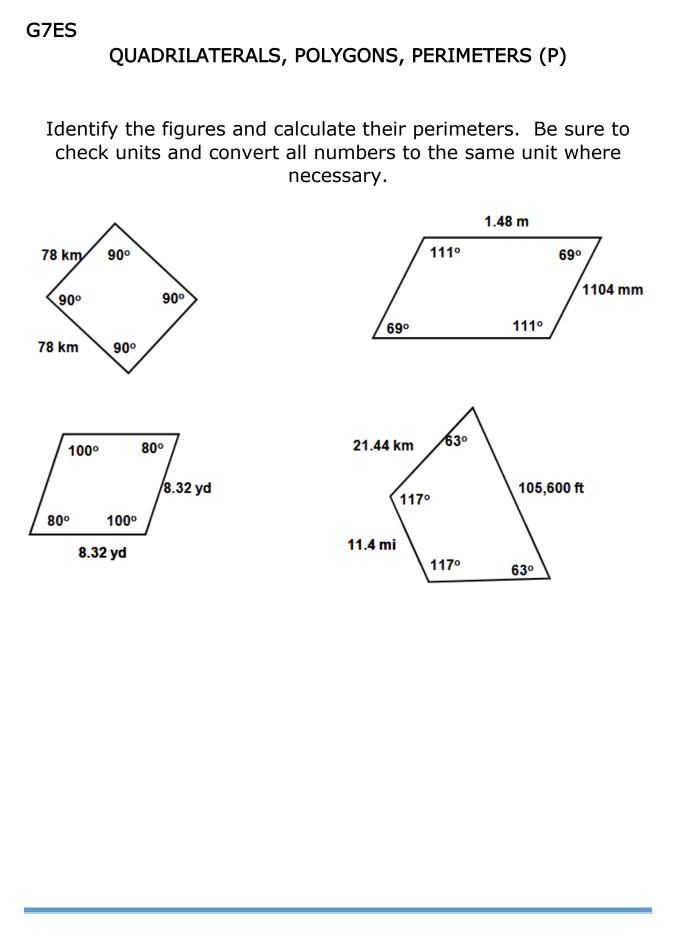
#### G7EA

QUADRILATERALS, POLYGONS, PERIMETERS (P)

Identify the figures below and compute their Perimeters

Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.





Triad Math, Inc. © 2018

