



Craig Hane, Ph.D., Founder

## Workforce Development: Module 7

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## 1.1 Lessons Abbreviation Key Table

C = Calculator Lesson  
P = Pre-algebra Lesson  
A = Algebra Lesson  
G = Geometry Lesson  
T = Trigonometry Lesson  
S = Special Topics

**The number following the letter is the Lesson Number.**

E = Exercises with Answers: Answers are in brackets [ ].  
EA = Exercises Answers: (only used when answers are not on the same page as the exercises.)  
ES = Exercises Supplemental: Complete if you feel you need additional problems to work.

## 1.2 Exercises Introduction

### Why do the Exercises?

Mathematics is like a "game." The more you practice and play the game the better you will understand and play it.

The Foundation's Exercises, which accompany each lesson, are designed to reinforce the ideas presented to you in that lesson's video.

It is unlikely you will learn math very well by simply reading about it or listening to Dr. Del, or anyone else, or watching someone else doing it.

You WILL learn math by "doing math."

It is like learning to play a musical instrument, or write a book, or play a sport, or play chess, or cooking.

You will learn by practice.

Repetition is the key to mastery.

You will make mistakes. You will sometimes struggle to master a concept or technique. You may feel frustration sometimes **"WE ALL DO."**

But, as you learn and do math, you will begin to find pleasure and enjoyment in it as you would in any worthwhile endeavor. Treat it like a sport or game.

**These exercises are the KEY to your SUCCESS!**

**ENJOY!**

## C13 LESSON: DEG RAD GRAD THREE ANGLE MEASURES

There are three measures of an angle acceptable by the TI 30XA calculator.

Degree **DEG**  $1/360$  of a circle

Gradian **GRAD**  $1/400$  of a circle

Radian **RAD**  $1/2\pi$  of a circle with radius 1. (57.3 DEG)

In our Practical Math Foundation we will only use the **DEG** which is what automatically comes up when you turn on the calculator.

The **DRG** Key changes the choice of unit.

If you enter a number in the **DEG** mode and then press the **2nd DRG** Keys, you will transform the number to the new unit.

For example, enter 180 as **DEG**, then transform into **RAD** (3.1416) and **GRAD** (200)

Or; enter 1 in **RAD** mode, and transform into 57.3 Degrees.

We will only use **DEG** in the Foundation training.

**RAD** will also be used in Tiers 4 and up. It is the "natural" measurement of an angle for trig and calculus.

## C13E

### DEG RAD GRAD THREE ANGLE MEASURES

1. DEG stands for?
2. What fraction of a circle is one degree?
3. What are the other two angle measures on the TI 30XA calculator?
4. Which measure comes up when you turn on the calculator?
5. How do you switch to the other two measures?
6. How do you convert Degrees to **RADs** and **GRADs**?
7. How many **RADs** are 90 degrees?
8. How many **GRADs** are 90 degrees?
9. What will we use exclusively in the Foundations Course to measure angles?

**Answers are on C13EA, page 46.**

**Take the C13 Quiz.**

## C13EA

### DEG RAD GRAD THREE ANGLE MEASURES

Answers: [ ]'s

1. DEG stands for? [Degree °]
2. What fraction of a circle is one degree? [1/360]
3. What are the other two angle measures on the TI 30XA calculator? [RAD and GRAD]
4. Which measure comes up when you turn on the calculator? [DEG]
5. How do you switch to the other two measures?  
[Press the DRG key once for RAD again for GRAD and again for DEG]
6. How do you convert Degrees to RADs and GRADs?  
[Enter the degrees and press the 2nd DEG key for RADs and press 2nd DEG key again for GRADs]
7. How many RADs are 90 degrees? [1.57]
8. How many GRADs are 90 degrees? [100]
9. What will we use exclusively in the Foundations Course to measure angles? [DEG Degrees]

**Take the C13 Quiz or review.**

## C14 LESSON: $\text{SIN}$ $\text{SIN}^{-1}$

These two keys are used to compute the Sine of an angle, and the angle, if you know its SIN.

This is used in Trigonometry, and also for some interesting formulas in Geometry.

We will always use the Degree, **DEG**, measure of an angle in the Foundation course.

Enter the angle, say,  $\theta$ , and press SIN

Example: 45 SIN yields .707

SIN ( $\theta$ ) is always between -1 and 1.

SIN<sup>-1</sup> is the "inverse" of the SIN, 2nd SIN

If SIN ( $\theta$ ) = N, then SIN<sup>-1</sup>(N) =  $\theta$

Example: SIN<sup>-1</sup>(.707) = 45°

SIN<sup>-1</sup>(N) only works for N between -1 and 1.

**NOTE: SIN 135 = 0.707...in general, SIN (180° -  $\theta$ ) = SIN ( $\theta$ )**



**C14E****SIN SIN<sup>-1</sup>****Answers: [ ]'s**

1. **SIN (45°) = ?** [0.707]
2. **SIN (0°) = ?** [0]
3. **SIN (10°) = ?** [0.174]
4. **SIN (30°) = ?** [0.500]
5. **SIN (60°) = ?** [0.866]
6. **SIN (75°) = ?** [0.966]
7. **SIN (85°) = ?** [0.996]
8. **SIN (90°) = ?** [1]
9. **SIN (95°) = ?** [0.996]
11. **SIN (120°) = ?** [0.866]
12. **SIN<sup>-1</sup>(0.5) = ?** [30 degrees]
13. What angle X, has **SIN (X) = 0.4** ? [23.58 degrees]
14. **SIN<sup>-1</sup>(0.4) = ?** [23.58 degrees]
15. **SIN<sup>-1</sup>[SIN(50°)] = ?** [50 degrees]

**Take C14 Quiz or do more exercises, C14ES.**



## C15 LESSON: $\text{COS}$    $\text{COS}^{-1}$

These two keys are used to compute the Cosine of an angle, and the angle, if you know its  $\text{COS}$ .

This is used in Trigonometry and also for some interesting formulas in Geometry.

We will always use the Degree, **DEG**, measure of an angle in the Foundation course.

Enter the angle, say,  $\theta$ , and Press  $\text{COS}$

**Example:** 45  $\text{COS}$  yields .707

$\text{COS}$  ( $\theta$ ) is always between -1 and 1.

$\text{COS}^{-1}$  is the "inverse" of the  $\text{COS}$ , 2nd  $\text{COS}$

If  $\text{COS}$  ( $\theta$ ) = N, then  $\text{COS}^{-1}$ (N) =  $\theta$     N between -1 and 1

Example:  $\text{COS}^{-1}$ (.707) =  $45^\circ$

**NOTE:**  $\text{COS}$  135 = -.707    In general,  $\text{COS}$  ( $180^\circ - \theta$ ) = -  $\text{COS}$ ( $\theta$ )

You could verify:  $\text{COS}$ ( $90 - \theta$ ) = **SIN** ( $\theta$ ) for example.

**SIN** and  $\text{COS}$  are intimately related as you will learn in the Trigonometry section of Tier 2, and even more in Tier 4.

**C15E****COS    COS<sup>-1</sup>****Answers: [ ]'s**

1. **COS (45°) = ?** [0.707]
2. **COS (0°) = ?** [1]
3. **COS (10°) = ?** [0.985]
4. **COS (30°) = ?** [0.866]
5. **COS (60 °) = ?** [0.500]
6. **COS (75°) = ?** [0.259]
7. **COS (85°) = ?** [0.087]
8. **COS (90°) = ?** [0]
9. **COS (95°)= ?** [-0.087]
10. **COS<sup>-1</sup> (0.5) = ?** [60 degrees]
11. **What angle X, has COS (X) = .4?** [66.4 degrees]
14. **COS<sup>-1</sup>(.4) = ?** [66.4 degrees]
15. **COS<sup>-1</sup>[SIN(50°)] = ?** [40 degrees]

**Take the C15 Quiz or do some more exercise, C15ES.**



## C16 LESSON: TAN    $TAN^{-1}$

These two keys are used to compute the Tangent of an angle, and the angle, if you know its **TAN**

This is used in Trigonometry.

We will always use the Degree, **DEG**, measure of an angle in the Foundation course.

Enter the angle, say,  $\theta$ , and Press **TAN**

Example: 45 **TAN** yields 1

**TAN** ( $\theta$ ) can be any size

**TAN**<sup>-1</sup> is the "inverse" of the **TAN**, **2nd TAN**

If **TAN** ( $\theta$ ) = N, then **TAN**<sup>-1</sup>(N) =  $\theta$

Example: **TAN**<sup>-1</sup>(1) = 45°

**NOTE:** We will not use **TAN** in the Foundation Course.

**TAN** is also intimately related to **SIN** and **COS**.



## C16ES

**TAN    TAN-1**

**Answers: [ ]'s**

1. **TAN (90°) = ?** [Error]
2. **TAN (89.99°) = ?** [5730]
3. **TAN (-89.99°) = ?** [-5730]
4. **TAN (88°) = ?** [29]
5. **TAN (80 °) = ?** [6]
6. **TAN (60°) = ?** [2]
7. **TAN (30°) = ?** [1]
8. **TAN (10°) = ?** [0.176]
9. **TAN<sup>-1</sup> (0.577) = ?** [30°]
10. **What angle X, has TAN (X) = 1 ?** [45°]
11. **TAN<sup>-1</sup>(1) = ?** [45°]
12. **TAN<sup>-1</sup>[TAN(150°)] = ?** [-30°]
13. **TAN<sup>-1</sup>[TAN(-30°)] = ?** [-30°]

**Take the C16 Quiz or review.**



**A9 LESSON:** (1)  $\text{SIN } X^\circ = A$ ,  $-1 \leq A \leq 1$ , OR (2)  $\text{SIN}^{-1}X = A^\circ$ ,  $0 \leq A^\circ \leq 180^\circ$

*NOTE: Contrary to the audio, you cannot defer this lesson.*

Two easy equations. (Apply correct **Inverse** to both sides)

**Note:**  $X$  is **angle** measured degrees ( $^\circ$ ) in the first equation  
 $A$  is **angle** measured in degrees ( $^\circ$ ) in the second equation

**Note:** You don't need to even know what **SIN** means to solve the equation using the calculator.

Example:  $\text{SIN } X^\circ = .548$       Apply  $\text{SIN}^{-1}$  to both sides  
 $X^\circ = \text{SIN}^{-1}(.548) = 33.2^\circ$       **Note:** 2nd SIN yields  $\text{SIN}^{-1}$

Example:  $\text{SIN } X = .8765$        $X = 61.2^\circ$  [ $X$  is in  $^\circ$  ]

Example:  $\text{SIN}^{-1}X = 28^\circ$       Apply **SIN** to both sides and  
get  $X = \text{SIN}(\text{SIN}^{-1}X) =$   
 $\text{SIN}(28^\circ) = .469$

Example:  $(.75 + \text{COS}49^\circ)\text{SIN}^{-1}X = (14.23 + \text{SIN}35^\circ)^2$   
(Looks bad, but is really easy. Just do the numbers first.)

$\text{COS}49^\circ = .656$ ; so  $.75 + .656 = 1.41$  and

$\text{SIN}35^\circ = .574$ ; so  $(14.23 + .574)^2 = 219$  and so we get

$1.41\text{SIN}^{-1}X = 219$ , or  $\text{SIN}^{-1}X = 219/1.41 = 155^\circ$

Thus,  $X = \text{SIN } 155^\circ = .416$

Check:  $1.41 \times \text{SIN}^{-1}.416 = 1.41 \times 24.6 = 34.7$ , not 219.

Something wrong. Must wait until **Trig Lesson T2** to understand.

Preview hint:  $\text{SIN}155^\circ = \text{SIN } 25^\circ$

A9 (1)  $\text{SIN } X^\circ = A$ ,  $-1 \leq A \leq 1$ , or (2)  $\text{SIN}^{-1}X = A^\circ$ ,  $0 \leq A^\circ \leq 180^\circ$

Two easy equations. (Apply correct **Inverse** to both sides)

**Note:**  $X$  is **angle** measured degrees ( $^\circ$ ) in the first equation

$A$  is **angle** measured in degrees ( $^\circ$ ) in the second equation

**Note:** You don't need to even know what **SIN** means to solve the equation using the calculator.

Example:  $\text{SIN } X^\circ = .548$  Apply  $\text{SIN}^{-1}$  to both sides

$$X^\circ = \text{SIN}^{-1}(.548) = 33.2^\circ \quad \text{Note: } \underline{\text{2nd SIN}} \text{ yields } \text{SIN}^{-1}$$

Example:  $\text{SIN } X = .8765$

$$X = 61.2^\circ \text{ [X is in } ^\circ]$$

Example:  $\text{SIN}^{-1}X = 28^\circ$

$$\text{Apply SIN to both sides and get } X = \text{SIN}(\text{SIN}^{-1}X) = \text{SIN}(28^\circ) = .469$$

Example:  $(.75 + \text{COS}49^\circ)\text{SIN}^{-1}X = (14.23 + \text{SIN}35^\circ)^2$

(Looks bad, but is really easy. Just do the numbers first.)

$$\text{COS}49^\circ = .656 \quad \text{so} \quad .75 + .656 = 1.41 \text{ and}$$

$$\text{SIN}35^\circ = .574 \quad \text{so} \quad (14.23 + .574)^2 = 219 \text{ and so we get}$$

$$1.41\text{SIN}^{-1}X = 219, \text{ or } \text{SIN}^{-1}X = 219/1.41 = 155^\circ$$

$$\text{Thus, } X = \text{SIN } 155^\circ = .416$$

$$\text{Check: } 1.41 \times \text{SIN}^{-1}.416 = 1.41 \times 24.6 = 34.7, \text{ not } 219.$$

Something wrong. Must wait until Trig **Lesson T2** to understand.

Preview hint:  $\text{SIN}155^\circ = \text{SIN } 25^\circ$

## A9E

$$(1) \sin X^\circ = A, -1 \leq A \leq 1, \text{ or}$$

$$(2) \sin^{-1}X = A^\circ, 0 \leq A^\circ \leq 180^\circ$$

Two easy equations. (Apply correct **Inverse** to both sides)

**Note:**  $X$  is angle measured degrees ( $^\circ$ ) in the first equation

$A$  is **angle** measured in degrees ( $^\circ$ ) in the second equation

Solve for  $X$ , the Unknown. **Note:** The Algebra is easy. The arithmetic can be complicated, but is easy with the **TI-30XA**.

1.  $\sin X^\circ = 0.548$

2.  $\sin X^\circ = 0.8765,$

3.  $\sin^{-1}X = 28^\circ$

4.  $2.3\sin X^\circ = 1.92$

5.  $\sin X^\circ = 1.5$

6.  $\sin^{-1}(0.8765) = X^\circ$

7.  $\sin^{-1}(\sin(56^\circ)) = X$

8.  $\sin(\sin^{-1}(0.321)) = X$

9.  $\sin^{-1}(X^2) = 15^\circ$

10.  $\sin(3X^\circ) = 0.5$

## A9EA

$$(1) \sin X^\circ = A, -1 \leq A \leq 1, \text{ or}$$

$$(2) \sin^{-1}X = A^\circ, 0 \leq A^\circ \leq 180^\circ$$

Answers: [ ]

Two easy equations. (Apply correct **Inverse** to both sides)

**Note:** X is angle measured degrees ( $^\circ$ ) in the first equation

A is **angle** measured in degrees ( $^\circ$ ) in the second equation

Solve for X, the **Unknown**. Note; The Algebra is easy. The arithmetic can be complicated, but easy with the **TI-30XA**.

1.  $\sin X^\circ = 0.548$  [33.23 $^\circ$ ]
2.  $\sin X^\circ = 0.8765$  [61.22 $^\circ$ ]
3.  $\sin^{-1}X = 28^\circ$  [0.4695]
4.  $2.3\sin X^\circ = 1.92$  [56.6 $^\circ$ ]
5.  $\sin X^\circ = 1.5$  [No Solution, Impossible]
6.  $\sin^{-1}(0.8765) = X^\circ$  [61.22 $^\circ$ ]
7.  $\sin^{-1}(\sin(56^\circ)) = X$  [56 $^\circ$ ]
8.  $\sin(\sin^{-1}(0.321)) = X$  [0.321]
9.  $\sin^{-1}(X^2) = 15^\circ$  [0.5087]
10.  $\sin(3X^\circ) = 0.5$  [10 $^\circ$ ]

## A9ES

$$(1) \sin X^\circ = A, -1 \leq A \leq 1, \text{ or}$$

$$(2) \sin^{-1}X = A^\circ, 0 \leq A^\circ \leq 180^\circ$$

Answers: [ ]

1.  $\sin X^\circ = 0.765$  **[X = 49.9°]**
2.  $\sin X^\circ = 0.278$  **[X = 16.14°]**
3.  $\sin^{-1}(0.254) = X^\circ$  **[X = 14.71°]**
4.  $\sin^{-1}(X) = 45^\circ$  **[X = 0.707]**
5.  $\sin X^\circ = 2.89$  **[NO Solution]**
6.  $\sin(\sin^{-1}(0.5)) = X$  **[X = 0.5]**
7.  $\sin(125^\circ) = X$  **[X = 0.8191]**
8.  $64\sin(X^\circ) = 38.99$  **[X = 37.53°]**
9.  $\sin(\sin^{-1}(0.75)) = X$  **[X = 0.75]**
10.  $\sin^{-1}(\cos(60^\circ)) = X^\circ$  **[X = 30°]**
11.  $\sin(X^\circ) = 0.171$  **[X = ± 3.14°]**
12.  $\sin^{-1}(\cos(115)) = X$  **[X = -25]**

**A10 LESSON: (1)  $\cos X^\circ = A$ ,  $-1 \leq A \leq 1$ , OR (2)  $\cos^{-1}X = A^\circ$ ,  $0 \leq A \leq 180^\circ$**

Two easy equations. (Apply **Inverse** to both sides)

**Note:** X is **angle** measured degrees ( $^\circ$ ) first equation and  
A is **angle** measured in degrees ( $^\circ$ ) in second equation

**Note:** You don't need to even know what **COS** means to solve the equation using the calculator.

Example:  $\cos X^\circ = .548$                       Apply  $\cos^{-1}$  to both sides  
 $X^\circ = \cos^{-1}(.548) = 56.7^\circ$  [X was understood to  
be in  $^\circ$ ] **Note:** 2nd COS yields  $\cos^{-1}$

Example:  $\cos^{-1}X = 28^\circ$                       Apply **COS** to both sides  
 $X = \cos(\cos^{-1}X) = \cos(28^\circ) = .883$

Example:  $(.75 + \cos 49^\circ)\cos^{-1}X = (14.23 + \sin 35^\circ)^2$   
(Looks bad, but is really easy. Just do the numbers first.)

$\cos 49^\circ = .656$  so  $.75 + .656 = 1.41$  and  
 $\sin 35^\circ = .574$  so  $(14.23 + .574)^2 = 219$

So we have:  $1.41\cos^{-1}X = 219$  or  $\cos^{-1}X = 219/1.41 = 155$

Thus:  $X = \cos 155^\circ = -.906$

Check:  $1.41 \times \cos^{-1}(-.906) = 1.41 \times 155 = 219$

**Note:** We didn't have the same problem we had with the **SIN**.  
Why not? Have to wait until **Trig Lesson T3** for  
explanation.

## A10E

$$(1) \cos X^\circ = A, -1 \leq A \leq 1, \text{ or}$$

$$(2) \cos^{-1}X = A^\circ, 0 \leq A \leq 180^\circ$$

Two easy equations. (Apply **Inverse** to both sides)

Note:  $X$  is **angle** measured degrees ( $^\circ$ ) first equation and

$A$  is angle measured in degrees ( $^\circ$ ) in second equation

Solve for  $X$ , the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated, but easy with the **TI-30XA**.

1.  $\cos X^\circ = 0.548$

2.  $\cos^{-1}X = 28^\circ$

3.  $\cos X^\circ = 0.982$

4.  $\cos X^\circ = \sin 79^\circ$

5.  $\cos^{-1}X = \sin^{-1}(0.435)$

6.  $4\cos(3X^\circ) = 2.56$

7.  $2.3\cos^{-1}(\sin X^\circ) = 45^\circ$

8.  $(0.75 + \cos 49^\circ)\cos^{-1}X = (14.23 + \sin 35^\circ)^2$

9.  $\sin^{-1}(\sin(125^\circ)) = X^\circ$

10.  $\cos^{-1}(\cos(125^\circ)) = X^\circ$

## A10EA

$$(1) \cos X^\circ = A, -1 \leq A \leq 1, \text{ OR}$$

$$(2) \cos^{-1}X = A^\circ, 0 \leq A \leq 180^\circ$$

Answers: []

Two easy equations. (Apply Inverse to both sides)

**Note:** X is angle measured degrees ( $^\circ$ ) first equation and

A is angle measured in degrees ( $^\circ$ ) in second equation

Solve for X, the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated, but easy with the **TI-30XA**.

$$1. \cos X^\circ = 0.548 \quad [56.8^\circ]$$

$$2. \cos^{-1}X = 28^\circ \quad [0.8829]$$

$$3. \cos X^\circ = 0.982 \quad [10.9^\circ]$$

$$4. \cos X^\circ = \sin 79^\circ \quad [11^\circ]$$

$$5. \cos^{-1}X = \sin^{-1}(0.435) \quad [0.9004]$$

$$6. 4\cos(3X^\circ) = 2.56 \quad [16.7^\circ]$$

$$7. 2.3\cos^{-1}(\sin X^\circ) = 45^\circ \quad [70.4^\circ]$$

$$8. (0.75 + \cos 49^\circ)\cos^{-1}X = (14.23 + \sin 35^\circ)^2 \quad [-0.9125]$$

$$9. \sin^{-1}(\sin(125^\circ)) = X^\circ \quad [55^\circ]$$

$$10. \cos^{-1}(\cos(125^\circ)) = X^\circ \quad [125^\circ]$$



## A10ES

$$(1) \cos X^\circ = A, -1 \leq A \leq 1, \text{ OR}$$

$$(2) \cos^{-1}X = A^\circ, 0 \leq A \leq 180^\circ$$

Answers: []

1.  $\cos X^\circ = 0.267$  [X = 74.5°]
2.  $\cos X^\circ = 0.6565$  [X = 48.97°]
3.  $\cos^{-1}(0.125) = X^\circ$  [X = 82.82°]
4.  $\cos^{-1}(X) = 45^\circ$  [X = 0.707]
5.  $\cos X^\circ = -0.725$  [X = 136.47°]
6.  $\cos X^\circ = -1.76$  [NO solution]
7.  $-3.75\cos(11^\circ) = X$  [X = -3.681]
8.  $\cos^{-1}(X) = 115^\circ$  [X = -0.4226]
9.  $\cos^{-1}(\sin(48^\circ)) = X^\circ$  [X = 42°]
10.  $\cos(3X^\circ) = -0.49$  [X = 39.78°]
11.  $\cos^{-1}(X/3) = 75^\circ$  [X = 0.7765]
12.  $\sin(16.5^\circ)\cos(X^\circ) = 0.119$  [X = 65.23°]