Craig Hane, Ph.D., Founder

Workforce Development:

Module 9

1.1 Lessons Abbreviation Key Table

C = Calculator Lesson
P = Pre-algebra Lesson
A = Algebra Lesson
G = Geometry Lesson
T = Trigonometry Lesson
S = Special Topics

The number following the letter is the Lesson Number.

E = Exercises with Answers: Answers are in brackets **[ ]**.
EA = Exercises Answers: (only used when answers are not on the same page as the exercises.)
ES = Exercises Supplemental: Complete if you feel you need additional problems to work.

1.2 Exercises Introduction

Why do the Exercises?

Mathematics is like a "game." The more you practice and play the game the better you will understand and play it.

The Foundation's Exercises, which accompany each lesson, are designed to reinforce the ideas presented to you in that lesson's video.

It is unlikely you will learn math very well by simply reading about it or listening to Dr. Del, or anyone else, or watching someone else doing it.

You WILL learn math by "doing math."

It is like learning to play a musical instrument, or write a book, or play a sport, or play chess, or cooking.

You will learn by practice.

Repetition is the key to mastery.

You will make mistakes. You will sometimes struggle to master a concept or technique. You may feel frustration sometimes "**WE ALL DO**."

But, as you learn and do math, you will begin to find pleasure and enjoyment in it as you would in any worthwhile endeavor. Treat it like a sport or game.

These exercises are the KEY to your SUCCESS!

ENJOY!

INTRODUCTION TO TRIGONOMETRY

Trigonometry, **Trig**, is the study of triangles.

Trig consists of several powerful tools which will empower you to solve virtually any solvable problem with triangles including the ones discussed in Lesson G19.

It begins with the basic Trig Functions, **SIN**, **COS**, and **TAN**.

These are the "**power tools**" that let us solve problems.

In the old days, there were extensive Trig Tables that were used. It was arduous to learn and apply these tables.

Today, with the power tool of the TI 30XA, we can solve virtually any triangle problem in a matter of minutes or less.

Actually, in some ways Trig is easier than geometry.

We will learn how to use the three Trig Functions, and also, we will learn two very powerful theorems which make these tools even more valuable:

The **Law of Sines** (Lesson T6)

The Generalized **Pythagorean Theorem** commonly called: **The Law of Cosines** (Lesson T7)

Trigonometry then has many extensions into analytical geometry, complex numbers, calculus, and functional analysis which have profound effects in science, engineering and technology.

T1 LESSON: TRIG FUNCTIONS SIN COS TAN

In any **Right Triangle**, there are **Six Ratios** of side lengths. They come in sets of three where one set is just the reciprocal of the other set.

See the triangle below: a/c, b/c, and a/b are one set.

c is called the **Hypotenuse**, or **Hyp**.

b is called the **Adjacent side** (to angle 1), or **Adj**

a is called the **Opposite side** (to angle 1, or **Opp**

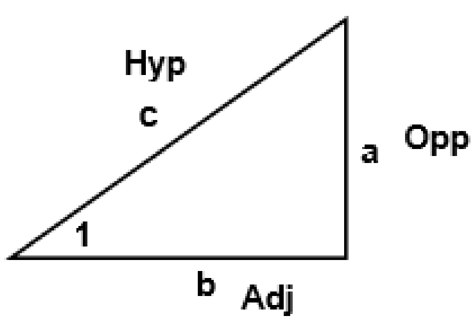
So, the Ratios are **Opp/Hyp**, **Adj/Hyp**, **Opp/Adj**

These three ratios are the three **Trig functions of angle 1**.

SIN(1) = Opp/Hyp
COS(1) = Adj/Hyp
TAN = Opp/Adj = **SIN**(1)/**COS**(1)

Angle 1 will always be measured in **degrees** **°** in this Foundation Course.

In advanced applications of Trig, angle 1 is measured in Radians, RAD.



SIN (1) = a/c = Opp/Hyp

COS (1) = b/c = Adj/Hyp

TAN (1) = a/b = Opp/Adj

When turn on the TI 30XA, **DEG** always comes up.

SIN^{-1} **COS**$^{-1}$ **TAN**$^{-1}$

Enter any number between -1 and 1, and find the angle whose **SIN** it is.

Ditto for **COS** and **TAN**. In other words, if

SIN (1) = a, then (1) = **SIN**$^{-1}$(a)

WARNING: See **T5** for some special information about **SIN**$^{-1}$

T1 Trig Functions   SIN   COS   TAN   Problems

Find **SIN**(1), **COS**(1), **TAN** (1) given **angle** (1) in **degrees** °

Find **Angle** (1), Given **SIN**(1), **COS**(1) using **SIN**$^{-1}$ and **COS**$^{-1}$

Problems:	**Angle 1**	**SIN(1)**	**COS(1)**	**TAN(1)**
in °				
30°	0.5	0.866	0.577	
45°	0.707	0.707	1	
60°	0.866	0.5	1.732	
17°	0.292	0.956	0.306	
38°	0.616	0.788	0.781	
52.7°	0.795	0.606	1.313	
68°	0.927	0.375	2.48	
85°	0.996	0.087	11	
90°	1	0	Error	
100°	0.985	-0.174	-5.68	
115°	0.906	-0.423	-2.15	
135°	0.707	-0.707	-1	
145°	0.574	-0.819	-0.7	
150°	0.5	-0.866	-0.577	
176°	0.07	-0.998	-0.07	

Problems: Find **angle** 1 if

	Angle (1)
SIN(1) = 0.7865	51.9°
SIN(1) = 0.5	30°
SIN(1) = -0.654	-40.8°
COS(1) = 0.7865	38.1°
COS(1) = 0.5	60°
COS(1) = -0.654	130.8°
TAN(1) = 0.7865	38.2°
TAN(1) = 0.5	26.6°
TAN(1) = -0.654	-33.2

Note: **SIN**$^{-1}$(**SIN**(120°) = 60°

COS^{-1}(**COS**(120°) = 120°

Note: These problems are repeated on the T1E page as well.

T1E

TRIG FUNCTIONS SIN COS TAN

Find **SIN**(1), **COS**(1), **TAN**(1) given **angle** (1) in degrees ^{o}
Find **Angle** (1), Given **SIN**(1), **COS**(1) using $\mathbf{SIN^{-1}}$ and $\mathbf{COS^{-1}}$

Angle 1	**SIN**(1)	**COS**(1)	**TAN**(1)
30^o			
45^o			
60^o			
17^o			
38^o			
52.7^o			
68^o			
85^o			
90^o			
100^o			
115^o			
135^o			
145^o			
150^o			
176^o			

Evaluate

$SIN^{-1}[COS(30^o)] = ?$
$COS^{-1}[COS(30^o)] = ?$
$SIN^{-1}[COS(120^o)] = ?$
$COS^{-1}[SIN(120^o)] = ?$
$COS^{-1}[SIN(60^o)] = ?$
$COS^{-1}[SIN(45^o)] = ?$
$TAN^{-1}[SIN(90^o)] = ?$
$SIN[COS^{-1}(.5)] = ?$
$SIN[COS^{-1}(.867)] = ?$
$COS[SIN^{-1}(.867)] = ?$
$SIN[COS^{-1}(1)] = ?$
$SIN[COS^{-1}(0)] = ?$
$SIN[COS^{-1}.707)] = ?$
$TAN[SIN^{-1}(.707)] = ?$
$TAN[COS^{-1}(.707)] = ?$

Find **angle** (1) if

Angle (1)

SIN(1) = 0.7865
SIN(1) = 0.5
SIN(1) = -0.654
COS(1) = 0.7865
COS(1) = 0.5
COS(1) = -0.654
TAN(1) = 0.7865
TAN(1) = 0.5
TAN(1) = -0.654

T1EA

TRIG FUNCTIONS SIN COS TAN

Find **SIN**(1), **COS**(1), **TAN**(1) given angle (1) in **degrees** ^{o}

Find **Angle** (1), Given **SIN**(1), **COS**(1) using **SIN**$^{-1}$ and **COS**$^{-1}$

Angle 1	SIN(1)	COS(1)	TAN(1)	Evaluate	
30º	0.5	0.866	0.577	SIN^{-1}[COS(30º)] = ?	60º
45º	0.707	0.707	1	COS^{-1}[COS(30º)]= ?	30º
60º	0.866	0.5	1.732	SIN^{-1}[COS(120º)] = ?	-30º
17º	0.292	0.956	0.306	COS^{-1}[SIN(120º)] = ?	30º
38º	0.616	0.788	0.781	COS^{-1}[SIN(60º)] = ?	30º
52.7º	0.795	0.606	1.313	COS^{-1}[SIN(45º)] = ?	45º
68º	0.927	0.375	2.475	TAN^{-1}[SIN(90º)] = ?	45º
85º	0.996	0.087	11.43	SIN[COS^{-1}(.5)] = ?	0.867
90º	1	0	Error	SIN[COS^{-1}(.867)] = ?	0.5
100º	0.985	-0.174	-5.68	COS[SIN^{-1}(.867)] = ?	0.5
115º	0.906	-0.423	-2.15	SIN[COS^{-1}(1)] = ?	0
135º	0.707	-0.707	-1	SIN[COS^{-1}(0)] = ?	1
145º	0.574	-0.819	-0.7	SIN[COS^{-1}.707)] = ?	0.707
150º	0.5	-0.866	-0.577	TAN[SIN^{-1}(.707)] = ?	1
176º	0.07	-0.998	-0.07	TAN[COS^{-1}(.707)] = ?	1

Find angle (1) if	Angle (1)
SIN(1) = 0.7865	51.9º
SIN(1) = 0.5	30º
SIN(1) = -.654	-40.8º
COS(1) = 0.7865	38.1º
COS(1) = 0.5	60º
COS(1) = -0.654	130.8º
TAN(1) = 0.7865	38.2º
TAN(1) = 0.5	26.6º
TAN(1) = -0.654	-33.2

T1ES

TRIG FUNCTIONS SIN COS TAN

1. $x = 30°$ Find sin(x) , cos(x), and tan(x)
2. $x = 60°$ Find sin(x), cos(x), and tan(x)
3. Find $\cos^{-1}(\sin(60°)) = ?$
4. If $\cos(x) = 0.5$ Find angle x
5. $x = 45°$ Find sin(x) cos(x) and tan(x)
6. $x = 15°$ Find sin(x), cos(x), and tan(x)
7. Find $\sin^{-1}(\cos(30°)) = ?$
8. If $\sin(x) = 0.315$ Find angle x
9. $x = 90°$ Find sin(x), cos(x), and tan(x)
10. $x = 150°$ Find sin(x), cos(x), and tan(x)
11. Find $\sin(\cos^{-1}(0.5)) = ?$
12. If $\tan(x) = 0.425$ Find angle x
13. $x = 117°$ Find sin(x), cos(x), and tan(x)
14. $x = 34.5°$ Find sin(x), cos(x), and tan(x)
15. Find $\sin^{-1}(\tan(17°)) = ?$
16. If $\sin(x) = -0.5$ Find angle x
17. $x = 100°$ Find sin(x), cos(x), and tan(x)
18. $x = 0°$ Find sin(x), cos(x), and tan(x)
19. Find $\tan^{-1}(\cos(70°)) = ?$
20. If $\tan(x) = -0.245$ Find angle x

T1ESA

TRIG FUNCTIONS SIN COS TAN Answers: [ ]

1. $x = 30°$ Find sin(x) , cos(x), and tan(x)

   [**sin(30°) = 0.5, cos(30°) = 0.866, tan(30°) = 0.577**]

2. $x = 60°$ Find sin(x), cos(x), and tan(x)

   [**sin(60°) = 0.866, cos(60°) = 0.5, tan(60°) = 1.732**]

3. Find $\cos^{-1}(\sin(60°)) = ?$

   [**30°**]

4. If $\cos(x) = 0.5$ Find angle x

   [**x = 60°**]

5. $x = 45°$ Find sin(x) cos(x) and tan(x)

   [**sin(45°) = 0.707, cos(45°) = 0.707, tan(45°) = 1**]

6. $x = 15°$ Find sin(x), cos(x), and tan(x)

   [sin(15) = 0.259, cos(15) = 0.966, tan(15) = 0.268]

7. Find $\sin^{-1}(\cos(30°)) = ?$

   [**x = 60°**]

8. If $\sin(x) = 0.315$ Find angle x

   [**x = 18.4°**]

9. $x = 90°$ Find sin(x), cos(x), and tan(x)

   [**sin(90°) = 1, cos(90°) = 0, tan(90°) = undefined**]

10. $x = 150°$ Find sin(x), cos(x), and tan(x)

    [**sin(150°) = 0.5, cos(150°) = -0.866, tan(150°) = -0.577**]

11. Find $\sin(\cos^{-1}(0.5)) = ?$

    [**0.866**]

12. If $\tan(x) = 0.425$ Find angle x

[$x = 23.03°$]

13. $x = 117°$ Find $\sin(x)$, $\cos(x)$, and $\tan(x)$

[$\sin(117°) = 0.891$, $\cos(117°) = -0.454$, $\tan(117°) = -1.96$]

14. $x = 34.5°$ Find $\sin(x)$, $\cos(x)$, and $\tan(x)$

[$\sin(34.5°) = 0.566$, $\cos(34.5°) = 0.824$, $\tan(34.5°) = 0.687$]

15. Find $\sin^{-1}(\tan(17°)) = ?$

[17.8°]

16. If $\sin(x) = -0.5$ Find angle x

[$x = 210°$, $330°$ or $-30°$]

17. $x = 100°$ Find $\sin(x)$, $\cos(x)$, and $\tan(x)$

[$\sin(100°) = 0.985$, $\cos(100°) = -0.174$, $\tan(100°) = -5.67$]

18. $x = 0°$ Find $\sin(x)$, $\cos(x)$, and $\tan(x)$

[$\sin(0°) = 0$, $\cos(0°) = 1$, $\tan(0°) = 0$]

19. Find $\tan^{-1}(\cos(70°)) = ?$

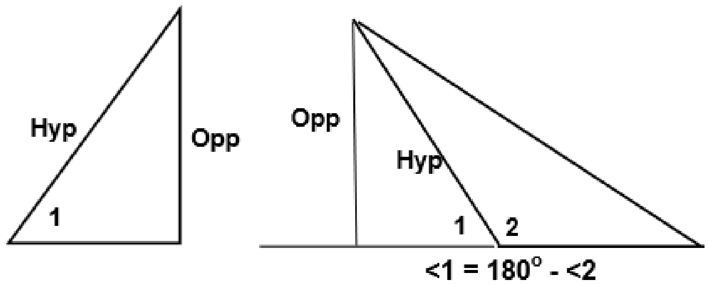
[18.88°]

20. If $\tan(x) = -0.245$ Find angle x

[$x = 166.2°$, $346.2°$, or $-13.8°$]

T2 LESSON: SIN X   SINE OF X   X IS AN ANGLE (DEGREES ^{o})

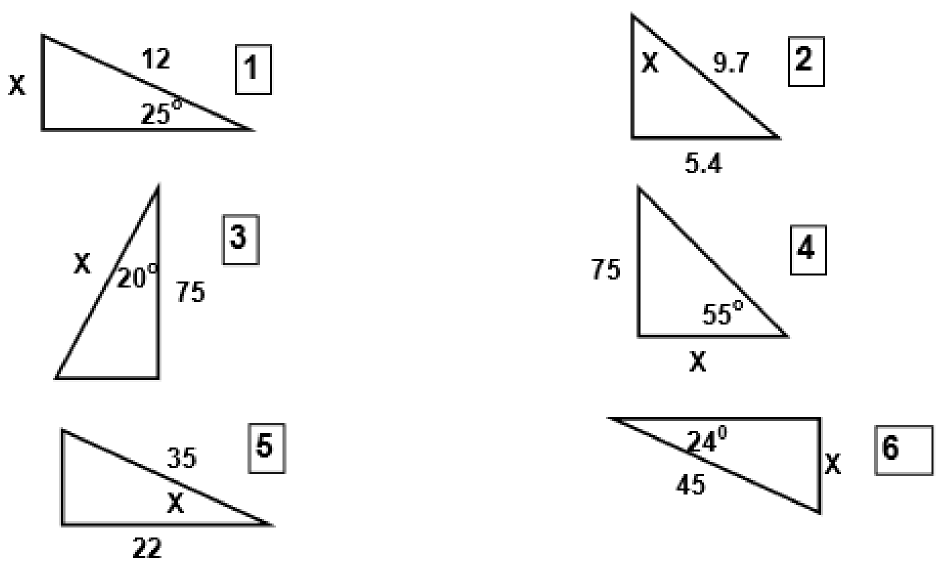
We will extend the definition of **SIN** to include all angles from 0^{o} to 180^{o}. In Tier 3 we will extend the definition to include all angles both positive and negative.



SIN(1) = **Opp/Hyp**   **SIN**(2) = **Opp/Hyp** = **SIN**(180^{o} - <2)

If know two out of three, find the third, **Opp**, **Hyp**, (1)

Opp = **SIN**(1)x**Hyp**	**Opp** = **SIN**(2)x**Hyp**
Hyp = **Opp/SIN**(1)	**Hyp** = **Opp/SIN**(2)
(1) = **SIN**$^{-1}$(**Opp/Hyp**)	(2) = **SIN**$^{-1}$(**Opp/Hyp**)



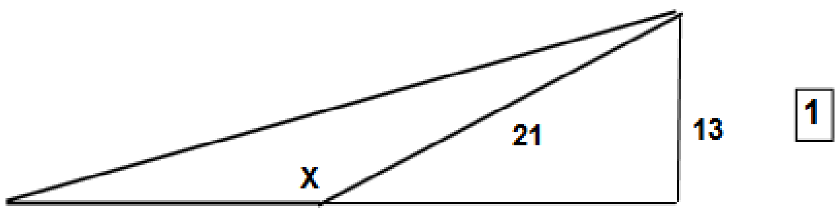
Answers: 1. 5.1 4. 52.5
2. 33.8^{o} 5. 51.1^{o}
3. 79.8 6. 18.3

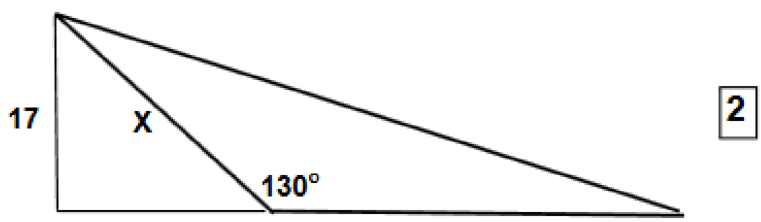
T2 SIN Problems

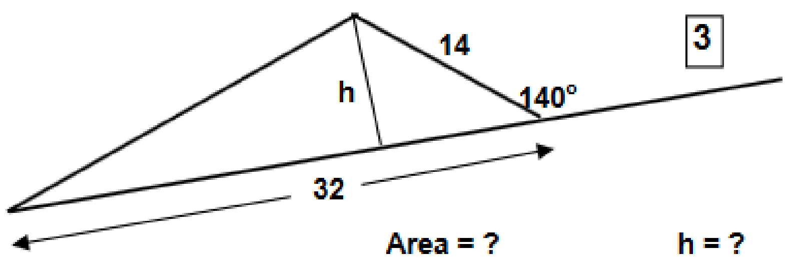
Always set up the Equation first. Then solve.

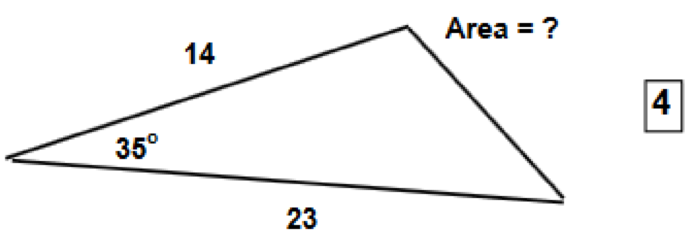
Use the **Pythagorean Theorem** if necessary.

NOTE: Why is **Area** = .5ab**SIN**(<ab) correct?









Answers:

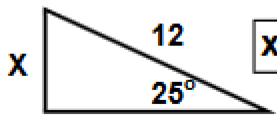
1. 141.8°
2. 22.2
3. A =144, h = 9
4. 92.3

T2E

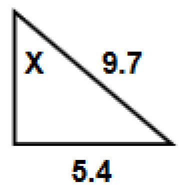
SIN X   SINE OF X

X is an **angle** (**degrees** °)

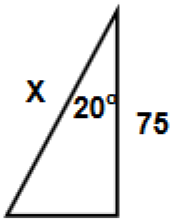
Find **x** in each of the following exercises.



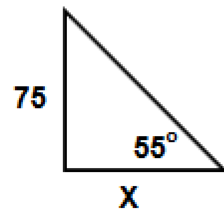
x = ?



x = ?

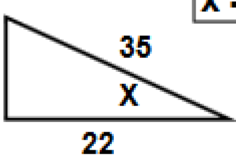


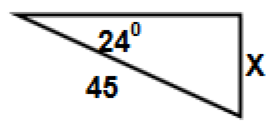
x = ?



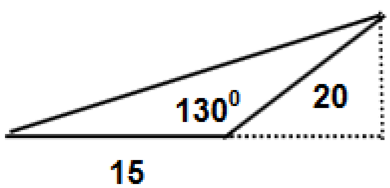
x = ?

x = ?°





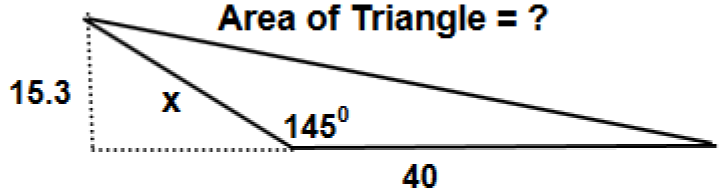
x = ?



X = ?

Area of Triangle = ?

Area of Triangle = ?



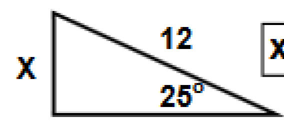
x = ?

T2EA

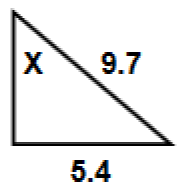
SIN X SINE OF X

X is an **angle** (**degrees** ^{o})

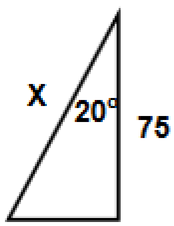
Find **x** in each of the following exercises



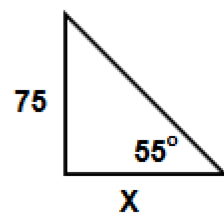
$x = 5.1$



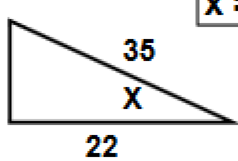
$x = 33.8^{o}$



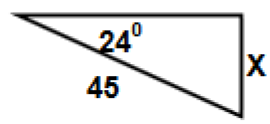
$x = 79.8$



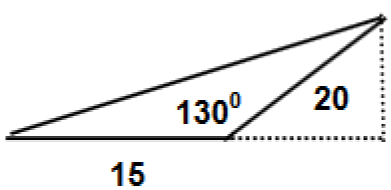
$x = 52.5$



$x = 51^{o}$



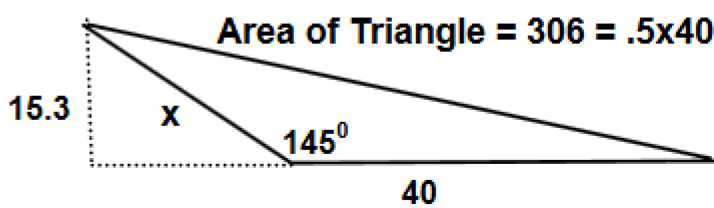
$x = 18.3$



X = 15.3

Area of Triangle = 115 = .5x15x20xSIN(130^{0}) = .5x15x15.3

Area of Triangle = 306 = .5x40x15.3 = .5x40x26.7xSIN(145^{0})

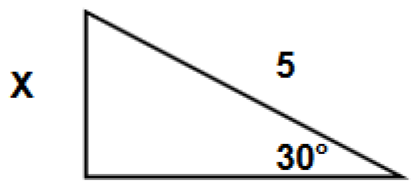


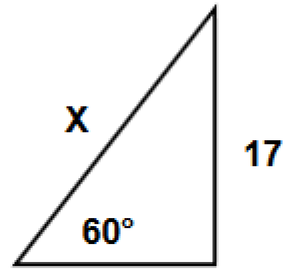
x = 26.7

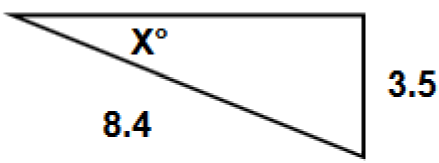
T2ES

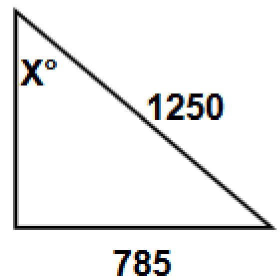
SIN X   SINE OF X

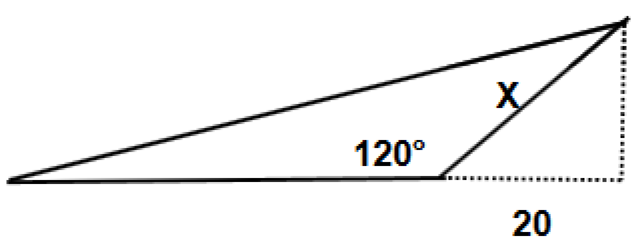
Find X in the following exercises.





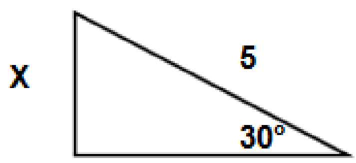




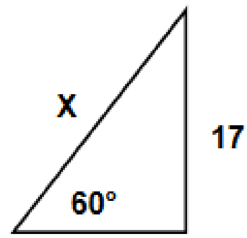


SIN X SINE OF X

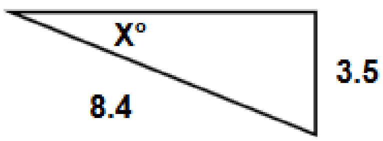
Find X in the following exercises.



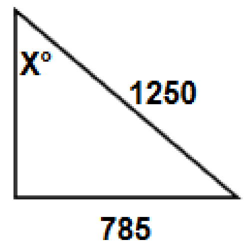
x = 2.5



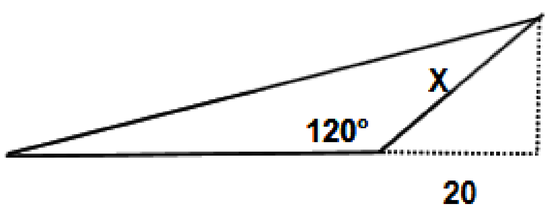
x = 19.6



x = 24.6°



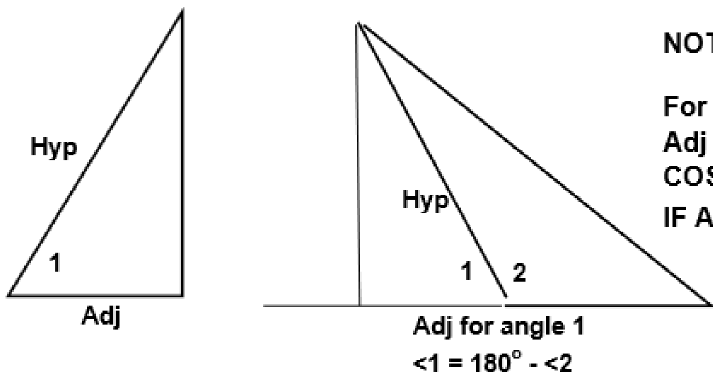
x = 38.9°



x = 40

T3 LESSON: COS X COSINE OF X. X IS AN ANGLE (DEGREES ^{O})

We will extend the definition of **COS** to include all angles from 0^{o} to 180^{o}. In Tier 3 we will extend the definition to include all angles both positive and negative.



NOTICE

For COS only
Adj is NEGATIVE
COS(2)< 0
IF Angle (2) >90°

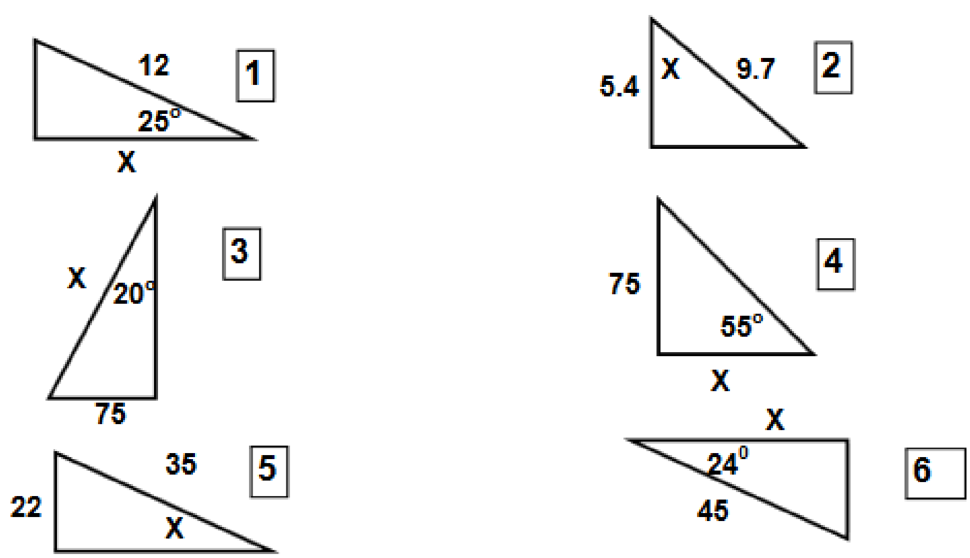
COS(1) = **Adj/Hyp** **COS**(2) = **Adj/Hyp** **COS**(2) = -**COS**(180^{o} - <1)

If you know two out of three, find the third, **Opp**, **Hyp**, (**1**)

Adj = **COS**(1)x**Hyp** **Adj** = **COS**(2)x**Hyp**

Hyp = **Adj/COS**(1) **Hyp** = **Adj/COS**(2)

(**1**) = **COS**$^{-1}$(**Adj/Hyp**) (2) = **COS**$^{-1}$(**Adj/Hyp**) **Adj** < 0



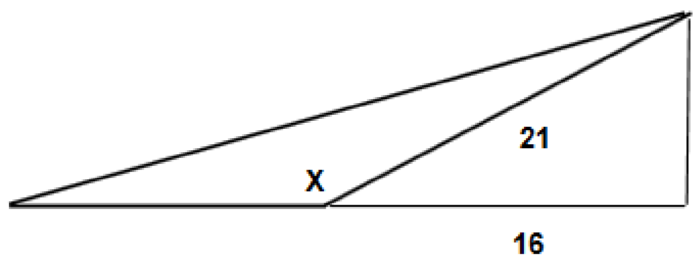
Answers: 1. 10.9 2. 56.2° 3. 219
4. 52.5 5. 38.9° 6. 41.1

T3 COS Problems

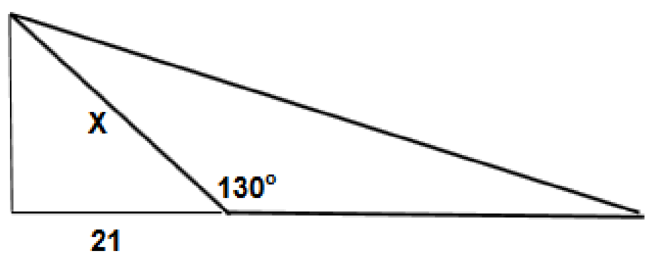
Always set up the Equation first. Then solve.

Use the **Pythagorean Theorem** if necessary.

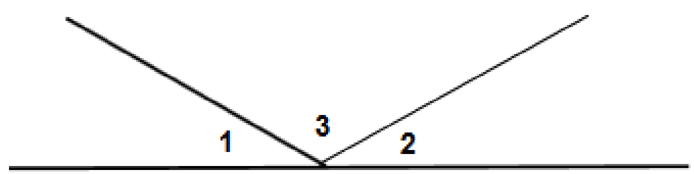
NOTE: **COS**(1) < 0 **IF** 180° > (1) > 90°



1



2



<1 = <2 and <1 + <3 + <2 =180°

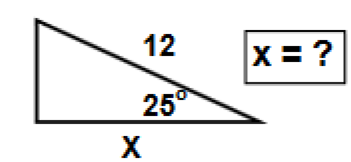
<1 = 35°	What does **COS**(<2 + <3) = ?	**3**
<3 = 120°	What are **COS**(1) and **COS**(2 + 3) ?	**4**
<2 = 30°	What are **SIN**(3) and **COS**(3) ?	**5**

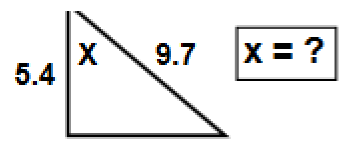
Answers: 1. 137° 2. 32.7 3. -0.819
4. 0.866, -0.866 5. 0.866, -0.5

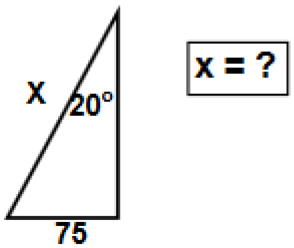
T3E

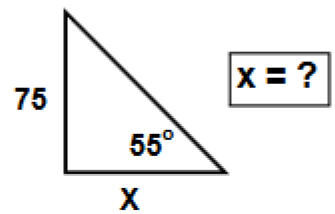
COS X COSINE OF X. X IS AN ANGLE (DEGREES ^{O})

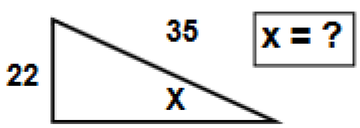
Find **x** in the following exercises.

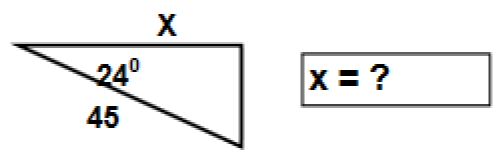


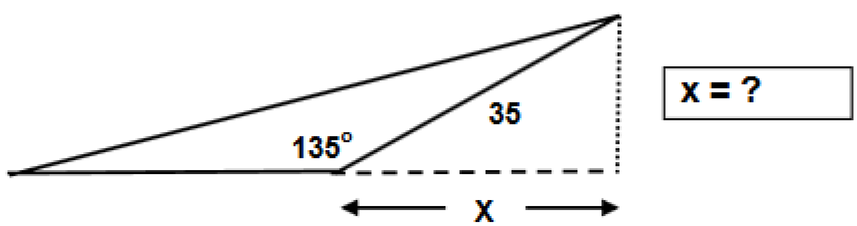








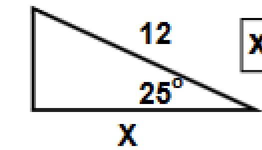




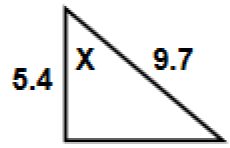
T3EA

COS X COSINE OF X. X IS AN ANGLE (DEGREES ^{O})

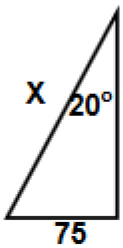
Find **x** in the following exercises.



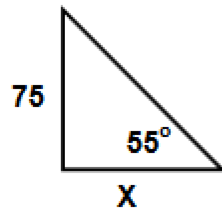
x = 10.9



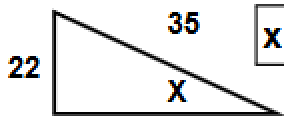
x = 56.2°



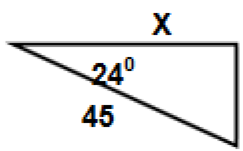
x = 219



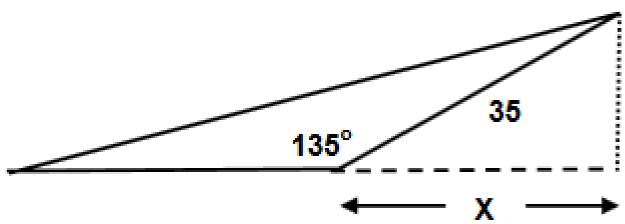
x = 52.5



x = 38.9°



x = 41.1

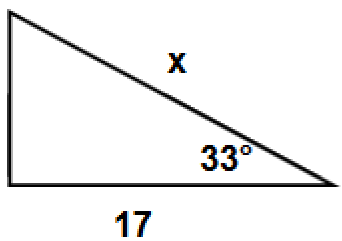


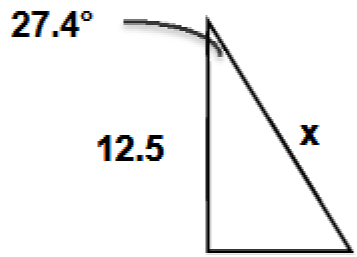
x = 24.7

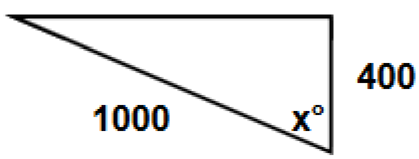
T3ES

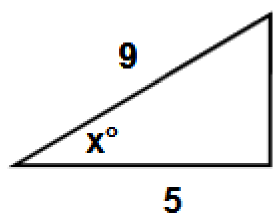
COS X COSINE OF X. X IS AN ANGLE (DEGREES ^{o})

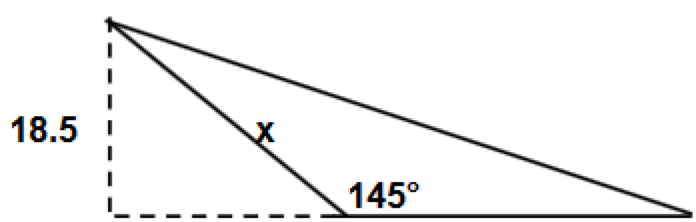
Find X in the following exercises.





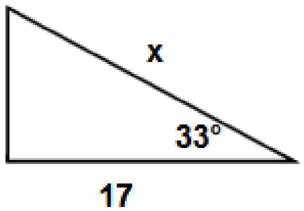




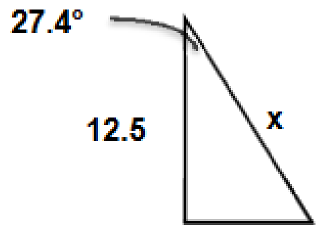


COS X COSINE OF X. X IS AN ANGLE (DEGREES °)

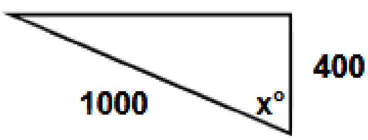
Find X in the following exercises.



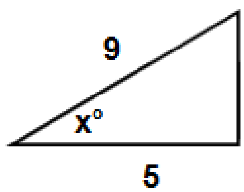
x = 20.3



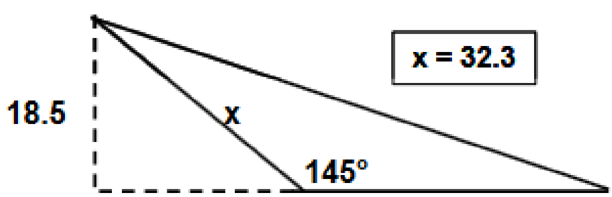
x = 14.1



x = 66.4°



x = 58.3°

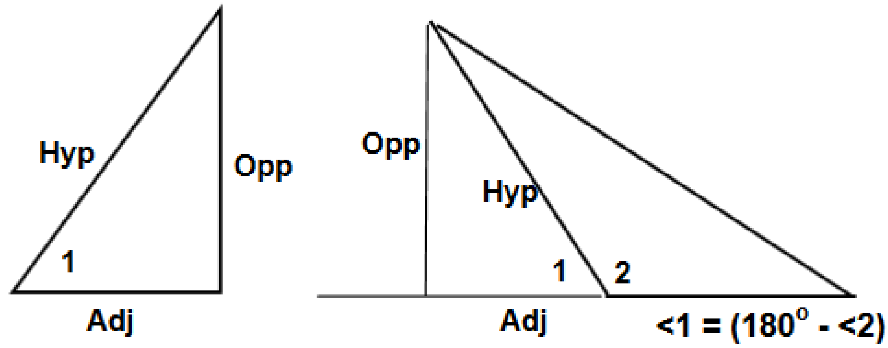


x = 32.3

T4 LESSON: TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{O})

TAN X can take on all values positive and negative

TAN X is **Not** defined at **X** = -90° or 90° (**Error**)



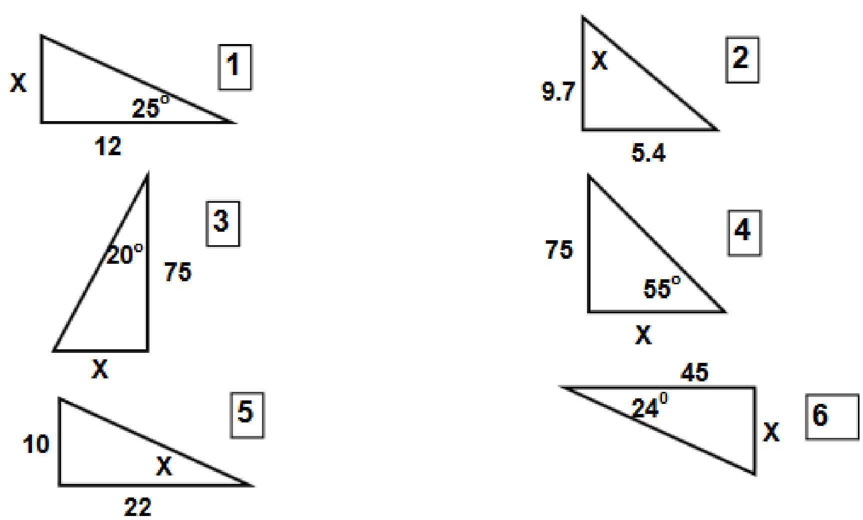
TAN(1) = **Opp/Adj** **TAN**(2) = **Opp/Adj** = -**TAN**(180° - <2)

If know two out of three, find the third, **Opp**, **Adj**, (1)

Opp = **TAN**(1)x**Adj** **Opp** = **TAN**(2)x**Adj** **Adj** < 0

Adj = **Opp/TAN**(1) **Adj** = **Opp/TAN**(2) **Adj** < 0

(1) = **TAN**$^{-1}$(**Opp/Adj**) (2) = **TAN**$^{-1}$(**Opp/Adj**)

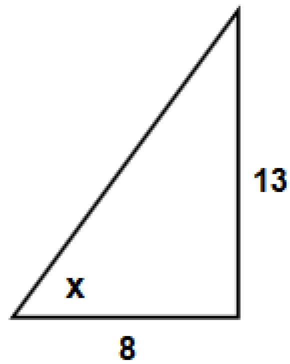


Answers: 1. 5.6 2. 29.1° 3. 27.3
4. 52.5 5. 24.4° 6. 20.0

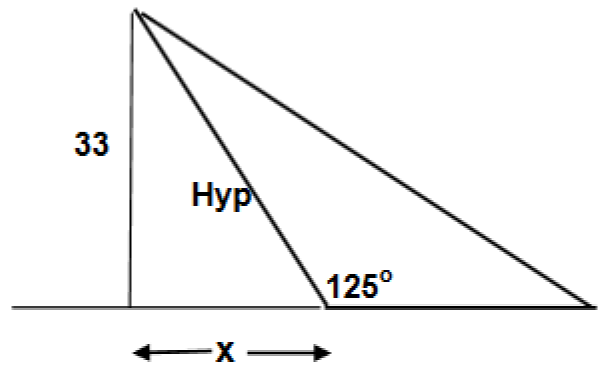
T4E

TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{o})

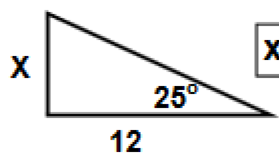
Find X in each of the following exercises.



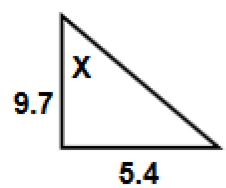
x = ?^{o}



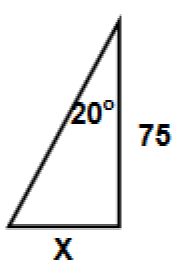
x = ?



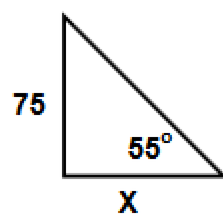
x = ?



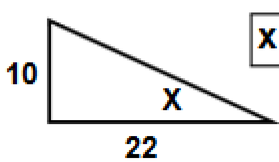
x = ?



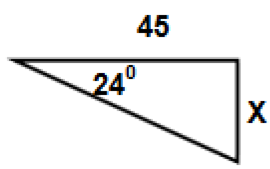
x = ?



x = ?



x = ?

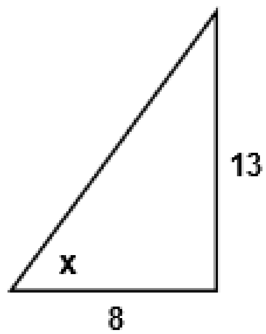


x = ?

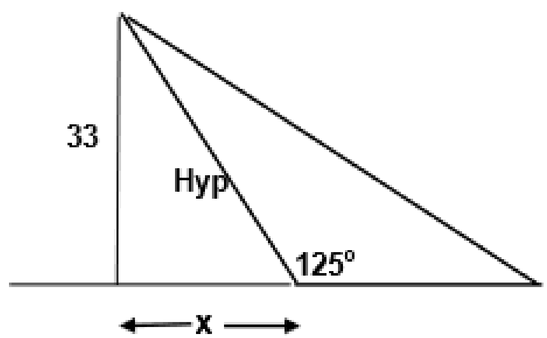
T4EA

TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{O})

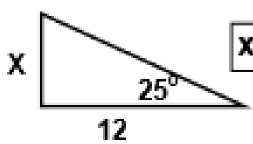
Find X in each of the following exercises.



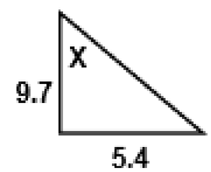
$x = 58.4^{o}$



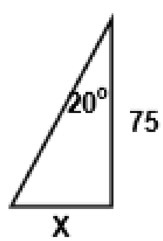
$x = 23.1$



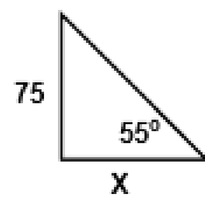
$\boxed{x = 5.6}$



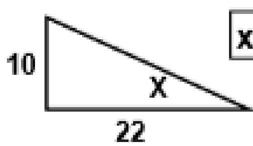
$\boxed{x = 29.1^{o}}$



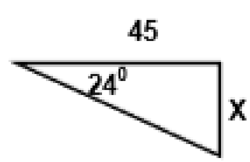
$\boxed{x = 27.3}$



$\boxed{x = 52.5}$



$\boxed{x = 24.4^{o}}$

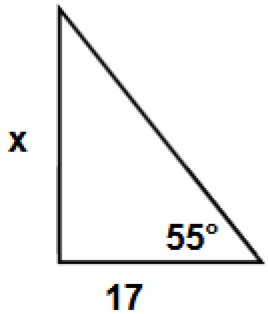


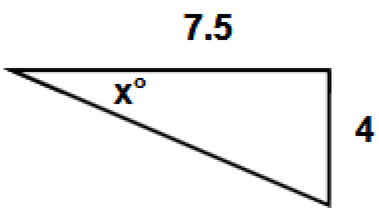
$\boxed{x = 20.0}$

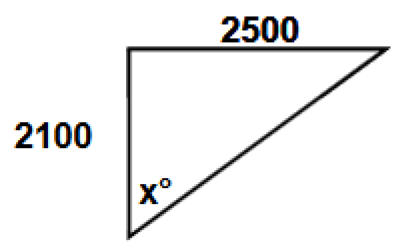
T4ES

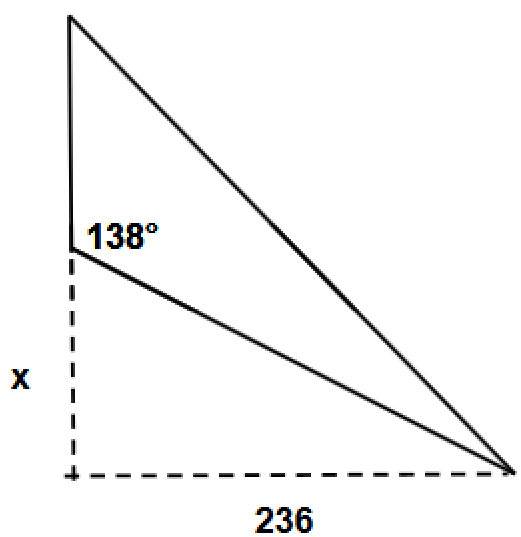
TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{O})

Find X in each of the following exercises.





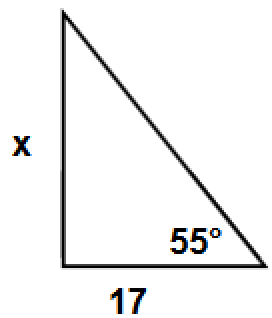




T4ESA

TAN X TANGENT OF X. X IS AN ANGLE (DEGREES ^{O})

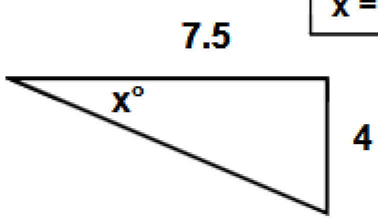
Find X in each of the following exercises.

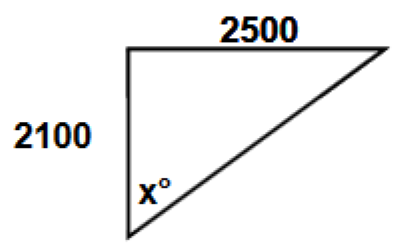


x = 24.3

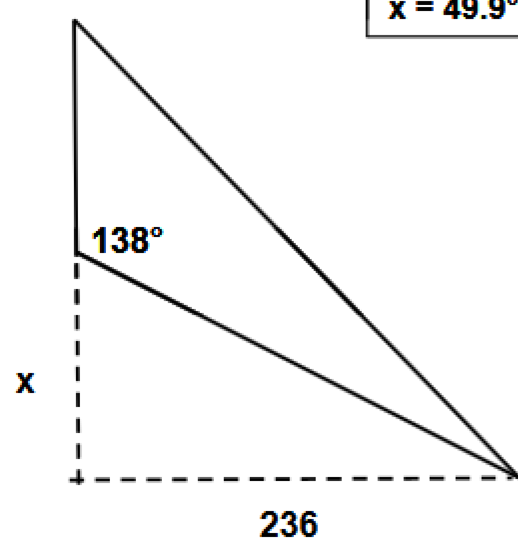
x = 4.55

x = 28.1°





x = 49.9°



x = 262.1

T5 LESSON: WARNING ABOUT SIN^{-1}

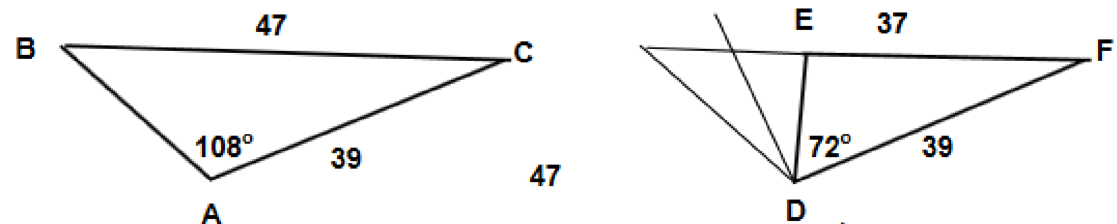
We are interested in **angles**, <A, from 0° to 180°

SIN(<A) = **SIN**(180° - <A)   (see Table below)

So, if we have a **triangle** with an **angle** <A > 90°, with **SIN**(<A), then its **SIN**$^{-1}$ will be wrong.

See below for example:

Suppose we know **SIN**(<A) = .95105, yet **SIN**$^{-1}$(.95105) = 72°



SIN(108°) = SIN(72°)   **SIN^{-1}[SIN(108°)] = 72°**

Angle <A	SIN(<A)	SIN^{-1}	Angle <A	COS(<A)	COS^{-1}
0	0.000	0	0	1.000	0
10	0.174	10	10	0.985	10
20	0.342	20	20	0.940	20
30	0.500	30	30	0.866	30
40	0.643	40	40	0.766	40
50	0.766	50	50	0.643	50
60	0.866	60	60	0.500	60
70	0.940	70	70	0.342	70
80	0.985	80	80	0.174	80
90	1.000	90	90	0.000	90
100	0.985	80	100	-0.174	100
110	0.940	70	110	-0.342	110
120	0.866	60	120	-0.500	120
130	0.766	50	130	-0.643	130
140	0.643	40	140	-0.766	140
150	0.500	30	150	-0.866	150
160	0.342	20	160	-0.940	160
170	0.174	10	170	-0.985	170
180	0.000	0	180	-1.000	180

T5E

WARNING ABOUT SIN^{-1}

When dealing with **angles** whose measure is between 90º and 180º, what happens with the **SIN** and **COS** which can lead to confusion?

If **X** is an angle between 90º and 180º how do **SIN** and **COS** behave?

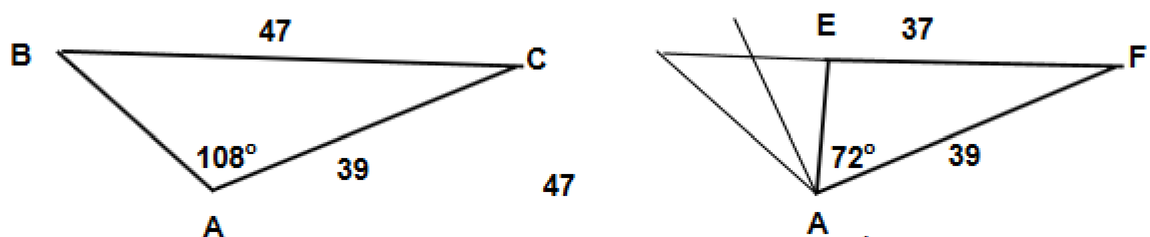
Answer: Give examples:

? For **COS**

? For **SIN**

Suppose we know **SIN**(<A) = .95105, which triangle could this apply to?

Answer: ?



$SIN(108^{\circ}) = SIN(72^{\circ})$ $SIN^{-1}[SIN(108^{\circ})] = 72^{\circ}$

Suppose we know **COS**(<A) = .3090, which triangle could this apply to?

Answer: ?

WHY?

Answer: ?

T5EA

WARNING ABOUT SIN^{-1}

When dealing with angles whose measure is between 90° and 180°, what happens with the **SIN** and **COS** which can lead to confusion?

If **X** is an angle between 90° and 180° how do **SIN** and **COS** behave?

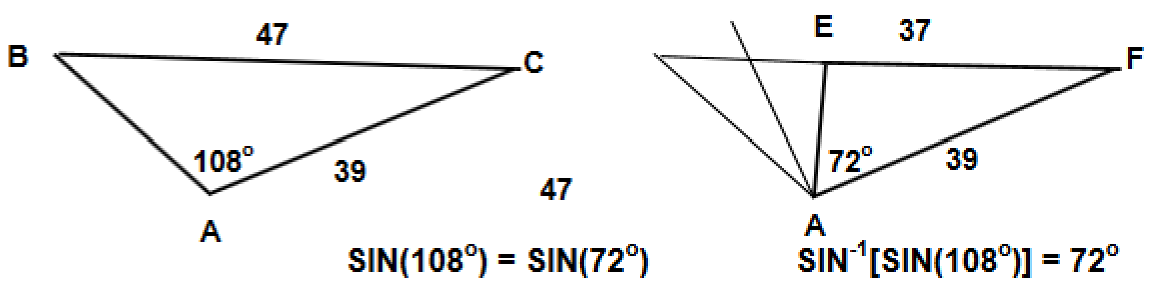
Answer: Give examples:

COS(X°) = - **COS**(180° - X°) **COS**(137°) = -**COS**(43°)

SIN(X°) = **SIN**(180° - X°) **SIN**(137°) = **SIN**(43°)

Suppose we know **SIN**(<A) = .95105, which triangle could this apply to?

Answer: Both



Suppose we know **COS**(<A) = .3090, which triangle could this apply to?

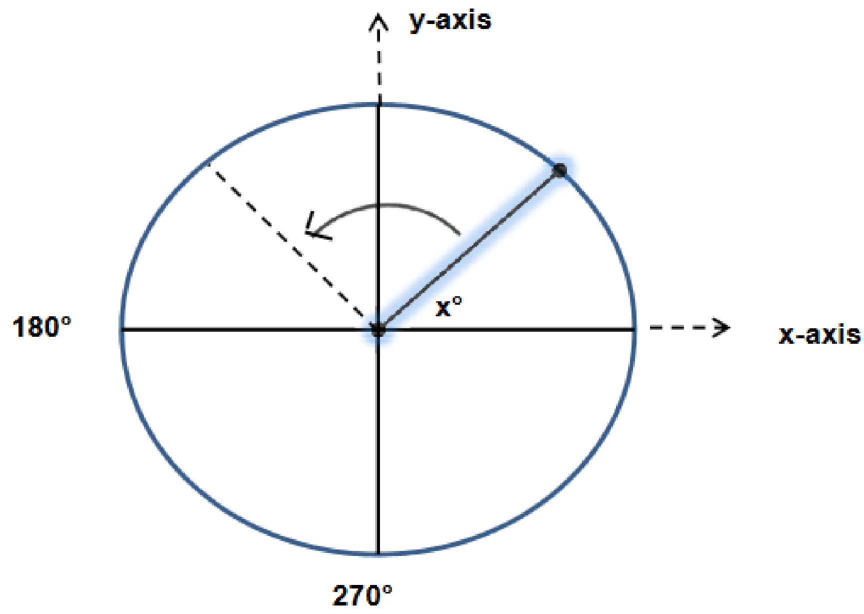
Answer: Only Triangle **AEF**

WHY?

Answer: **COS** (108°) = -.3090

T5ES

WARNING ABOUT SIN^{-1}

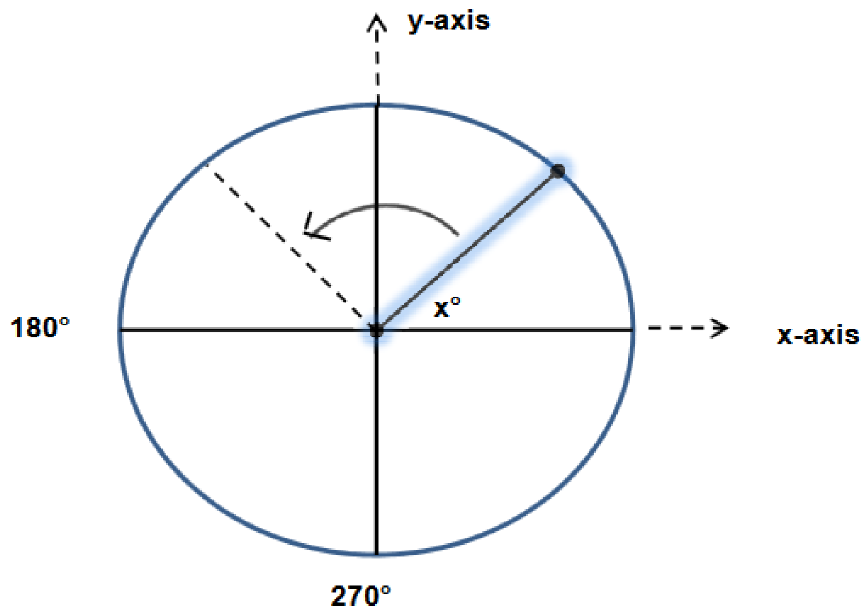


In the diagram above, the highlighted line rotates around in a circle in the xy-plane, starting at 0° and ending at 360°. Fill in the + or -

	0-90°	90-180°	180-270°	270-360°
sin(x)				
cos(x)				
tan(x)				

T5ESA

WARNING ABOUT SIN^{-1}



In the diagram above, the highlighted line rotates around in a circle in the xy-plane, starting at 0° and ending at 360°. Fill in the + or -

	0-90°	90-180°	180-270°	270-360°
sin(x)	positive	positive	negative	negative
cos(x)	positive	negative	negative	positive
tan(x)	positive	negative	positive	negative

T6 LESSON: LAW OF SINES

Problem: Suppose you have a triangle with two angles measuring 40° and 100° and the side opposite the 40° angle is 16 inches.

What is the length, X, of the side opposite the 100° angle? Look at the figure below.

Clearly X is larger than 16 in. Hmmm…maybe it is just proportional to the angles: How about:

$$X = (100^\circ/40^\circ)x16 = (5/2)x16 = 40 \text{ ?}$$

Construct such a triangle and measure it, and you find it measures about 241/2 inches. SO; no, this doesn't work.

Hmmm…what could we do? How about trying some type of correction factor? How about taking the **SIN** of both angles?

SIN(100°)/**SIN**(40°)x16 = 24.5 **Eureka!** ??

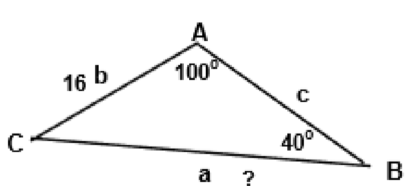
Could this always work? Answer: **YES**.

[**SIN**(<A)/**SIN**(<B)]xb = a, **ALWAYS**, for any angles.

Where **a** is opposite <A and **b** opposite <B

This is called the **Law of Sines**. We prove it in **Tier 3**.

We use it for practical problems. It makes "solving" **triangles** "child's play," especially with a **TI 30XA**.



Law of Sines

a/**SIN**(<A) = b/**SIN**(<B) = c/**SIN**(<C)

[**SIN**<A/**SIN**<B] x b = a

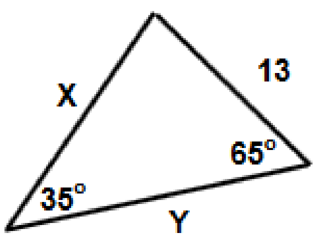
T6 Law of Sines Problems

If you know two angles and an opposite side, you can find them all.

If you know two sides and an opposite angle you can find them all. Sometimes two possibilities.

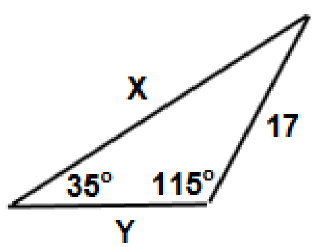
Makes solving problems "child's play."

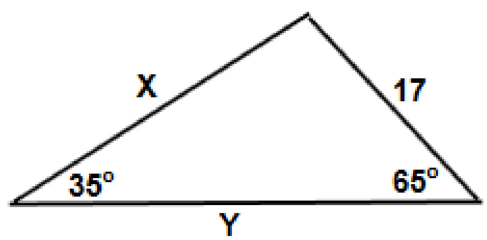
Still, if you know two sides and the included angle, we can't solve for the third side. Need one more tool.



$X/\sin(65^\circ) = 13/\sin(35^\circ)$, $X = 20.5$

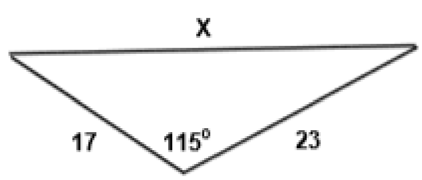
$Y/\sin(80^\circ) = 13/\sin(35^\circ)$, $Y = 22.3$





$X/\sin(115^\circ) = 17/\sin 35^\circ)$ $X = 26.9$ $Y = 14.8$	$X/\sin(65^\circ) = 17/\sin 35^\circ)$ $X = 26.9$ $Y = 29.2$

Observe: $115^\circ + 65^\circ = 180^\circ$ **Thus:** $\sin(115^\circ) = \sin(65^\circ)$



Find X

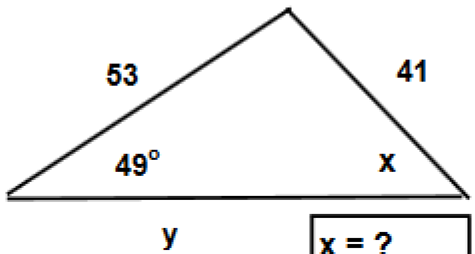
Got to recognize limitations

Need One More Tool

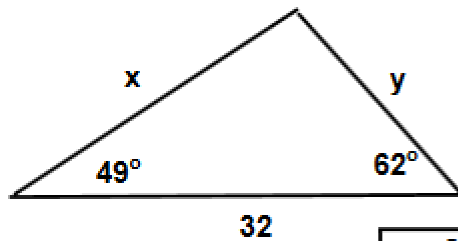
T6E

LAW OF SINES

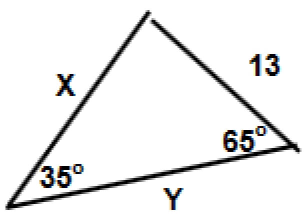
Find the Unknowns and answer questions.



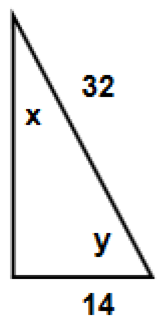
x = ?
y = ?



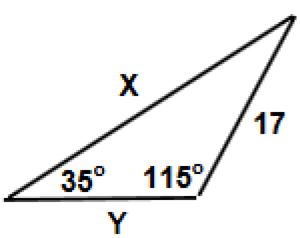
x = ?
y = ?



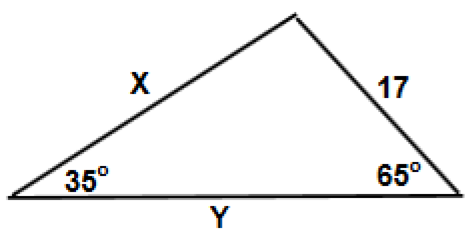
x = ?
y = ?



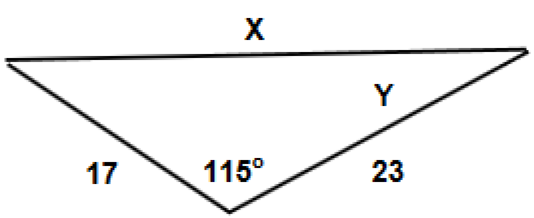
x = ?
y = ?



x = ?
y = ?



x = ?
y = ?

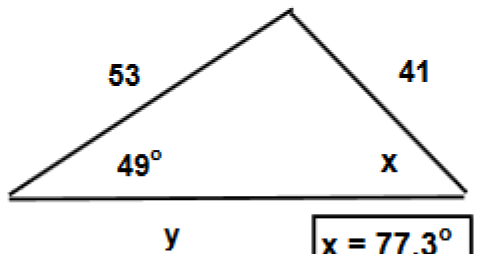


x = ?
y = ?

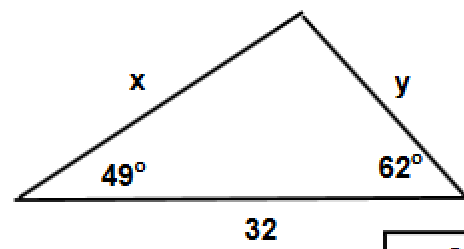
T6EA

LAW OF SINES

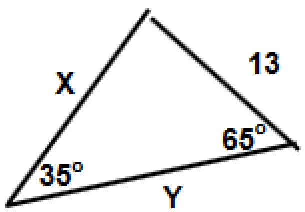
Find the unknowns and answer questions.



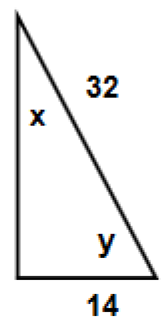
x = 77.3°
y = 43.8



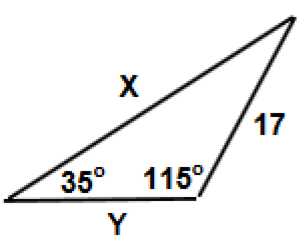
x = 30.3
y = 25.9



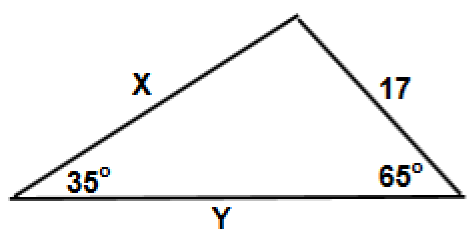
x =20.5
y = 22.3



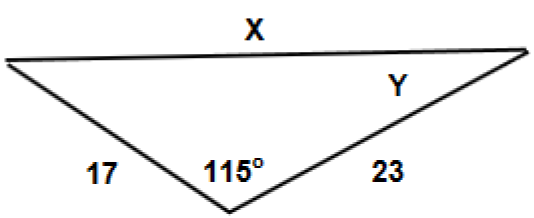
x = 25.9°
y = 64.1°



x = 26.9
y = 14.8



x = 26.9
y = 29.2

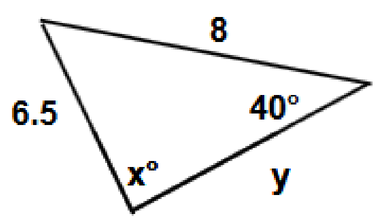


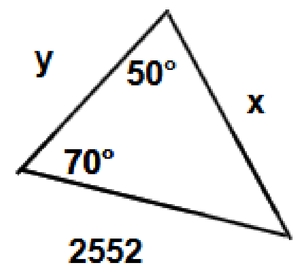
Can not find x and y with tools given so far.
See T7 for solution.

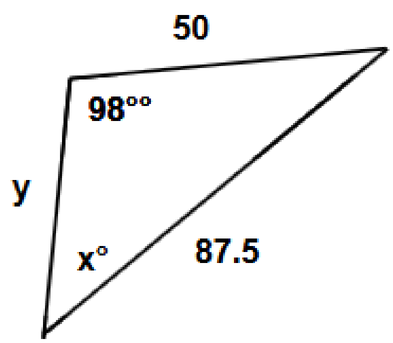
T6ES

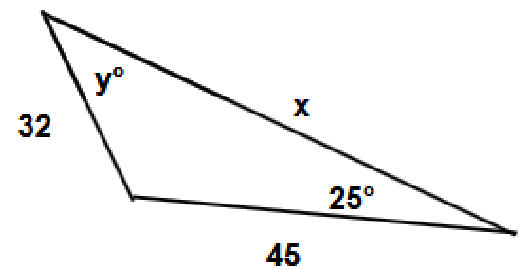
LAW OF SINES

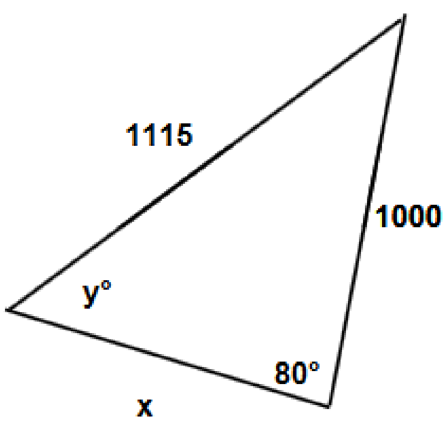
Find X and Y in the following exercises.







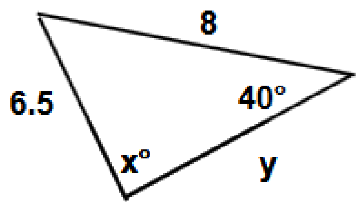




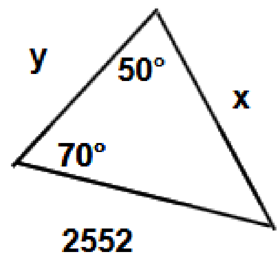
T6ESA

LAW OF SINES

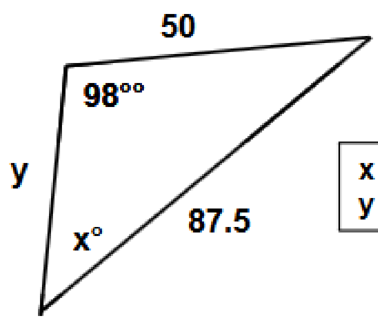
Find X and Y in the following exercises.



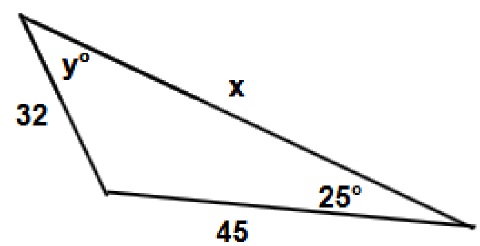
x = 52.3°
y = 6.18



x = 3130.5
y = 2885.1

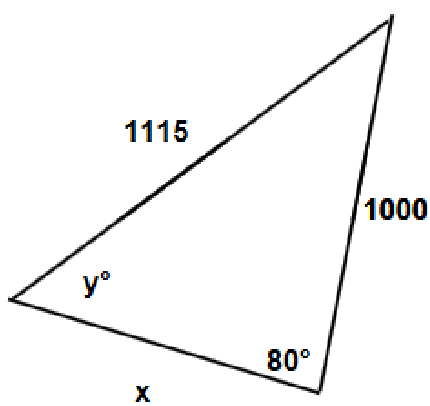


x = 34.5°
y = 72.4



x = 66.5
y = 36.5°

x = 697.1
y = 62°



T7 LESSON: LAW OF COSINES - GENERALIZED PYTHAGOREAN THEOREM

Suppose we know two sides and the included **angle** of a **triangle**. How can we calculate third side's length?

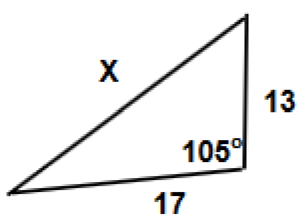
Easy if the **angle** is 90°. $c^2 = a^2 + b^2$

We need a "**correction factor**" for non-right **angles**,

$<a,b \quad c^2 = a^2 + b^2 - 2ab\mathbf{COS}(<a,b)$, works for all **triangles**.

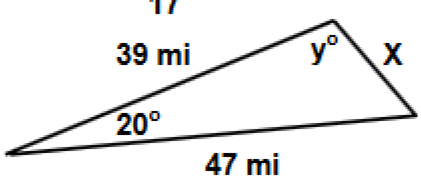
Also, let us find the **angles** when we only know the three sides of a **triangle**.

$<a,b = \mathbf{COS}^{-1}[(a^2 + b^2 - c^2)/(2ab)]$, where $<a,b$ is included angle.



$X^2 = 17^2 + 13^2 - 2x13x17x\mathbf{COS}105^o$

Thus, **X** = 23.9 **NOTE: COS**105° = -.2589, so **CF** is +



$X^2 = 39^2 + 47^2 - 2x39x47x\mathbf{COS}20^o$

Thus, **X** = 16.9 mi

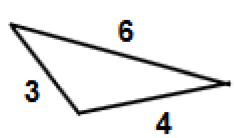
Now we can also calculate y° Use **Law of Sines**

y° = 72° or (180° - 72°) = 108°

Clearly from the diagram 108° is correct.

Find the Area of the 3, 4, 6 triangle using $A = .5ab\mathbf{SIN}(<a,b)$

First, we must calculate $<a,b$ where a = 3, b = 4



$<3,4 = \mathbf{COS}^{-1}[(3^2 + 4^2 - 6^2)/(2x3x4)] = 117.3^o$

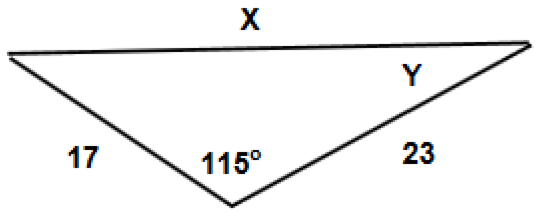
Area = $.5x\mathbf{SIN}(117.3^o)x3x4 = 5.33\ U^2$

T7E

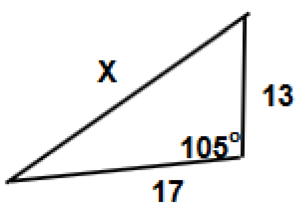
LAW OF COSINES - GENERALIZED PYTHAGOREAN THEOREM

Find the Unknowns

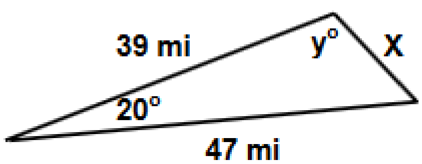
Start with the problem we could not solve in T6



x = ?
y = ?

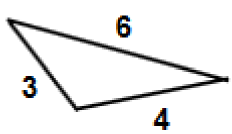


x = ?



x = ?
y = ?

Find the Area of this triangle

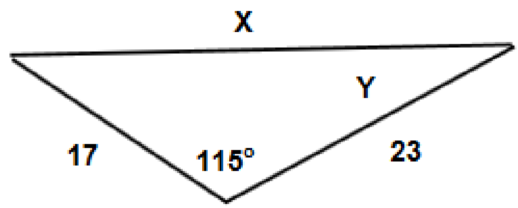


T7EA

LAW OF COSINES - GENERALIZED PYTHAGOREAN THEOREM

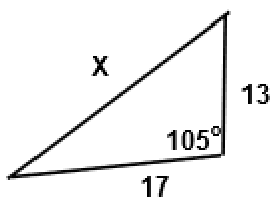
Find the Unknowns

Start with the problem we could not solve in T6



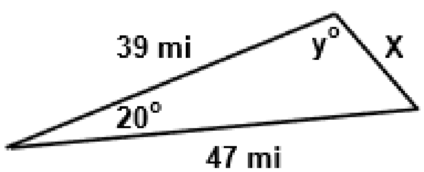
$x = 33.9$
$y = 27°$

$x^2 = 17^2 + 23^2 - 2x17x23x\mathbf{COS}(115^o)$
$y = \mathbf{SIN}^{-1}[\{\mathbf{SIN}(115^o)/33.9\}x17]$



$X^2 = 17^2 + 13^2 -2x13x17x\mathbf{COS}105^o$

Thus, **X** = 23.9 **NOTE**: **COS**105^o = -.2589, so, **CF** is **+**



$X^2 = 39^2 + 47^2 - 2x39x47x\mathbf{COS}20^o$

Thus, **X** = 16.9 mi

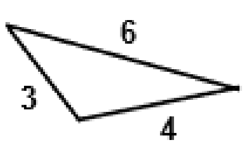
Now we can also calculate y^o

Use **Law of Sines**

Clearly from the diagram 108^o is correct. $y^o = 72^o$ or $(180^o - 72^o) = 108^o$

Find the Area of the 3, 4, 6 triangle using A = .5ab**SIN**(<a,b)

First, we must calculate <a,b where a = 3, b = 4



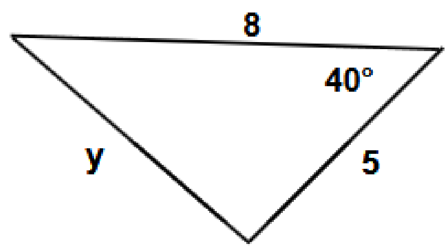
$<3,4 = \mathbf{COS}^{-1}[(3^2 + 4^2 -6^2)/(2x3x4)] = 117.3^o$

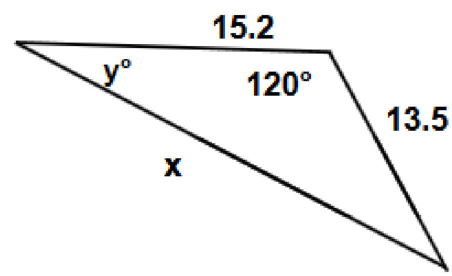
Area = $.5x\mathbf{SIN}(117.3^o)x3x4 = 5.33\ U^2$

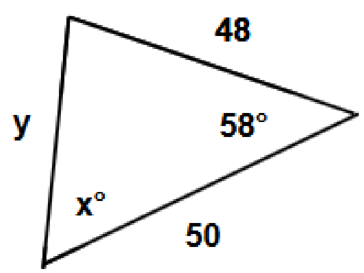
T7ES

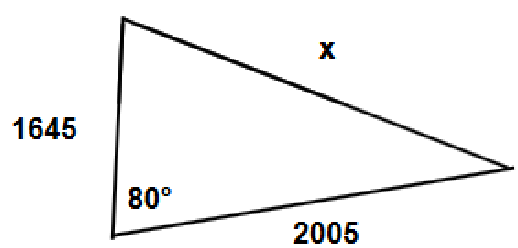
LAW OF COSINES

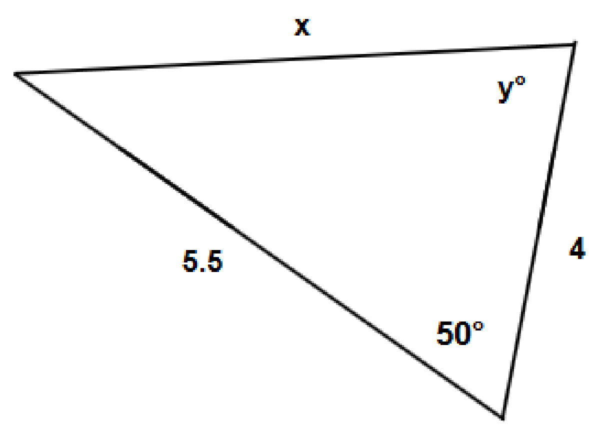
Find X and Y in the following exercises.





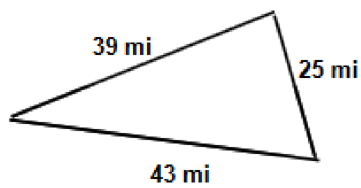


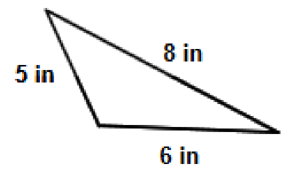


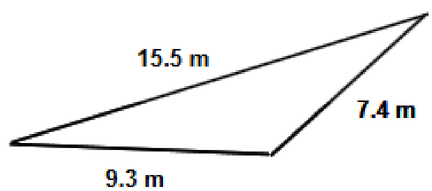


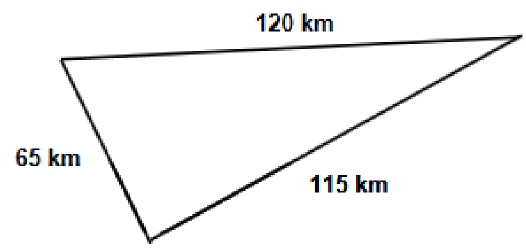
T7ES (cont.)

AREAS OF IRREGULAR TRIANGLES





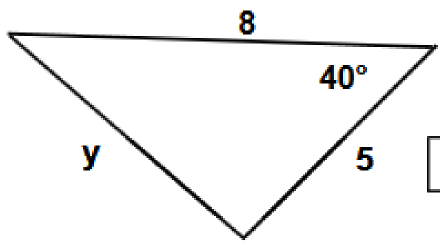




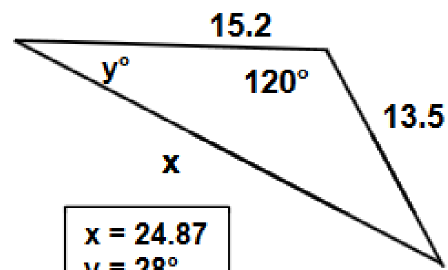
T7ESA

LAW OF COSINES

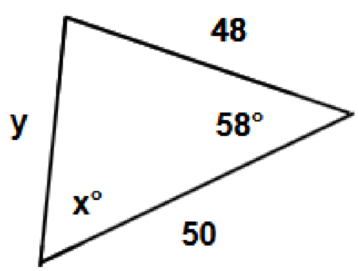
Find X and Y in the following exercises.



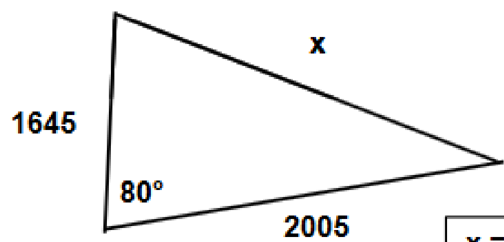
y = 5.26



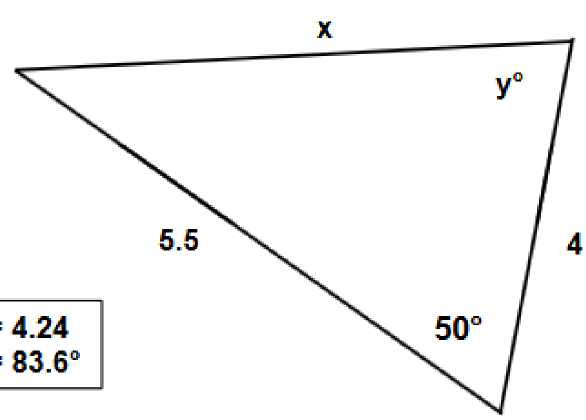
x = 24.87
y = 28°



x = 58.9°
y = 47.5

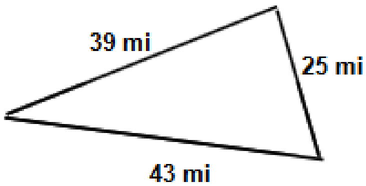


x = 2362.3

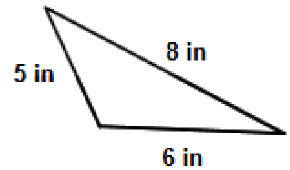


x = 4.24
y = 83.6°

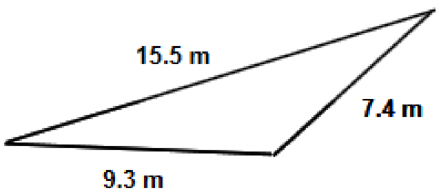
LAW OF COSINES



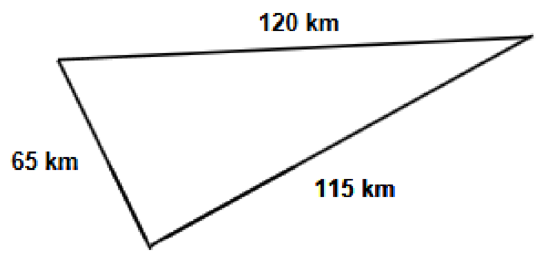
A = 482.1 mi^2



A = 14.98 in^2



A = 23.9 m^2



A = 3,659 km^2

T8 LESSON: TRIGONOMETRY BEYOND PRACTICAL MATH

Trigonometry is a huge extremely important subject with profound applications in science and engineering.

The **Trig Functions** are called the **Circle Functions** and are defined for **ALL** angles, both positive and negative.

Trig Functions are very important in **calculus**.

Trig Functions are probably best understood in the context of the Complex Number System.

Trig Functions are the basis of modern spectrometry via what is called the **Fourier Transform**.

The **Trig Functions** are periodic and that is what makes them so important in any type of **cyclical behavior** such as vibration analysis, and music.

So next, you will need to understand the **Trig functions** via graphs in analytical geometry (Tier 3).

Then one needs to learn about them in the context of the Complex Number System. That is when many of the famous **Trig Identities** will become very natural and understandable. What I consider the most important equation in all of mathematics makes this clear (Tier 4).

Then one needs to learn about their behavior utilizing the **calculus**. It is truly amazing (Tier 5).

Ultimately, they are profound in Functional Analysis and modern physics such as **Quantum Theory** (Tier 9).

T8E

TRIGONOMETRY BEYOND PRACTICAL MATH

Trigonometry is a huge extremely important subject with profound applications in science and engineering and advanced mathematics.

If you are planning to study math beyond Practical Math, then you should be aware of some of the future applications of **Trigonometry**.

List as many things you have heard about where **Trig** will be useful and applicable.

If you study other resources such as Wikipedia you will probably come up with other applications in addition to those I have pointed out.

Please accept my best wishes for your future success.

I hope mathematics will be rewarding to you in your future endeavors, and enjoyable too.

Thank you for studying this **Foundations Course**.

Dr. Del.

T8EA

TRIGONOMETRY BEYOND PRACTICAL MATH

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Ultimately, they are profound in **Functional Analysis** and modern physics such as **Quantum Theory** (Tier 9).