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Workforce Development:

Intermediate Math for Industry

1.1 Lessons Abbreviation Key Table

C = Calculator Lesson
P = Pre-algebra Lesson
A = Algebra Lesson
G = Geometry Lesson
S = Special Topics

The number following the letter is the Lesson Number.

E = Exercises with Answers: Answers are in brackets **[ ]**.
EA = Exercises Answers: (only used when answers are not on the same page as the exercises.)
ES = Exercises Supplemental: Complete if you feel you need additional problems to work.

1.2 Exercises Introduction

Why do the Exercises?

Mathematics is like a "game." The more you practice and play the game the better you will understand and play it.

The Foundation's Exercises, which accompany each lesson, are designed to reinforce the ideas presented to you in that lesson's video.

It is unlikely you will learn math very well by simply reading about it or listening to Dr. Del, or anyone else, or watching someone else doing it.

You WILL learn math by "doing math."

It is like learning to play a musical instrument, or write a book, or play a sport, or play chess, or cooking.

You will learn by practice.

Repetition is the key to mastery.

You will make mistakes. You will sometimes struggle to master a concept or technique. You may feel frustration sometimes "**WE ALL DO**."

But, as you learn and do math, you will begin to find pleasure and enjoyment in it as you would in any worthwhile endeavor. Treat it like a sport or game.

These exercises are the KEY to your SUCCESS!

ENJOY!

INTRODUCTION TO ALGEBRA

Algebra is a "technology" for finding unknown numbers, X, Y, Z, etc., from known numbers A, B, C, etc. In our Foundation course, we will only deal with one unknown number, usually denoted X, but we could denote it with any symbol.

The Algebra technique is to create an Equation involving the unknown number X and the known numbers A, B, C, etc., based on their known relationships and then "solving" the equation for the unknown, and checking the answer.

Step 1 is to "**create**" the equation between X and the knowns.

Step 2 is to "**solve**" this equation by finding out what value of X makes the equation true when substituted for X.

Step 3 is to "**verify**" or "**check**" the solution by making the substitution.

Simple Example: [Word Problem] Three years from now Mary will be twice as old as Joe who is 7 years old today. How old is Mary now?

Step 1. Let X be Mary's age today. This is the unknown we want to find. In three years Mary will be X + 3 years old. In three years Joe will be 7 + 3 = 10 years old. So, we are given that in three years X + 3 = 2x10 = 20

Step 2. Solve the equation. By trial and error, it appears 17 might be the answer.

Step 3. Check. Substitute 17 for X. 17 + 3 = 20. So, 17 is the answer.

Now, in general, it is not too hard to do Step 1. Define what X stands for and then relate the given facts to X and create an equation.

Step 2 can be very easy; or, very difficult, to solve. In the Foundation course, we will deal with equations that arise in many common situations, and these are usually easy to solve.

Step 3 is quite easy with a calculator.

A1 LESSON: FOUR WAYS TO SOLVE AN ALGEBRA EQUATION

Suppose you have an equation with one unknown, **X**. How can you solve it?

There are essentially four ways.

1. **Guess the answer**. Check to see if you are right. This is a good way with really simple equations. It can be the best way with very complicated equations **IF** you have a computer to help. This is then called **Numerical Analysis**.

2. **Apply a Formula**. This is fine **IF** you know an appropriate formula. This is useful if you are solving the same type of equation frequently and have the formula available. However, it can be quite difficult to find or remember the correct formula. Formulas are often given in Handbooks for special situations.

3. **Apply a Process**. This is the best way for certain equations, and it is how we will solve most of our equations in this Foundation course, and in the real world.

4. **Apply a Power Tool**. This is the best way for complex equations. One great tool for this is Mathematica. This is how engineers solve most of their equations. But, you must learn to use this tool first. We will cover it extensively in the upper Tiers in our advanced training. If also applies to other types of equations.

In our Foundation course, we will learn to **Apply a Process**. This usually is easier than trying to find the correct formula. It is faster and more accurate for certain types of equations we will be dealing with.

A1E

Four Ways to Solve an Algebra Equation

Suppose you have an equation with one unknown, X. How can you solve it?

What are the Four Ways to solve an equation?

1.

2.

3.

4.

Which way will be utilize and learn in the Foundations Course? Why?

A1EA

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Which way will be utilize and learn in the Foundations Course? Why?

In our Foundation course, we will learn to <u>Apply a Process</u>. This usually is easier than trying to find the correct formula. It is faster and more accurate for certain types of equations we will be dealing with.

A1ES

Four Ways to Solve an Algebra Equation

1. In the PMF, what do we want to know about an Algebra Equation?

2. Why do we normally use Applying a Process instead of Applying a Formula to solve algebra equations?

A1ESA

Four Ways to Solve an Algebra Equation   Answers: [ ]

1. In the PMF, what do we want to know about an Algebra Equation?

[**We want to see if we can find the value of the unknown in the equation, most generally denoted by X!**]

2. Why do we normally use Applying a Process instead of Applying a Formula to solve algebra equations?

[**Applying a Formula only works for special types of problems and specific formulas, and requires a good deal of memorization. Applying a Process allows us to work with many types of equations with needing to memorize specific formulas!**]

A2 LESSON: THE RULE OF ALGEBRA

Suppose we have an equation to solve. It will have a Left Side (**LS**) and a Right Side (**RS**) either of which might contain the unknown, **X**, and other known numbers. (Any letter could be the unknown.)

Equation: **LS** = **RS** Can switch sides **RS** = **LS**

THE RULE of Equation Solving is: You may do the same thing to both sides of the equation and obtain a new equation:

1. **LS** + A = **RS** + A, **LS** - A = **RS** – A Add or Subtract a Number to both sides of the equation.
2. **LS**xA = **RS**xA, **LS**÷A = **RS**÷A Multiply or Divide a Number
3. 1/**LS** = 1/**RS** Invert both sides
4. $(\mathbf{LS})^2 = (\mathbf{RS})^2$ Square both sides
5. √**LS** = √**RS** Square Root Both Sides
6. SIN (**LS**) = SIN (**RS**) Take the SIN of both sides.
7. Any legitimate math operation to both sides.

The idea is to apply a sequence of operations or transformations to both sides until you arrive at:

X = Number "The Solution"

Then <u>**check your answer**</u> by substituting this Number into the Equation in place of **X** and see that both sides are equal.
We will see many examples of this in the lessons to follow. These are the kinds of Equations you will be solving in the "real world."

A2E

THE RULE OF ALGEBRA

Suppose we have an equation to solve. It will have a Left Side (**LS**) and a Right Side (**RS**) either of which might contain the unknown, **X**, and other known numbers. (Any letter could be the unknown.)

Equation: **LS** = **RS** Can switch sides **RS** = **LS**

1. What is **THE RULE** of Equation Solving?

2. Give examples of applying this Rule.

3. Describe the process you will use to solve an equation using this Rule.

4. After you have a solution: **X** = Number, what should you always do, especially if the answer is important?

A2EA

THE RULE OF ALGEBRA

Suppose we have an equation to solve. It will have a Left Side (**LS**) and a Right Side (**RS**) either of which might contain the unknown, **X**, and other known numbers. (Any letter could be the unknown.)

Equation: **LS** = **RS** Can switch sides **RS** = **LS**

1. **THE RULE** of Equation Solving is: *You may do the same thing to both sides of the equation and obtain a new equation:*

2. Examples:

1) **LS** + **A** = **RS** + **A**, **LS** - **A** = **RS** - A (add or subtract a number to both sides of the equation)
2) **LS**x**A** = **RS**x**A**, **LS**÷**A** = **RS**÷**A** (multiply or divide a number)
3) 1/**LS** = 1/RS (invert both sides)
4) $(\mathbf{LS})^2 = (\mathbf{RS})^2$ (square both sides)
5) $\sqrt{\mathbf{LS}} = \sqrt{\mathbf{RS}}$ (square root both sides)
6) SIN (**LS**) = SIN (**RS**) (take the SIN of both sides)
7) Any legitimate math operation to both sides.

3. The idea is to apply a sequence of operations or transformations to both sides until you arrive at:

X = Number "**The Solution**"

4. Then <u>check your answer</u> by substituting this Number into the Equation in place of **X** and see that both sides are equal.

We will see many examples of this in the lessons to follow. These are the kinds of Equations you will be solving in the "real world."

THE RULE OF ALGEBRA

1. When solving an equation, any math operation (adding, subtracting, multiplying, etc.) done to one side of the equation must be ___________________________. *fill in the blank*

2. If we solved the equation $X + 3 = 8$, and got $X = 6$, what IMPORTANT STEP would help us realize we made a mistake?

A2ESA

THE RULE OF ALGEBRA Answers: [ ]

1. When solving an equation, any math operation (adding, subtracting, multiplying, etc.) done to one side of the equation must be [done to the other side of the equation].

2. If we solved the equation X + 3 = 8, and got X = 6, what IMPORTANT STEP would help us realize we made a mistake?
   [**If we checked our solution by plugging it back into the original equation we would see that X = 6 gives 9 = 8, which is obviously incorrect!**]

A3 LESSON: $X + A = B$ THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

$X + A - A = B - A$ [subtract **A** from both sides] [transpose **A**]

Thus: $X = B - A$ since $A - A = 0$ and $X + 0 = X$

Example: $X + 2 = 5$ [subtract **2** from both sides]

Solution: $X = X + 2 - 2 = 5 - 2 = 3$

Example: $X - 7 = -13$ [add **7** to both sides]

Solution: $X = X - 7 + 7 = -13 + 7 = -6$ [we have transposed **7**]

Example: $8.13 = -7.19 + X$

Same as: $X - 7.19 = 8.13$ [since can switch sides]

Solution: Add 7.19 to both sides. $X = 15.32$ (use calculator)

Example: $X + (-18.4) = +\sqrt{37.9}$

Same as: $X - 18.4 = 6.16$ [take square root +(-) = -]

$X = X - 18.4 + 18.4 = 6.16 + 18.4 = 24.56 = 24.6$

[add 18.4]

Example: $X - \mathbf{SIN}(37^o) = [COS(68^o)]2$ [do not be intimidated

$\mathbf{SIN}(37^o) = .6018$ $COS(68^o) = .3746$ $(.3746)^2 = .1403$

SO: $X - .6018 = .1403$ and

THUS: $X = .7421$

A3E

X + A = B THIS IS AN EASY LINEAR EQUATION

$X + A - A = B - A$ [subtract **A** from both sides] [transpose **A**]

Thus: $X = B - A$ **since** $A - A = 0$ **and** $X + 0 = X$

Solve for **X,** the Unknown

1. $X + 42 = 59$
2. $X - 17 = -43$
3. $8.13 = -17.19 + X$
4. $X + (-28.4) = +\sqrt{87.9}$
5. $6.5 - X = 23.5$
6. $5432 = X + 4375$
7. $X - \sqrt{675} = \sqrt{9876}$
8. $X - 3/4 = 9/13$
9. $6/7 = 8/11 - X$
10. $0.00035 + X = 0.0017$
11. $X - \mathbf{SIN}(37^o) = [\mathbf{COS}(68^o)]^2$
12. $\mathbf{COS}(48^o) = \mathbf{TAN}(78^o) - X$
13. $(13.4 + 9.7)^2 + X = 87.4^2$

A3EA

X + A = B THIS IS AN EASY LINEAR EQUATION Answers: [ ]

X + A - A = B - A [subtract A from both sides] [transpose A]

Thus: X = B – A since A - A = 0 and X + 0 = X

1. X + 42 = 59	[17]
2. X - 17 = -43	[-26]
3. 8.13 = -17.19 + X	[25.32]
4. X + (-28.4) = +√87.9	[37.8]
5. 6.5 - X = 23.5	[-17]
6. 5432 = X + 4375	[1057]
7. X - √675 = √9876	[125.4]
8. X - 3/4 = 9/13	[75/52=123/52]
9. 6/7 = 8/11 – X	[-10/77]
10. 0.00035 + X = 0.0017	[0.00135]
11. X - SIN(37^o) = [COS(68^o)]^2	[0.742]
12. COS(48^o) = TAN(78^o) – X	[4.035]
13. $(13.4 + 9.7)^2 + X = 87.4^2$	[7105.2]

A3ES

$X + A = B$ THIS IS AN EASY LINEAR EQUATION

Answers: [ ]

1. $X + 54 = 100$	[**X = 46**]
2. $8.7 - X = 4.9$	[**X = 3.8**]
3. $X + (-0.567) = 3.14$	[**X = 3.707**]
4. $X + \sqrt{25} = 10$	[**X = 5**]
5. $17^2 - X = 100$	[**X = 189**]
6. $X - \sin(30°) = 1$	[**X = 1.5**]
7. $X - 5/6 = 4/5$	[**X = 1.633**]
8. $7/6 = 8/5 - X$	[**X = 0.433**]
9. $0.3017^4 + X = 0.0012^2$	[**X = -0.0083**]
10. $[\cos(180°)]^2 - X = \sin(270°)$	[**X = 2**]
11. $\pi - X = \pi/2$	[**X = π/2**]
12. $(2^3 + X) - 4 = (2^2 + 3^2)$	[**X = 9**]

A4 LESSON: AX = B THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

$X = AX/A = B/A$ [divide both sides by **A**] **Note**: $A/A = 1$

Example: $3X = 12$

Solution: $X = 3X/3 = 12/3 = 4$ [divide by **3** both sides always]

Example: $2.16X = -56.3$

Solution: $X = -56.3/2.16 = -26.0648 = -26.1$

Example: $-37.8 = -6.78X$

Solution: $-6.78X = -37.8$ [switch sides]

Then: $X = (-37.8)/(-6.78) = 5.6$ [divide by -6.78]

Example: $(3.85)^2X = \sqrt{349}/\ \mathbf{SIN}(79^0)$ [easy does it!]

$(3.85)^2 = 14.8$ $\sqrt{349} = 18.7$ $\mathbf{SIN}\ (79^0) = .982$

So: $14.8X = 18.7/.982 = 19.0$ $X = 1.29$ [divide by 14.8]

Always simplify the numbers first, and then solve the equation. The calculator makes this easy. Also, always **CHECK** your answer by plugging it back into the equation and being sure both sides are equal.

$(3.85)^2 x1.29 = 19.1$ $\sqrt{349}/\ \mathbf{SIN}(79^o) = 19.0$ [round off error]

A4E

AX = B THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

X = **AX/A** = **B/A** [divide both sides by **A**] **Note**: **A/A** = 1

Solve for **X,** the Unknown

1. 4**X** = 12
2. 2.16**X** = -56.3
3. -37.8 = -6.78**X**
4. 0.003**X** = 0.15
5. (4/5)**X** = 7/9
6. $(1+3)^2$**X** = √65
7. $(3.85)^2$X =√349/ **SIN**(79^0) {Easy does it!}
8. (1 + 2/3) = (7/12)**X**
9. 2345**X** = 9876
10. 54.5 = -87.7**X**
11. **COS**(32°)**X** = 3**SIN**(32°)
12. **X** = 3**TAN**(32°)

A4EA

AX = B THIS IS AN EASY LINEAR EQUATION Answers: [ ]

What can you do to both sides to get closer to a solution?

X = **AX/A** = **B/A** [divide both sides by **A**] **Note**: **A/A** = 1

Solve for **X,** the Unknown

1. 4**X** = 12 [**3**]
2. 2.16**X** = -56.3 [**-26.1**]
3. -37.8 = -6.78**X** [**5.58**]
4. 0.003**X** = 0.15 [**50**]
5. (4/5)**X** = 7/9 [**35/36 = 0.97**]
6. $(1+3)^2$**X** = √65 [**0.5**]
7. $(3.85)^2$**X** =√349/ **SIN**(79^0) [**1.28**]
8. (1 + 2/3) = (7/12)**X** [**20/7 = 26/7 = 2.86**]
9. 2345**X** = 9876 [**4.2**]
10. 54.5 = -87.7**X** [**-0.62**]
11. **COS**(32^o)**X** = 3**SIN**(32^o) [**1.875**]
12. **X** = 3**TAN** (32^o) [**1.875**]

A4ES

AX = B THIS IS AN EASY LINEAR EQUATION Answers: [ ]

1. $5X = 27.25$ [**X = 5.45**]
2. $67 - 2 = 13X$ [**X = 5**]
3. $5.1X - 3 = 2.1$ [**X = 1**]
4. $9 = 3X + 17$ [**X = - 2.6**]
5. $(5^2)X = 1000$ [**X = 40**]
6. $TAN(30°)X = 18$ [**X = 31.18**]
7. $(\sqrt{169})X = 26$ [**X = 2**]
8. $(-7/8) = (-8/5)X$ [**X = 0.5469**]
9. $[SIN(60°)]^2X = 3$ [**X = 4**]
10. **In the equation AX = B, when solving it we would divide B by A. Notice how dividing B by A is the same as MULTIPLYING B by (1/A).** In the equation, $(2/3)X = 2$, we would solve by dividing 2 by (2/3). If we want to think in terms of multiplication, what we would multiply 2 by instead?

    [**We would think of multiplying 2 by the reciprocal of 2/3, which is 3/2.**]

11. $(\sqrt{36})[COS(60°)]^2 = SIN(270°)X$ [**X = -1.5**]
12. $3X + 3X + 3X = -0.62612$ [**X = -0.0696**]

A5 LESSON: AX+B = CX+D THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

Get all the X terms on one side and numbers on other side.

$AX - CX = D - B$ or $(A - C)X = D - B$ [distributive law]

$X = (D - B)/(A - C)$ [divide both sides by (A - C)]

Example: $3X + 7 = 5 - 7X$

Solution: $3X + 7X = 5 - 7$ **or** $10X = -2$ **or** $X = -2/10 = -.5$

Example: $-18.3X + 4.6X - 22.4 = 13.9X - 45.4 + 3.9X$

$-18.3X + 4.6X -13.9X - 3.9X = -45.4 + 22.4$

$(-18.3 + 4.6 -13.9 -3.9)X = -31.5X = -23.0$

$X = -23.0/-31.5 = .730$

Once again…always do the numerical calculations first.

Example: $(2.13)^2X - \mathbf{LOG}(345) = 1/\mathbf{COS}(12.5^o) + \sqrt{(5 + 1/.15)}X$

$(2.13)^2 = 4.54$ $\mathbf{LOG}\ (345) = 2.54$

$\mathbf{COS}(12.5^o) = .976$ $1/.976 = 1.024$

and: $\sqrt{(5 + 1/.15)} = \sqrt{(5 + 6.67)} = 3.42$ [easy w/calculator]

$4.54X - 2.54 = 1.024 + 3.42X$

or: $(4.54 - 3.42)X = 1.024 + 2.54$

$1.12X = 3.56$

$X = 3.56/1.12 = 3.18$ [you check the answer]

$(2.13)^2x3.18 - \mathbf{LOG}\ (345) = 11.9 = 1/\mathbf{COS}\ (12.5^o) +$

$\sqrt{}\ (5 + 1/.15)3.18$

A5E

$AX + B = CX + D$ THIS IS AN EASY LINEAR EQUATION.

Solve for X, the Unknown. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30XA**.

1. $3X + 7 = 5 - 7X$
2. $3.2X - 9 = 4.1X + 7.8$
3. $-12X - 98 = 23X + 76$
4. $0.002X - 0.015 = 0.0087 - 0.005X$
5. $(3/4)X - 2/7 = (4/5)X + 3/8$
6. $\mathbf{SIN}(28^o)X - 1.4 = \mathbf{COS}(28^o)X + 2.3$
7. $-18.3X + 4.6X - 22.4 = 13.9X - 45.4 + 3.9X$
8. $(2.13)^2X - \mathbf{LOG}(345) = 1/\mathbf{COS}(12.5^o) + \sqrt{(5 + 1/0.15)}\, X$
9. $2\ 5/6X - 7.1 = 7\ 2/3X + 3.2$
10. $(1/7)X + 2/3 = (3/8)X - 4/9$
11. $2.4 - 3.5X = 7.8 - 1.2X$
12. $(\mathbf{LOG}54)X + 45^2 = \mathbf{SIN}(45^o) - (4.5)^2X$
13. $X - \mathbf{LN}(60) = 3 - 2X$
14. $45 - 17X = 8X + 76$

A5EA

AX + B = CX + D **This is an easy Linear Equation** Answers: [ ]

Solve for **X**, the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30XA**.

1. $3X + 7 = 5 - 7X$ [**-0.2**]
2. $3.2X - 9 = 4.1X + 7.8$ [**-18.7**]
3. $-12X - 98 = 23X + 76$ [**-4.97**]
4. $0.002X - 0.015 = 0.0087 - 0.005X$ [**3.39**]
5. $(3/4)X - 2/7 = (4/5)X + 3/8$ [**-13 3/14 = -185/14 = -13.21**]
6. $\mathbf{SIN}(28^o)X - 1.4 = \mathbf{COS}(28^o)X + 2.3$ [**-8.95**]
7. $-18.3X + 4.6X - 22.4 = 13.9X - 45.4 + 3.9X$ [**0.73**]
8. $(2.13)^2X - \mathbf{LOG}(345) = 1/\mathbf{COS}(12.5^o) + \sqrt{(5 + 1/.15)}X$ [**3.18**]
9. $2\ 5/6X - 7.1 = 7\ 2/3X + 3.2$ [**-2.13**]
10. $(1/7)X + 2/3 = (3/8)X - 4/9$ [**4 92/117 = 560/117 =4.79**]
11. $2.4 - 3.5X = 7.8 - 1.2X$ [**-2.35**]
12. $(\mathbf{LOG}54)X + 45^2 = \mathbf{SIN}(45^o) - (4.5)^2X$ [**-92.09**]
13. $X - \mathbf{LN}(60) = 3 - 2X$ [**2.37**]
14. $45 - 17X = 8X + 76$ [**-1.24**]

A5ES

AX + B = CX + D This is an easy Linear Equation Answers: [ ]

1. $4x - 17 = -35 - 5X$ [**X = -2**]
2. $25 + 3.5X = -25 + 7.5X$ [**X = 12.5**]
3. $6^2X - 24 = 36 + 18X$ [**X = 3.333**]
4. $0.375 + 4.25X = 1.525 - 8.125X$ [**X = 0.0929**]
5. $SIN(45°)X - 4 = 12 - COS(45°)X$ [**X = 11.31**]
6. $(\sqrt{144})X - 2^4 = 3^3 + (\sqrt{36})X$ [**X = 7.167**]
7. $LOG(15)X + 1 = LN(25) + 2X$ [**X = -2.693**]
8. $1/COS(0°) -4X = -1/SIN(90°) + (3/4)X$ [**X = 0.421**]
9. $\pi X - 2/3\pi = 3\pi X - 8/3\pi$ **HINT: What can be removed from both sides of the equation?**

   [**Since Pi is on either side of the equation, it can be removed.**]
   [**X = 1**]

10. $2TAN(45°)X + 2X - 0.375 = SIN(12.5°)X - \sqrt{0.025}$

    [**X = 0.0573**]
11. $(1/4)^2X - 25.67 = 27X + 6.022$ [**X = -1.176**]
12. $[LN(25-7.4)]^2X - 17 = 1/LOG(2) - 3COS(37°)X$

    [**X = 1.19**]

A6 LESSON: $A/X = C/D$ THIS IS AN EASY LINEAR EQUATION

What can you do to both sides to get closer to a solution?

Flip both sides: $X/A = D/C$ **then** $X = A\times(D/C)$

Example: $3/X = 12/5$

Solution: $X/3 = 5/12$ **then** $X = 3\times(5/12) = 1.25$

Example: $2.16/X = -56.3$ **then** $X/2.16 = 1/-56.3$

Solution: $X = 2.16/-56.3 = -.038$ (check: $2.16/-.038 = -56.8$)

Example: $-37.8 = -6.78/X$

Solution: $-6.78/X = -37.8$ (switch sides)

Then: $X = (-6.78)/(-37.8) = .18$ (flip and multiply by -6.78)

Example: $(3.85)^2/X = \sqrt{349}/ \mathbf{SIN}(79^0)$

$(3.85)^2 = 14.8$ $\sqrt{349} = 18.7$ $\mathbf{SIN}\ (79^0) = .982$

So: $14.8/X = 18.7/.982 = 19.0$ **or** $X = 14.8/19.0 = .78$

Always simplify the numbers first, and then solve the equation. Also, always **CHECK** your answer by plugging it back into the equation and being sure both sides are equal.

$(3.85)^2/.78 = 19.0$ $\sqrt{349}/ \mathbf{SIN}(79^o) = 19.0$

A6E

$A/X = C/D$ THIS IS AN EASY LINEAR EQUATION.

Flip both sides: $X/A = D/C$ **then** $X = A\times(D/C)$

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30XA**.

1. $3/X = 12/5$
2. $2.16/X = -56.3$
3. $-37.8 = -6.78/X$
4. $(3.85)^2/X = \sqrt{349}/\ \mathbf{SIN}(79^0)$

Always simplify the numbers first and then, solve the equation. Also, always **CHECK** your answer by plugging it back into the equation and being sure both sides are equal.

5. $\mathbf{SIN}(23^o)/X = \mathbf{COS}(54^o)$
6. $23^2 = (12.5)^2/X$
7. $(3/4)/X = 9/16$
8. $\mathbf{LOG}(4235)/X = \mathrm{LN}\ 435$
9. $10.5/X = 9.8/4.1$
10. $(5^2 + 7^2)/X = 1/(0.05)^2$
11. $\mathbf{COS}(37^o)/\mathbf{SIN}(37^o) = 1/X$

A6EA

A/X = C/D This is an easy Linear Equation Answers: [ ]

Flip both sides: X/A = D/C **then** X = Ax(D/C)

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30XA.**

1. $3/X = 12/5$ [**1.25**]
2. $2.16/X = -56.3$ [**-0.038**]
3. $-37.8 = -6.78/X$ [**0.179**]
4. $(3.85)^2/X = \sqrt{349}/$ **SIN**(79^0) [**0.779**]

Always simplify the numbers first, and then solve the equation. Also, always **CHECK** your answer by plugging it back into the equation and being sure both sides are equal.

5. **SIN**$(23^o)/X$ = **COS**(54^o) [**0.665**]
6. $23^2 = (12.5)^2/X$ [**0.295**]
7. $(3/4)/X = 9/16$ [**1 1/3 = 4/3 = 1.33**]
8. **LOG**$(4235)/X$ = **LN** 435 [**0.597**]
9. $10.5/X = 9.8/4.1$ [**4.39**]
10. $(5^2 + 7^2)/X = 1/(0.05)^2$ [**0.185**]
11. **COS**(37^o)/**SIN**$(37^o) = 1/X$ [**0.754**]

A6ES

A/X = C/D   This is an easy Linear Equation   Answers: [ ]

1. 4/X = 1 [**X = 4**]
2. 10/X = 2/4 [**X = 20**]
3. 17/X = 1/17 [**X = 289**]
4. SIN(30°)/X = 1/COS(60°) [**X = 0.25**]
5. 25.3/X = -98.1/27.6 [**X = -7.12**]
6. (√225)/X = 12/19 [**X = 23.75**]
7. 23.6/-0.025 = 1112/X [**X = -1.178**]
8. SIN(56°)/X = COS(27°) [**X = 0.93**]
9. TAN(75°)/COS(23.5°) = SIN(14°)/X [**X = 0.0594**]
10. LOG(92)/X = 15/LN(25) [**X = 0.4214**]
11. π/X = 1/2 [**X = 2π**]
12. -COS(180°)/2X = 43SIN(25°)/3.643 [**X = 0.1002**]

A7 LESSON: $AX^2 = B$ THIS IS AN EASY NON-LINEAR EQUATION

What can you do to both sides to get closer to a solution?

$X^2 = B/A$ (divide by A) **now** take the square root both sides

$X = \sqrt{(B/A)}$ [**Note:** Answer could be **+** or **-**]

Example: $X^2 = 387$ $X = 19.7$ **or** -19.7 [$\sqrt{387} = 19.7$]

Example: $\mathrm{SIN}(125^o)X^2 = (5.4 + 3.4)^2$ (simplify numbers first)

$\mathrm{SIN}(125^o) = .819$ $(5.4 + 3.4)^2 = (8.8)^2 = 77.4$

So: $.819X^2 = 77.4$ **or** $X^2 = 77.4/.819$ **or** $X^2 = 94.55$

So: $X = 9.7$

Check: $\mathrm{SIN}(125^o)x(9.7)^2 = 77.07$ [close enough due to **r/o**]

Note: $X = \sqrt{94.55} = 9.724$ to more digits

Then: $\mathrm{SIN}(125^o)x(9.724)^2 = 77.5$

Always be aware of how many digits are really significant and the unavoidable round off (**r/o**) error. Ask yourself: How accurate or precise can I measure, or do I need to measure?

A7E

$AX^2 = B$ THIS IS AN EASY NON-LINEAR EQUATION

$X^2 = B/A$ (divide by A) **now** take the square root both sides

$X = \sqrt{(B/A)}$ [**Note:** Answer could be **+** or **-**]

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30XA**.

1. $X^2 = 387$
2. $\mathbf{SIN}(125^\circ)X^2 = (5.4 + 3.4)^2$
3. $X^2 = 23^2$
4. $X^2 = (\sqrt{78})^2$
5. $X^2 = \mathbf{LOG}(98)$
6. $\mathbf{SIN}(34^\circ) = \mathbf{COS}(23^\circ)X^2$
7. $(3/4)X^2 = 9/16$
8. $X^2 = 16A^2$
9. $X^2 = (\mathbf{SIN}(78^\circ))^2 + (\mathbf{COS}(78^\circ))^2$
10. $X^2 = \mathbf{COS}^{-1}[(3^2 + 4^2 - 6^2)/2x3x4]$
11. $X^2 = \sqrt{81}$

A7EA

$AX^2 = B$ THIS IS AN EASY NON-LINEAR EQUATION Answers:[ ]

$X^2 = B/A$ (divide by A) **now** take the square root both sides

$X = \sqrt{(B/A)}$ [**Note:** Answer could be **+** or **-**]

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30XA.**

1. $X^2 = 387$	[**19.7**]
2. $\mathrm{SIN}(125^o)X^2 = (5.4 + 3.4)^2$	[**9.7**]
3. $X^2 = 23^2$	[**23**]
4. $X^2 = (\sqrt{78})^2$	[**√78**]
5. $X^2 = \mathrm{LOG}(98)$	[**1.41**]
6. $\mathrm{SIN}(34^o) = \mathrm{COS}(23^o)X^2$	[**0.779**]
7. $(3/4)X^2 = 9/16$	[**0.866**]
8. $X^2 = 16A^2$	[**4A**]
9. $X^2 = (\mathrm{SIN}(78^o))^2 + (\mathrm{COS}(78^o))^2$	[**1**]
10. $X^2 = \mathrm{COS}^{-1}[(3^2 + 4^2 - 6^2)/2x3x4]$	[**10.8**]
11. $X^2 = \sqrt{81}$	[**3**]

A7ES

$AX^2 = B$ **THIS IS AN EASY NON-LINEAR EQUATION** Answers:[ ]

1. $X^2 = 81$ [X = ±9]
2. $X^2 = 169$ [X = ±13]
3. $3X^2 = 45$ [X = ±3.87]
4. $X^2 = 275^2$ [X = ±275]
5. $\sin(35°)X^2 = 65$ [X = ±10.645]
6. $(3/7)X^2 = (19/8)$ [X = ±2.354]
7. $\log(8.756)X^2 = \ln(253)$ [X = ±2.423]
8. $X^2 = \pi^2$ [X = ±π]
9. $3X^2 = \sqrt{121}$ [X = ±1.915]
10. $X^2 = \sin(65°) - \cos(45°)$ [X = ±0.4463]
11. $4X^2 = (2^4 + 3^3 + 4^2)^2$ [X = ±29.5]
12. $X^2 = (3\pi^2)^2$ [$X = \pm 3\pi^2$]

A8 LESSON: $A\sqrt{X} = B$ THIS IS AN EASY NON-LINEAR EQUATION

What can you do to both sides to get closer to a solution?

$\sqrt{X} = B/A$ (divide by A) **now** take the square both sides

$X = (B/A)^2$ [**Note:** Answer will be positive]

Example: $\sqrt{X} = 387$ $X = 149{,}769$ which is $(387)^2$

How many digits are significant…**probably 3.**
150,000 is good enough.

Example: $\text{SIN}(125^o)\sqrt{X} = (5.4 + 3.4)^2$ (simplify numbers first)

$\text{SIN}(125^o) = .819$ $(5.4 + 3.4)^2 = (8.8)^2 = 77.4$

So: $.819\sqrt{X} = 77.4$ **or** $\sqrt{X} = 77.4/.819$ **or** $\sqrt{X} = 94.55$

or $X = 8940$

Check: $\text{SIN}(125^o)\text{x}\sqrt{8940} = 77.4$

Always be aware of how many digits are really significant and the unavoidable round off (**r/o**) error. Ask yourself: How accurate or precise can I measure, or do I need to measure?

A8E

$A\sqrt{X} = B$ This is an easy non-Linear Equation

$\sqrt{X} = B/A$ (divide by A) **now** take the square both sides

$X = (B/A)^2$ [**Note:** Answer will be positive]

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but is easy with the **TI-30XA**.

1. $\sqrt{X} = 387$
2. $\sqrt{X} = -23.5$
3. $\sqrt{X} = 7/8$
4. $3.5\sqrt{X} = 98.2$
5. $78 = 4.2\sqrt{X}$
6. $\sqrt{X} = 6^2$
7. $\sqrt{X} = \sqrt{17}$
8. $\mathbf{SIN}(125^o)\sqrt{X} = (5.4 + 3.4)^2$ (simplify numbers first)
9. $\sqrt{X} = \mathbf{LOG}(6754)$
10. $\sqrt{X} = \mathbf{SIN}^2(65^o) + \mathbf{COS}^2(65^o)$

A8EA

$A\sqrt{X} = B$ This is an easy non-Linear Equation

Answers:[ ]

$\sqrt{X} = B/A$ (divide by A) **now** take the square both sides

$X = (B/A)^2$ [**Note:** Answer will be positive]

Solve for **X,** the **Unknown**. **Note:** The Algebra is easy. The arithmetic can be complicated but easy with the **TI-30XA**.

1. $\sqrt{X} = 387$	[**149,769**]
2. $\sqrt{X} = -23.5$	[**552.25**]
3. $\sqrt{X} = 7/8$	[**0.766 or 49/64**]
4. $3.5\sqrt{X} = 98.2$	[**787**]
5. $78 = 4.2\sqrt{X}$	[**345**]
6. $\sqrt{X} = 6^2$	[**1296**]
7. $\sqrt{X} = \sqrt{17}$	[**17**]
8. $\mathbf{SIN}(125^\circ)\sqrt{X} = (5.4 + 3.4)^2$	[**8937**]
9. $\sqrt{X} = \mathbf{LOG}(6754)$	[**14.67**]
10. $\sqrt{X} = \mathbf{SIN}^2(65^\circ) + \mathbf{COS}^2(65^\circ)$	[**1**]

A8ES

$A\sqrt{X} = B$ This is an easy non-Linear Equation

Answers: [ ]

1. $\sqrt{X} = 9$ [**X = 81**]
2. $\sqrt{X} = 3/4$ [**X = 9/16**]
3. $2.5\sqrt{X} = 10$ [**X = 16**]
4. $\sqrt{X} = \text{COS}(30°)$ [**X = 0.75**]
5. $\sqrt{X} = \sqrt{225}$ [**X = 225**]
6. $\sqrt{X} = \text{COS}(75°)/\text{LOG}(25)$ [**X = 0.0343**]
7. $\sqrt{X} = \text{COS}(45°) + \text{SIN}(45°)$ [**X = 2**]
8. $(\sqrt{X})^2 = (30.25)^2$ [**X = 915.0625**]
9. $\sqrt{X} = [\text{COS}(12.5°) + \text{TAN}(12.5°)]/\text{SIN}(12.5°)$ [**X = 30.636**]
10. $\sqrt{25}\sqrt{X} = 2000$ [**X = 160000**]
11. $\sqrt{(16X)} = 24$ **HINT: $\sqrt{(16X)} = \sqrt{16}\sqrt{X}$** [**X = 36**]
12. $\text{SIN}(87°)\sqrt{25X} = \text{LOG}(63)$ [**X = 0.3604**]

INTRODUCTION TO GEOMETRY

The Foundation Course is dedicated to your learning how to solve practical math problems that arise in a wide variety of industrial and "real world" situations.

In addition to learning how to use the power tool called a scientific calculator, you need to learn material from three fields, Algebra, Geometry and Trigonometry.

Geometry is the "Centerpiece" of math that you will use in most problems. It is all about physical space in one, two, and three dimensions: Lines, Flat Surfaces and 3-D objects.

Algebra is a tool that is often used along with Geometry to solve problems.

You use Geometry to set up an equation which you then solve for the unknown. The unknown might be a length, or some dimension you need to know, or area, or volume.

Trigonometry is a special subject used for triangles. There are occasions where you cannot solve a problem with just algebra and geometry alone and where you need trigonometry. It deals with triangles.

Geometry is one of humankind's oldest mathematical subjects along with numbers and algebra.

Geometry is the foundation of modern science and technology and much modern mathematics.

Mathematics is like a "contact" sport, or a game.

You learn by practicing and "doing."

Each Lesson will include a video discussion of the topic just as we did in Algebra.

Then you will be given Homework Problems to work.

You are encouraged to make up your own problems.

The more you "play" and the more questions you ask, the better you will learn.

When you think you are ready, take the Online Quiz.

This will give you an indicator if you have mastered the material. If not, go back and "play" some more.

Learning math is like climbing a ladder. If you do it one small step at a time, it is pretty easy. But, it is difficult to go from rung 4 to rung 9 directly.

This Foundation course has been designed to let you climb the ladder of math understanding in small steps.

But, **YOU** must do the climbing. Watching someone else climb isn't enough. Play the game.

G1 LESSON: WHAT IS GEOMETRY?

Mathematics is based on two fundamental concepts:

Numbers and Geometry

Numbers are used to count and measure things.

Geometry is used to model physical things.

There are actually several different kinds of geometry.

We will study the oldest of all geometries, **Euclidean**.

Euclidean Geometry is used in most practical situations.

We will study:

Points:	0 dimensional
Lines:	1 dimensional
Surface Objects:	2 dimensions
And:	3-D objects

We will learn how to analyze many geometric situations and then set up **Equations** to find the value of various unknowns. This could be how long something is, or how much area something is, or the volume of something.

Many of the practical problems one comes across in many walks of life involve some type of geometric object.

Historically, in our schools, emphasis has been placed on proving theorems (statements about geometric objects) with rigorous logic and step by step deductions.

This can be difficult and tedious, and sometimes seemingly meaningless. We will emphasis sound reasoning in the Foundation Course, but not formal "proofs."

G1E

WHAT IS GEOMETRY?

1. Math is based on what two fundamental concepts?
2. Numbers are used to?
3. **Geometry** is used to?
4. The oldest kind of **Geometry** is?
5. In **Geometry** we will study what four things?
6. What will we use to find unknowns in **Geometry**?
7. What kind of **Unknowns** might we wish to find?

G1EA

WHAT IS GEOMETRY? Answers: []

1. Math is based on what two fundamental concepts?

   [**Numbers and Geometry**]

2. Numbers are used to? [**Count and measure things**]

3. Geometry is used to? [**Model physical things**]

4. The oldest kind of geometry is? [**Euclidean**]

5. In Geometry we will study what four things?

   [**Points: 0 dimensional**]
   [**Lines: 1 dimension**]
   [**Surface Objects: 2 dimensions**]
   [**And: 3-D objects**]

6. What will we use to find Unknowns in Geometry?

   [**Equations and Algebra**]

7. What kind of Unknowns might we wish to find?

[**This could be how long something is, or how much area something is, or the volume of something.**]

Many of the practical problems one comes across in many walks of life involve some type of geometric object.

G2 LESSON: STRAIGHT LINES AND ANGLES

A **Point** is ideally a location in space with no length or width. It has zero area.

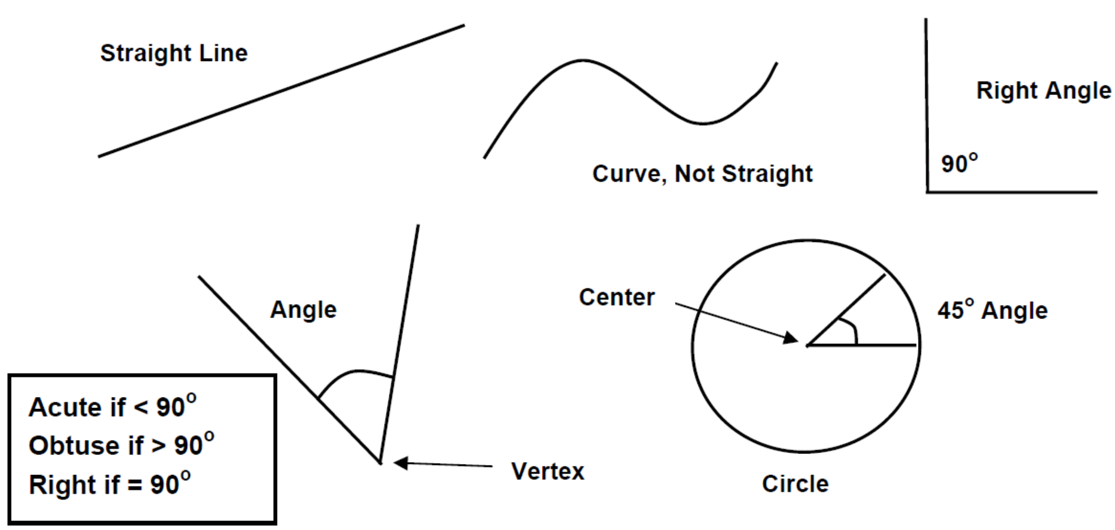
A **Plane** is a flat surface consisting of points. Think of a wall or blackboard as a plane. It is a surface with zero curvature.

A **Straight Line** (Segment) is the collection of points between two points that represents the shortest distance between them. It too has zero curvature. A **Straight Line** can be extended indefinitely.

The intersection of two lines (**straight**, unless I otherwise state), forms an **Angle** and their point of intersection is called the **Vertex.**

Angles are measured in **Degrees** (°) where there are 360° in a complete circle, a set of points equidistant from a point, center.

A **Right Angle** measures 90° and the two sides are **Perpendicular**.



G2E

STRAIGHT LINES AND ANGLES

1. What are: Point, Plane, and Straight Line?

2. What are an Angle and a Vertex?

3. How are Angles measured?

4. What is a Right Angle?

5. What are Acute and Obtuse Angles?

G2EA

STRAIGHT LINES AND ANGLES   Answers: []

1. What are: Point, Plane, and Straight Line?

   [A Point is ideally a location in space with no length or width. It has zero area.

   A Plane is a flat surface consisting of points. Think of a wall or blackboard as a plane. It is a surface with zero curvature.

   A Straight Line (Segment) is the collection of points between two points that represents the shortest distance between them.]

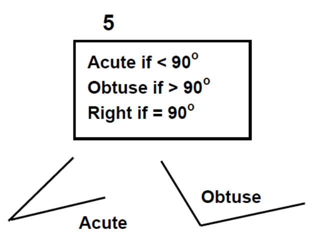
2. What are an Angle and a Vertex?

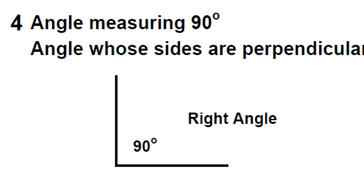
   [The intersection of two lines (straight, unless I otherwise state), forms an Angle and their point of intersection is called the Vertex.]

3. How are Angles measured?

   [Angles are measured in Degrees (o) where there are 360o in a complete circle, a set of points equidistant from a point, center.]

4. What is a Right Angle?   **[See Below Right]**
5. What are Acute and Obtuse Angles?   **[See Below Left]**





G3 LESSON: PARALLEL LINES

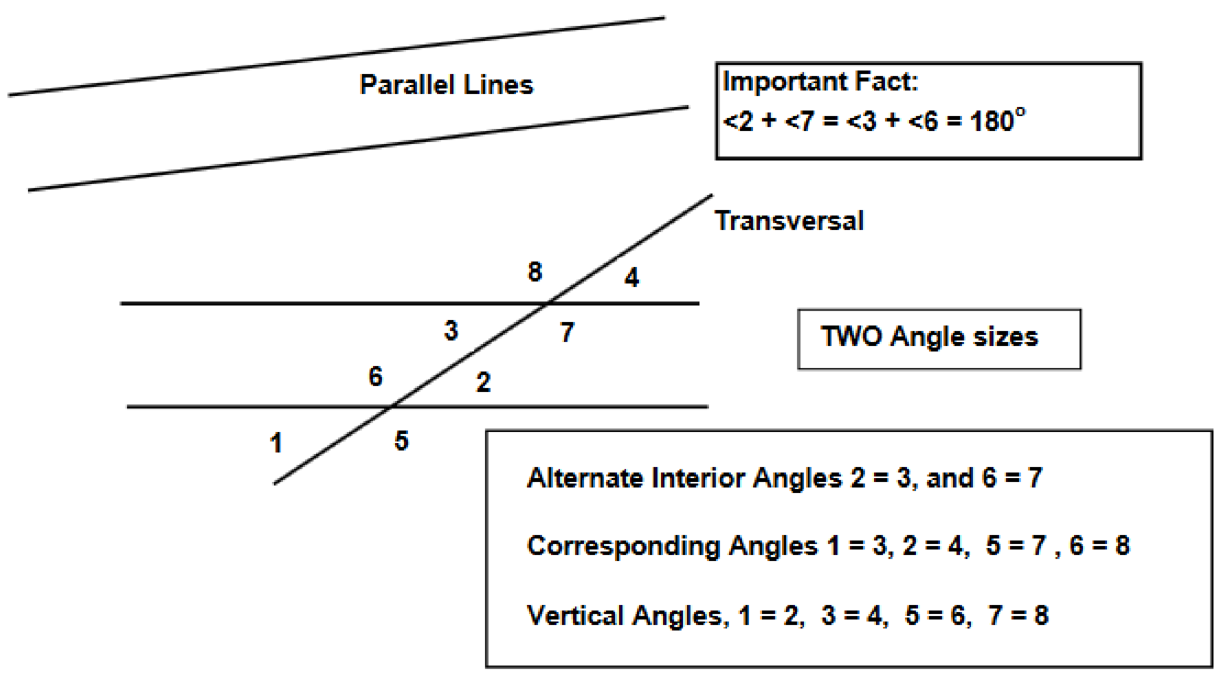
Two straight lines are **parallel** if they never intersect no matter how far they are extended in either direction.

The Fundamental Property in **Euclidean** Geometry is:

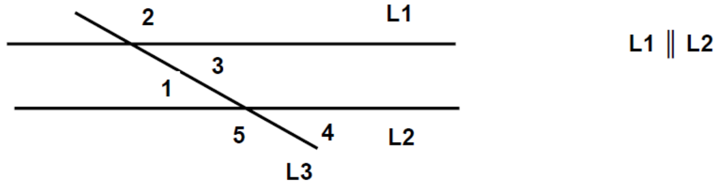
Given a straight line and an external point, there is exactly one straight line through this point parallel to the given line.

This is called the **Parallel Postulate** and it is not true for other **non-Euclidean** geometries.

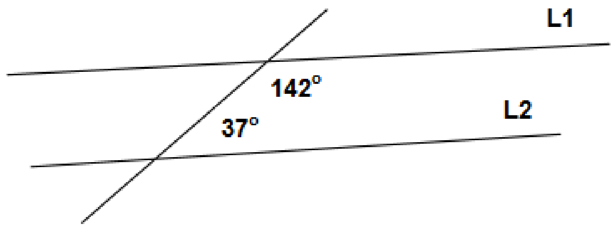
When two parallel lines are crossed by another straight line, called a **transversal**, eight angles are created in two sets of four equal-sized angles. This is a critical property.



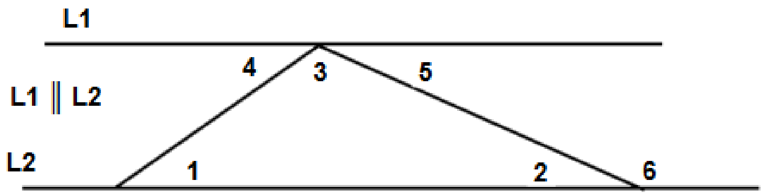
G3 Problems for Parallel Lines



1. Given <1=42^o, what are the sizes of <2 and <3?
2. Given <3=50^o, what are the sizes of <4 and <5?
3. Given <2=132^o, what are the sizes of <1 and <4?



4. Are L1 and L2 parallel?
5. What is the sum of two interior angles if the lines are parallel?



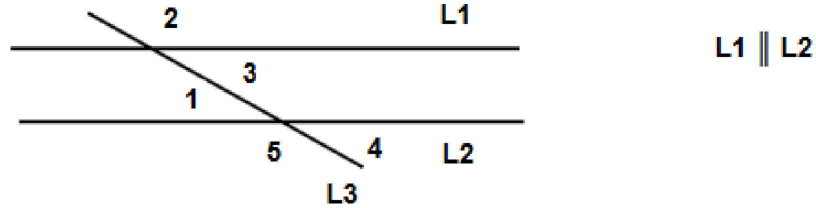
6. What is <4 + <3 +<5 =?
7. Which of the angles are equal <1, <2, <3, <4, <5=?
8. What is <1 + <2 + <3=?
9. If <1 = 43^o and <3 = 102^o then what does <6=?
10. In problem #9, what does <2=?

ANSWERS

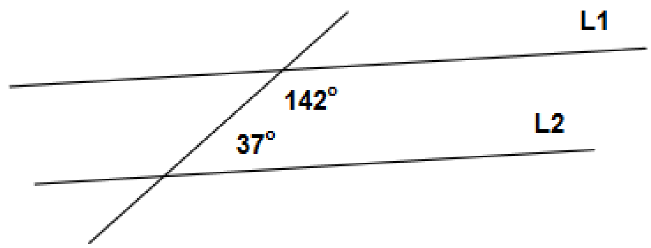
1. <2 = 138^o and <3 = 42^o
2. <4 = 50^o and <5 = 130^o
3. <1 = <4 = 48^o
4. No
5. 180^o
6. 180^o
7. <1 = <4 and <2 = <5
8. 180^o
9. 43^o + 102^o = 145^o
10. 180^o -145^o = 35^o

G3E

PARALLEL LINES

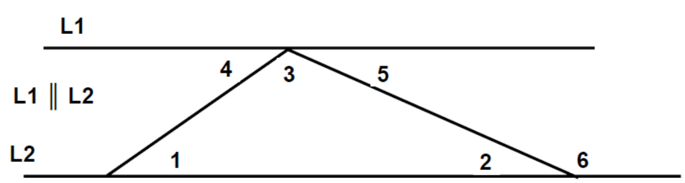


1. Given $\angle 1 = 38^o$, what are the sizes of <2 and <3?
2. Given $\angle 3 = 54^o$, what are sizes of <4 and <5?
3. Given $\angle 2 = 138^o$, what are sizes of <1 and <4?
4. Given:



Are L1 and L2 Parallel?

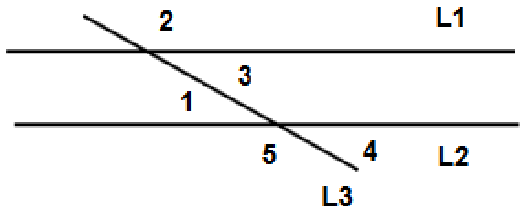
5. What is the sum of two opposite interior angles if the lines are parallel?



6. What is <4 + <3 + <5 =?
7. Which of these angles are equal <1, <2, <3, <4, <5?
8. What is <1 + <2 + <3 =?
9. If $\angle 1 = 42^o$ and $\angle 3 = 105^o$, what does <6 =?
10. In problem #9, what does <2 =?
11. The sum of the three angles of a triangle equal?

G3EA

PARALLEL LINES Answers: []



L1 ∥ L2

1. Given <1 = 38^o, what are the sizes of <2 and <3?

   [**<2 = 142^o and <3 = 38^o**]

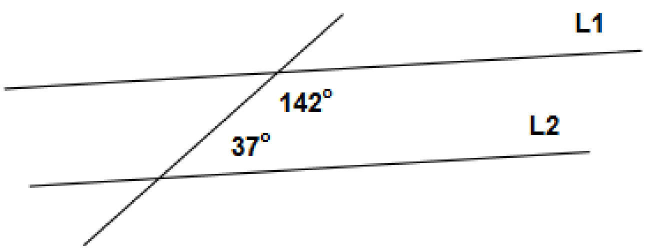
2. Given <3 = 54^o, what are sizes of <4 and <5?

   [**<4 = 54^o and <5 = 126^o**]

3. Given <2 = 138^o, what are sizes of <1 and <4?

   [**<1 = 42^o and <5 = 138^o**]

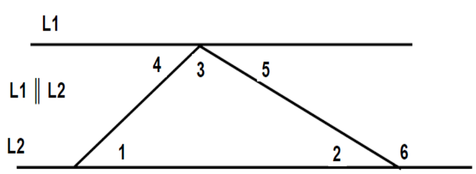
4. Given:



   Are L1 and L2 Parallel? [**NO because: 142 + 37 = 179 not 80**]

5. What is the sum of two interior angles if the lines are parallel?

   [**<5 + <6 = 180^o**]



6. What is <4 + <3 + <5 =? [**180^o**]

7. Which of these angles are equal <1, <2, <3, <4, <5?

   [**<1 = <4**]

PARALLEL LINES (cont'd) Answers: []

8. What is <1 + <2 + <3 =? [180^o]

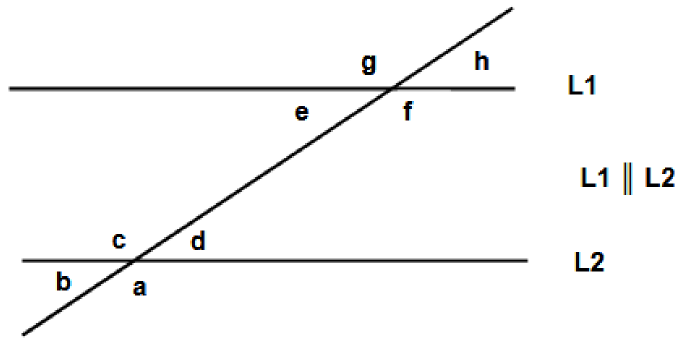
9. If <1 = 42^o and <3 = 105^o, what does <6 =? [**147^o**]

10. In problem #9, what does <2 =? [**33^o**]

11. The sum of the three angles of a triangle equal? [**180^o**]

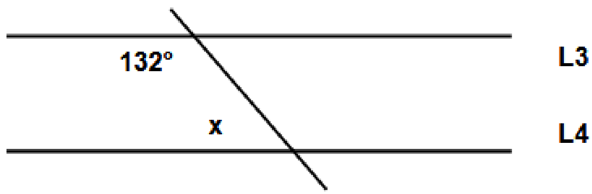
G3ES

PARALLEL LINES



1.) How many angles do you need to know in order to replace the letters in the diagram to the left?

2.) If <a = 115, find the rest of the remaining angles.

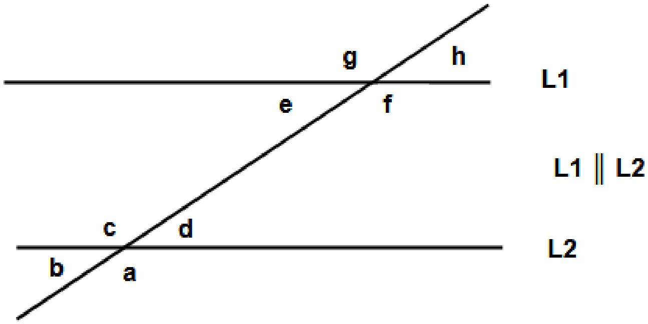


3.) If L3 and L4 are parallel, what must <x equal?

4.) If two lines are truly parallel, what will they never do?
Hint: Think about intersecting lines

G3ESA

PARALLEL LINES

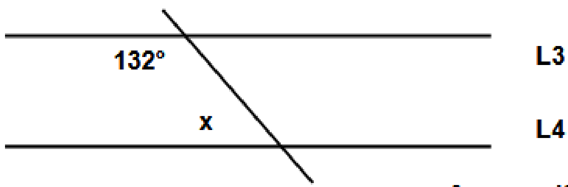


1.) How many angles do you need to know in order to replace the letters in the diagram to the left?

Answer: Only 1 angle.

2.) If <a = 115, find the rest of the remaining angles.

Answer: <b = <d = <e = <h = 65°, <a = <c = <f = <g = 115°



3.) If L3 and L4 are parallel, what must <x equal?

Answer: If L3 and L4 are parallel, <x and 132° must add up to 180°, therefore <x = 180 - 132 = 48°

4.) If two lines are truly parallel, what will they never do?
Hint: Think about intersecting lines

Answer: Two parallel lines will never touch.

G4 LESSON: TRIANGLES, DEFINITION, SUM OF ANGLES

A Triangle is a three-sided **polygon**, i.e., a geometric figure created by three intersecting straight lines. Thus, a triangle has three sides and three vertices.

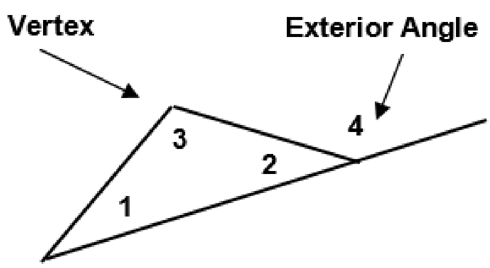
The sum of the three interior angles of a triangle is always 180°.
Exterior Angle = Sum of opposite Interiors

1 + 2 + 3 = 180° and 4 = 1 + 3

Triangles are often used to model a physical situation.

There are several types of triangles:

Right, Acute, Obtuse, Isosceles, and Equilateral. See below.



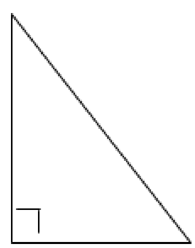
<1 + <2 + <3 = 180°

<4 = <1 + <3

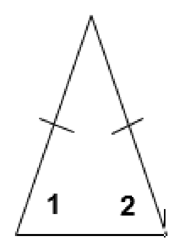
Acute All angles < 90°

Obtuse One angle > 90°

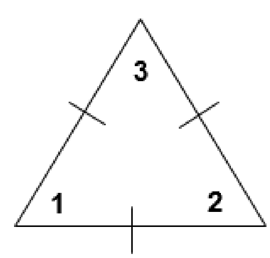
Right One angle = 90°



Right Triangle



Isosceles Triangle
<1 = <2
Two Equal Sides



Equilateral Triangle
<1 = <2 = <3 = 60°

Three Equal Sides

G4 Triangle Problems

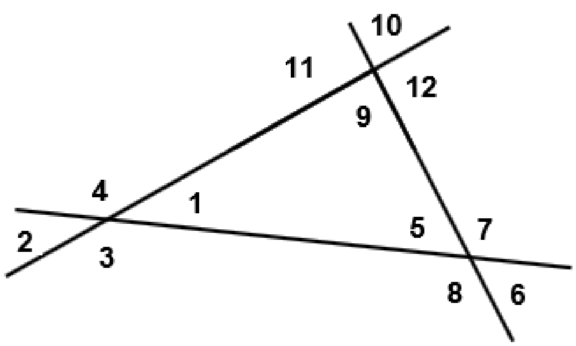
Finding unknown angles from known angles.

Each **vertex** of a triangle has four angles associated with it for a total of twelve angles for a triangle. There will be six values.

If you know any two angles from two different vertices, then you can calculate all the other angles.

This is demonstrated below.

Note 1: Angles do not have the < symbol



$1 + 5 + 9 = 180^o$

$4 = 3 = 180^o - 1$ $\quad 2 = 1$

$7 = 8 = 180^o - 5$ $\quad 6 = 5$

$11 = 12 = 180^o - 9$ $\quad 10 = 9$

Given any two angles from two vertices, we can calculate all the other angles.

Example 1 $\quad 1 = 40^o$ **and** $7 = 120^o$ $\quad$ **Find the other angles**

Answers $\quad 5 = 6 = 180^o - 120^o = 60^o$ $\quad 8 = 120^o$

$4 = 3 = 180^o - 40^o$ $\quad 2 = 40^o$

** $\quad 9 = 10 = 180^o - 1 - 5 = 180^o - 40^o - 60^o = 80^o$

$11 = 12 = 180^o - 80^o = 100^o$

Example 2 $\quad 9 = 75^o$ **and** $8 = 110^o$ **Find other angles**

Answers $\quad 5 = 6 = 180^o - 110^o = 70^o$ **and** $7 = 110^o$

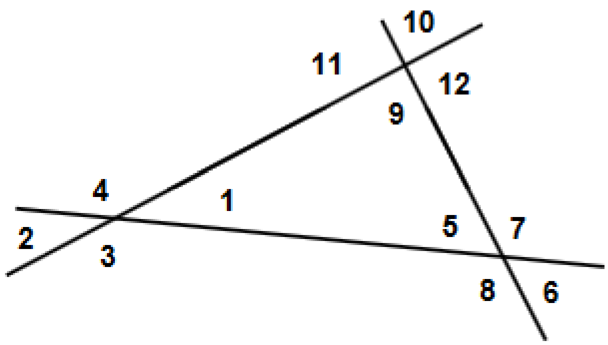
$11 = 12 = 180^o - 75^o = 105^o$ **and** $10 = 75^o$

** $\quad 2 = 1 = 180^o - 75^o - 70^o = 35^o$ **and** $4 = 3 = 180^o - 35^o = 145^o$

G4E

TRIANGLES

Find the unknown angles from known angles below.



$1 + 5 + 9 = 180^\circ$

$4 = 3 = 180^\circ - 1$ $2 = 1$

$7 = 8 = 180^\circ - 5$ $6 = 5$

$11 = 12 = 180^\circ - 9$ $10 = 9$

Given any two angles from two vertices, we can calculate all the other angles.

Exercise 1: $1 = 40^\circ$ and $7 = 120^\circ$ **Find the other angles.**

Exercise 2: $9 = 75^\circ$ and $8 = 110^\circ$ **Find the other angles.**

Exercise 3: $2 = 38^\circ$ and $10 = 70^\circ$ **Find the other angles.**

Exercise 4: $9 = 72^\circ$ and $6 = 68^\circ$ **Find the other angles.**

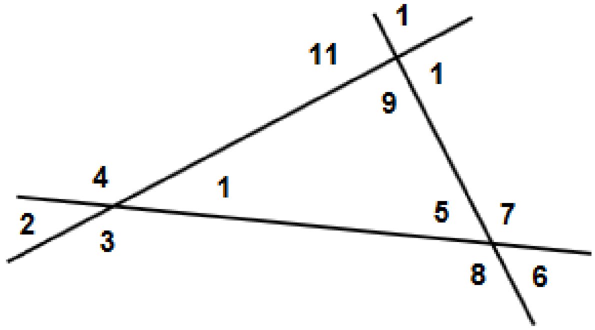
Exercise 5: $4 = 135^\circ$ and $7 = 118^\circ$ **Find the other angles.**

Exercise 6: $10 = 85^\circ$ and $12 = 95^\circ$ **Find the other angles.**

G4EA

TRIANGLES

Find the unknown angles from known angles below.



1 + 5 + 9 = 180°

4 = 3 = 180° - 1 2 = 1

7 = 8 = 180° - 5 6 = 5

11 = 12 = 180° - 9 10 = 9

Given any two angles from two vertices, we can calculate all the other angles.

Exercise 1 1 = 40° and 7 = 120° Find the other angles

Answers
5 = 6 = 180° - 120° = 60° 8 = 120°
4 = 3 = 180° - 40° = 140° 2 = 40°
11 = 12 = 180° - 80° = 100° 9 = 10 = 80°

Exercise 2 9 = 75° and 8 = 110° Find other angles

Answers
5 = 6 = 180° -110° = 70° and 7 = 110°
11 = 12 = 180° - 75° = 105° and 10 = 75°
2 = 1 = 180° - 75° - 70° = 35° and 4 = 3 = 180° - 35° = 145°

Exercise 3 2 = 38° and 10 = 70° Find the other angles

Answers
1 = 38° and 3 = 4 = 142°
9 = 70° and 11 = 12= 110°
5 = 6 = 72° and 7 = 8 = 108°

Exercise 4 9 = 72° and 6 =68° Find the other angles

Answers
5 = 6 = 68° and 7 = 8 = 112°
11 = 12 = 108° and 10 = 72°
2 = 1 = 40° and 4 = 3 = 140°

Exercise 5 4 = 135° and 7 =118° Find the other angles

Answers
5 = 6 = 62° and 7 = 8 = 118°
11 = 12 = 107° and 9 =10 = 73°
2 = 1 = 45° and 4 = 3 = 135°

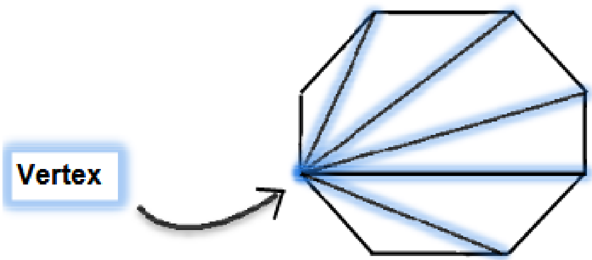
Exercise 6 10 = 85° and 12 = 95° Find the other angles

Answers
9 = 85° and 11 = 95°
Not enough information for the other angles.

TRIANGLES

Note: The interior angles of any polygon add up to the number of sides the shape has - 2 and then multiplied by 180.

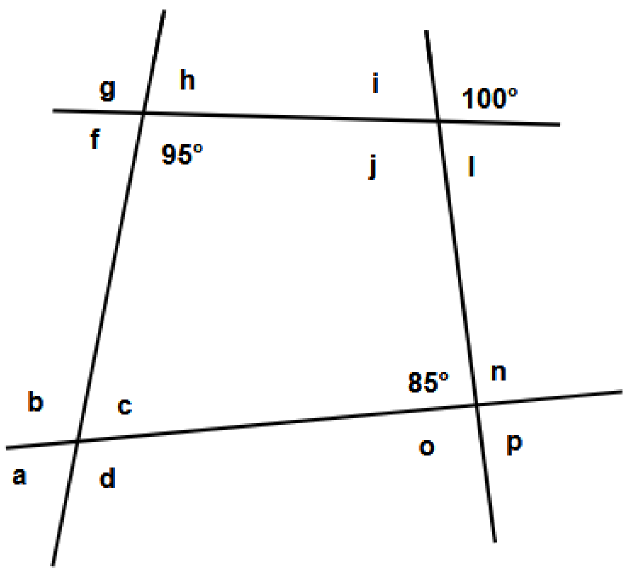
Ex. Triangles have 3 sides --> (3 - 2) times 180 = 180°
Ex. Rectangles have 4 sides --> (4 - 2) times 180 = 360°



1.) The reasoning behind this trick all comes back to triangles. How many degrees does a triangle's interior angles add up to?

2.) Now how many triangles can we break up this octagon into from a single vertex?

****Note: A vertex is just a corner made by two lines!****



3.) With the help of this trick, find the remaining angles in the diagram to the left.

TRIANGLES

Note: The interior angles of any polygon add up to the number of sides the shape has - 2 and then multiplied by 180.

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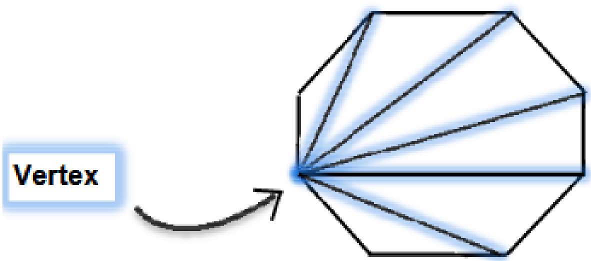


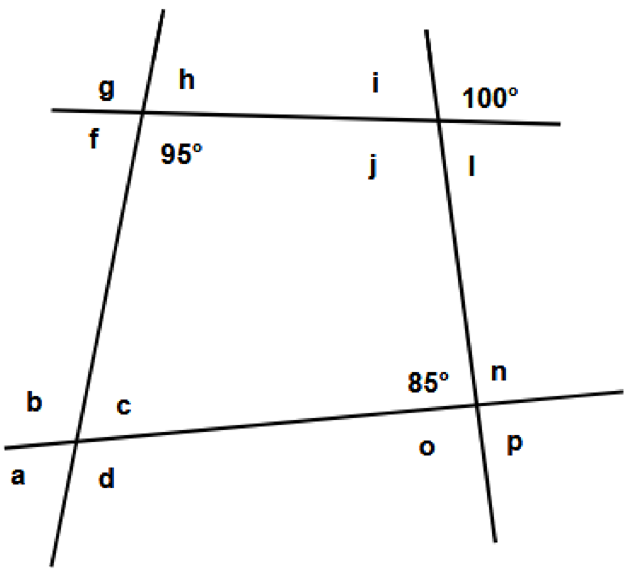
1.) The reasoning behind this trick all comes back to triangles. How many degrees does a triangle's interior angles add up to?

Answer: 180°

2.) Now how many triangles can we break up this octagon into from a single vertex?

****Note: A vertex is just a corner made by two lines!****

Answer: 6 triangles



3.) With the help of this trick, find the remaining angles in the diagram to the left.

Answer:
a = c = 80°
b = d = 100°
f = h = 85°
g = 95°
i = l = 80°
j = 100°
n = o = 95°
p = 85°

G5 LESSON: RIGHT TRIANGLES - PYTHAGOREAN THEOREM

A **Right Triangle** has one of its angles = 90°

The side opposite the **right angle** is called the **Hypotenuse.**

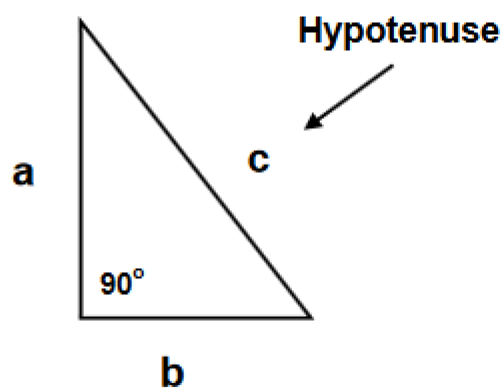
The sum of the **other two angles** will sum to 90°

The Lengths of the three sides of a **Right Triangle** are related by the **Pythagorean Theorem.**

If, they are **a**, **b**, and **C** where "**C**" is the **hypotenuse**, then:

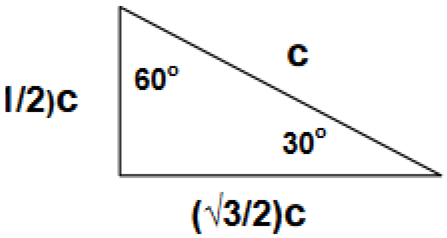
$$a^2 + b^2 = c^2$$

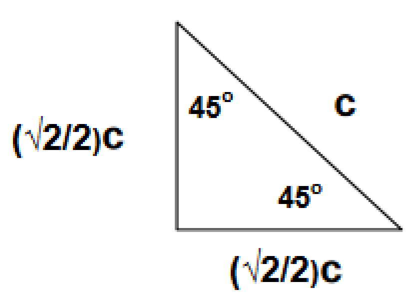
So, $c = \sqrt{a^2 + b^2}$; $b = \sqrt{c^2 - a^2}$; $a = \sqrt{c^2 - b^2}$



Pythogorean Theorem

$$a^2 + b^2 = c^2$$

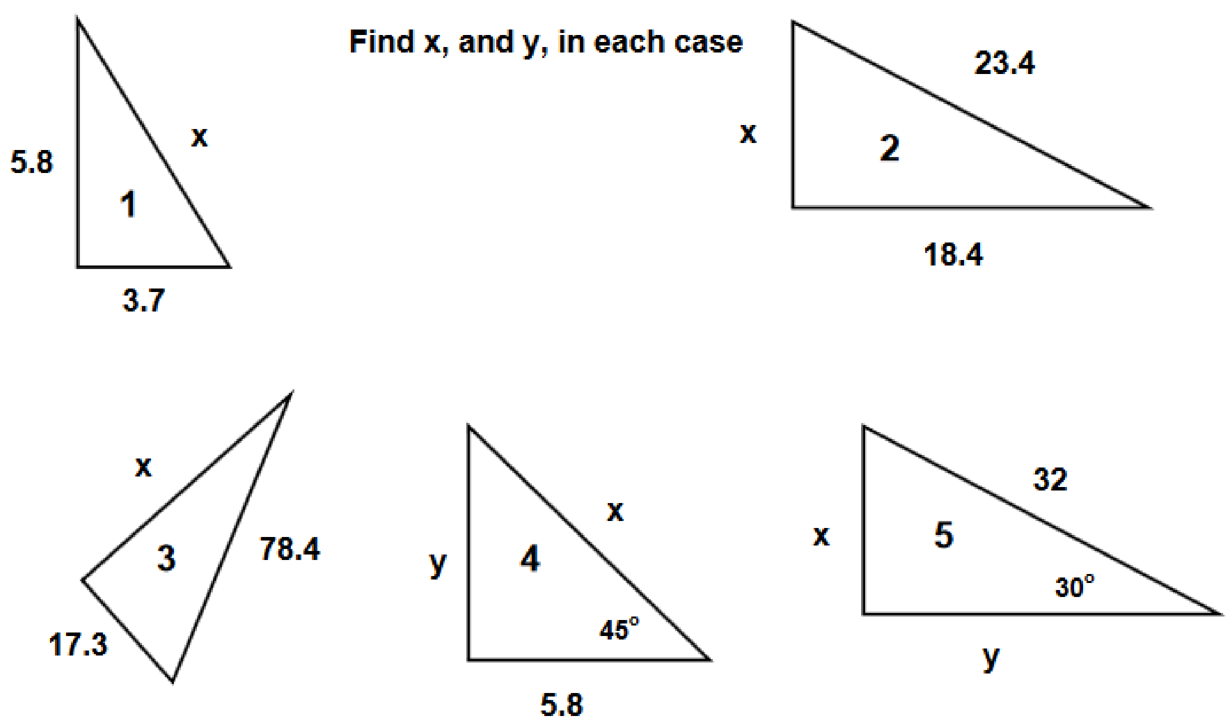




G5 Right Triangle Problems

Typically, you are given one or two sides or angles and want to figure out the other sides or angles.

Here are a few examples (You will typically use the **Pythagorean Theorem** and a calculator):



Answers 1. x = 6.9 2. x = 14.5 3. 76.5 4. y = 5.8, x = 8.2 5. x = 16, y = 27.7

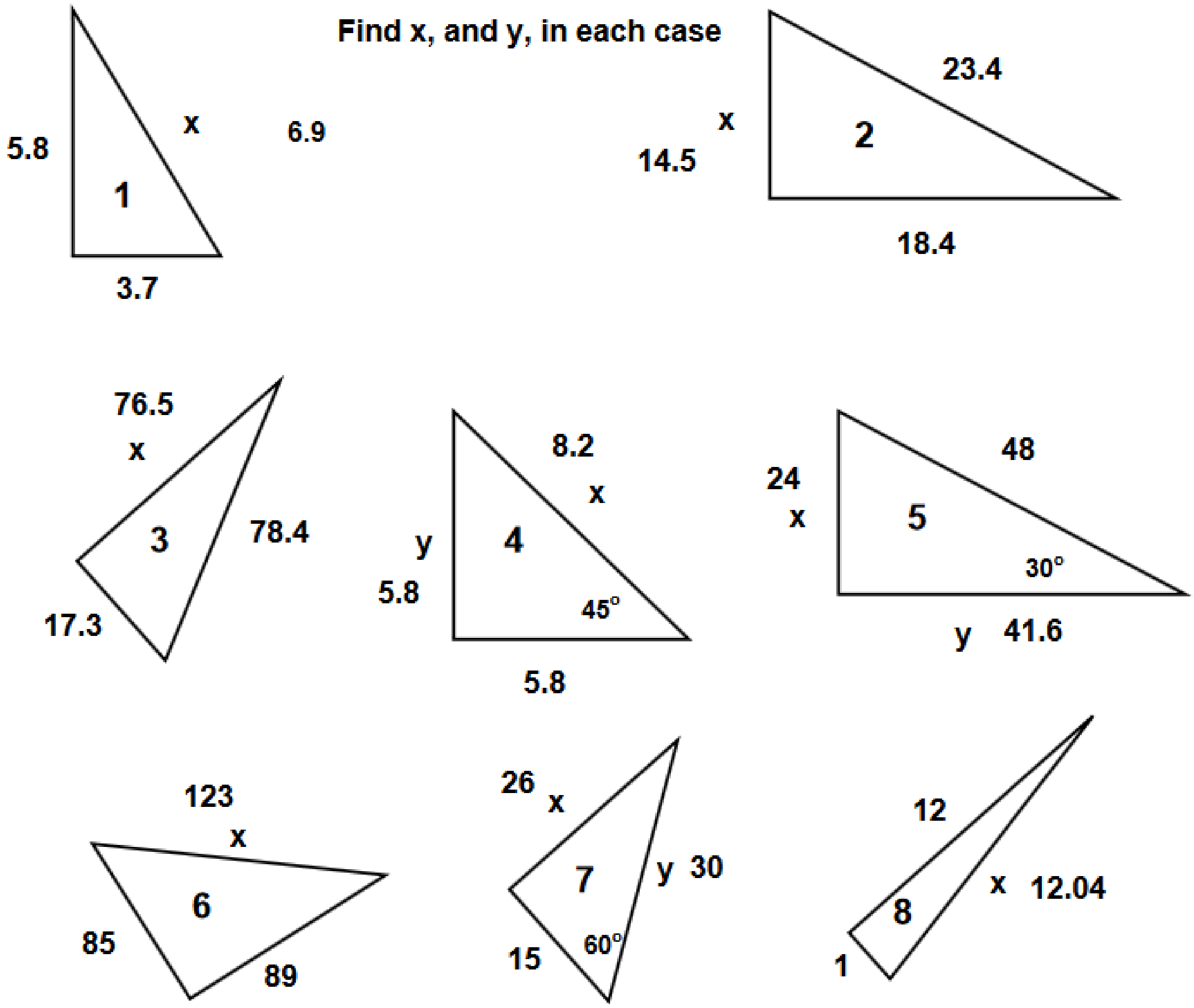
G5E

RIGHT TRIANGLES

Find **x** and **y** in each of the Exercises below.

You will typically use the **Pythagorean Theorem** and a calculator.

All triangles below are **right triangles**.



G5EA

RIGHT TRIANGLES

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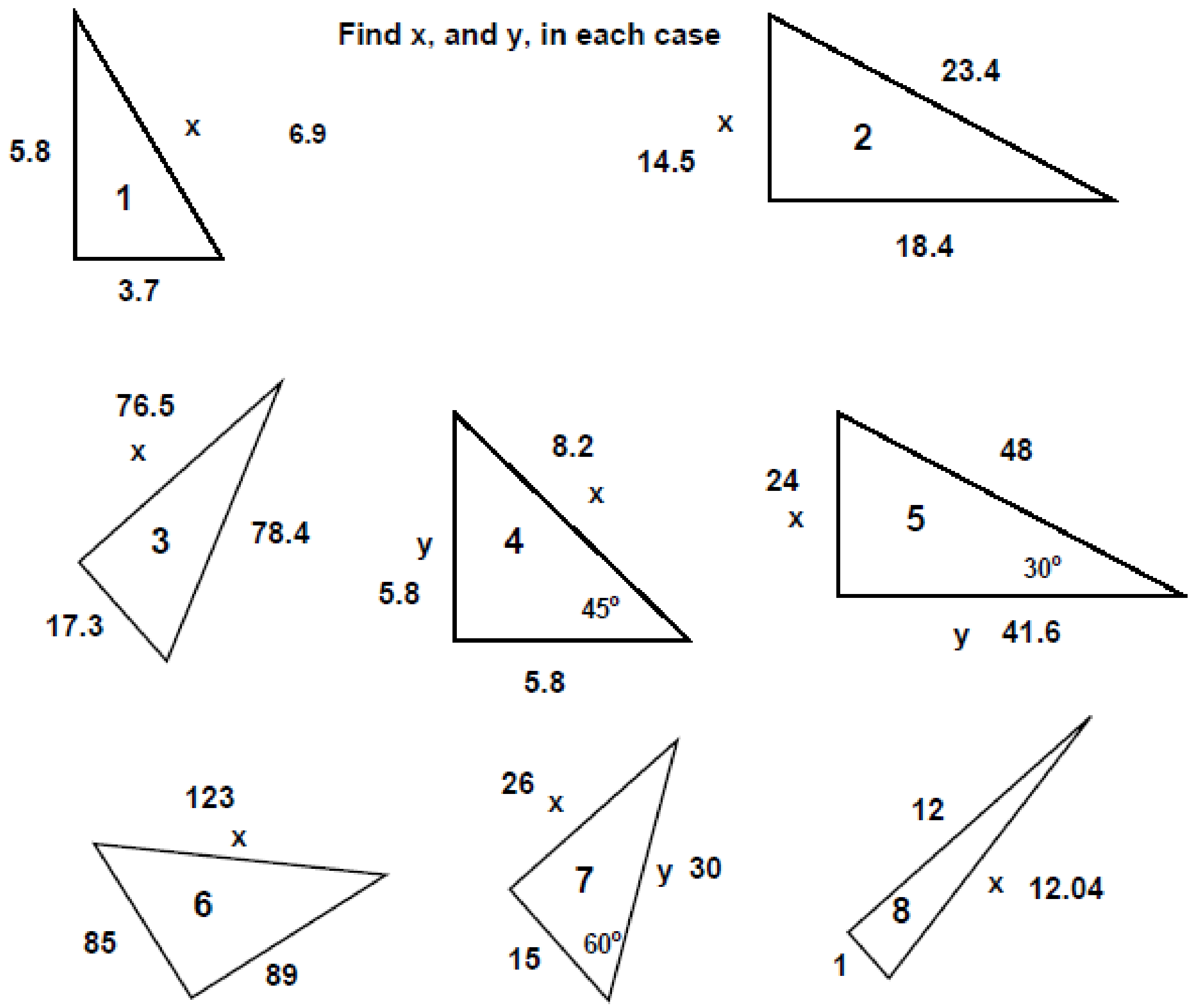
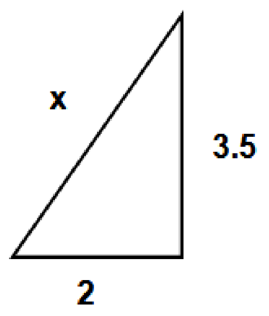
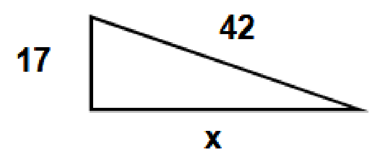


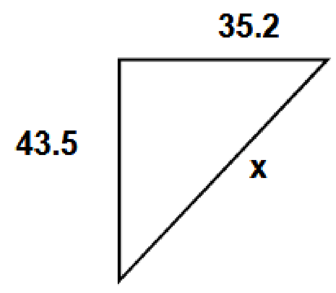
G5ES

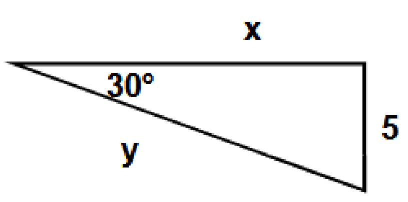
RIGHT TRIANGLES

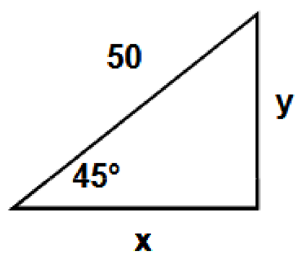
Find the unknowns, x and y.

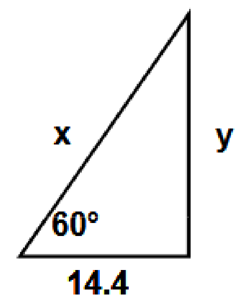






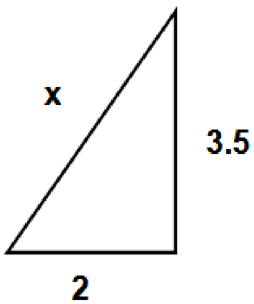




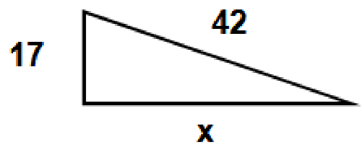


G5ESA

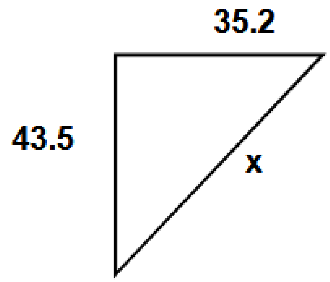
RIGHT TRIANGLES



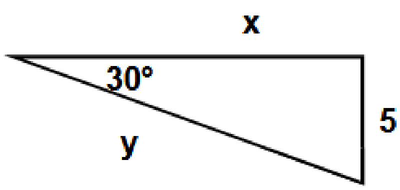
x = 4.03



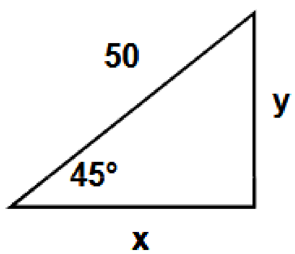
x = 38.41



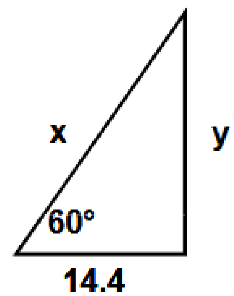
x = 55.95



x = 8.66, y = 10



x = 35.36, y = 35.36



x = 28.8, y = 24.94

G6 LESSON: SIMILAR TRIANGLES

Two Triangles are similar if they have equal angles.

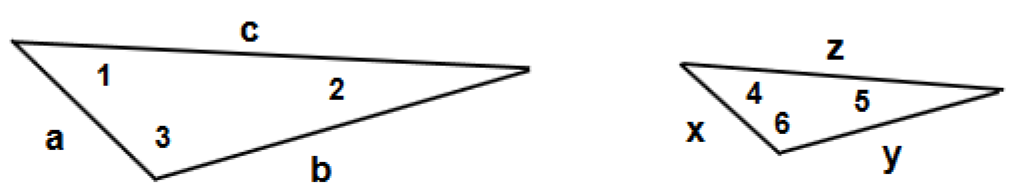
This means they have the same "**shape**" but may be of different sizes. If they also are the same size they are **congruent**.

Similar triangles appear frequently in practical problems.

Their corresponding ratios are equal, and that is what makes them so important and useful.

This is often the way you set up an **Equation** to find an **Unknown**.

Note: If **two** sets of angles are equal, the **third** must be equal also, and the triangles are similar.



Given: 1 = 4 ; 2 = 5 ; 3 = 6, Called corresponding angles

Corresponding sides are: a ↔ x ; b ↔ y ; c ↔ z

The Following Ratios are Equal

a/x = b/y = c/z and **x/a = y/b = z/c**

a/b = x/y	**a/c = x/z**	**b/c = y/z**
b/a = y/x	**c/a = z/x**	**c/b = z/y**

G6 Similar Triangles Problems

When you have **two equal ratios** with one **unknown** it is a simple algebra problem to solve for the unknown **X**.

X/a = b/c **and X** = a(b/c)   **X**/3 = 7/12 **and X** = 3x(7/12) = 1.75

a/**X** = b/c **and X** = a(c/b)   3/**X** = 7/12 **and X** = 3x(12/7) = 5.15

Find two similar triangles where the **unknown** is one side and you know three more sides, one of which is opposite the corresponding angle of the unknown.

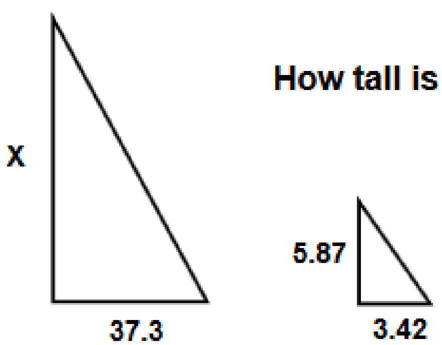
Given: 1 = 4 ; 2 = 5 ; 3 = 6 Find x

x/ 12.3 = 3.7/5.4 so x = 12.3(3.7/5.4) = 8.4

or x/3.7 = 12.3/5.4 so x = 3.7(12.3/5.4) = 8.4

Wrong: x/3.7 = 5.4/12.3 See why?

How tall is the Pole? The horizontal lines are shadows



x/5.87 = 37.3/3.42

x = 5.87(37.3.3/3.42)

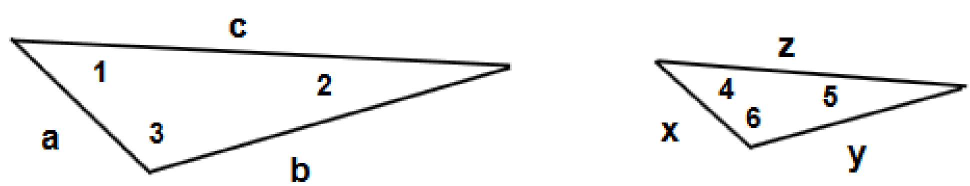
= 64.02 = 64.0

G6E

SIMILAR TRIANGLES

In each Exercise assume the triangles are similar.

Find lengths that you can.



Given: <1 = <4 and <2 = <5

1. What can you conclude about <3 and <6 and why?

2. What are the corresponding sides in pairs?

3. a = 12.3, b = 18.7, x = 5.4, y = ?, z = ?

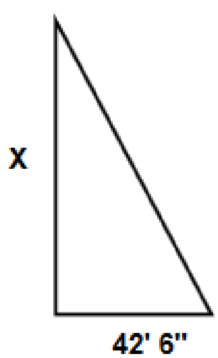
4. c = 1435, z = 765, y = 453, What can you figure?

5. a = .05, x = .02, y = .04, What can you figure?

6. c = 4, b = 3, x = 1.5, What can you figure?

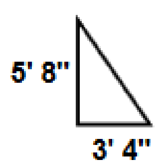
7. b = $2\frac{3}{8}$, x = 3/4, y = 4/5, What can you figure?

8. In Drawing below, how tall is the pole?



How tall is the Pole? The horizontal lines are shadows

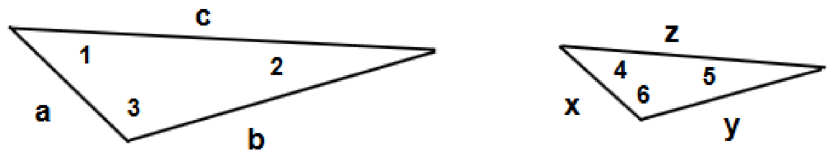
Hint: 1" = 1/12', So, 5'8" = ($5\frac{8}{12}$)'



G6EA

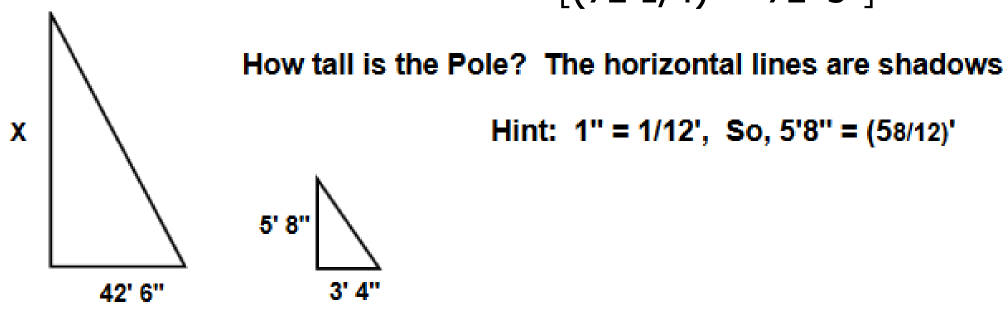
SIMILAR TRIANGLES Answers: []

In each Exercise assume the triangles are similar. Find lengths that you can.



Given: <1 = <4 and <2 = <5

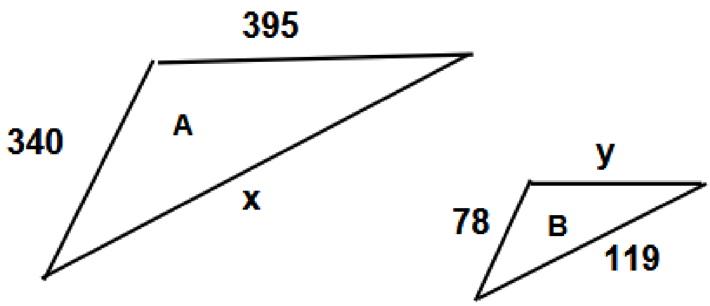
1. What can you conclude about <3 and <6 and why?
   [They are equal due to sum of angles of triangle equals 180°]
2. What are the corresponding sides in pairs?
   [a ↔ x, b ↔ y, c ↔ z]
3. a = 12.3, b = 18.7, x = 5.4, y = ?, z = ?
   [y = 8.2 Have not yet learned how to calculate z]
4. c = 1435, z = 765, y = 453 What can you figure?
   [b = 850]
5. a = 0.05, x = 0.02, y = 0.04 What can you figure?
   [b = 0.1]
6. c = 4, b = 3, x = 1.5 What can you figure?
   [Nothing with just similar triangles]
7. b = 2 3/8, x = 3/4 y = 4/5, What can you figure?
   [a = 2 29/128 = 2.23]
8. In drawing below, how tall is the pole?
   [(72 1/4)' = 72' 3"]



G6ES

SIMILAR TRIANGLES

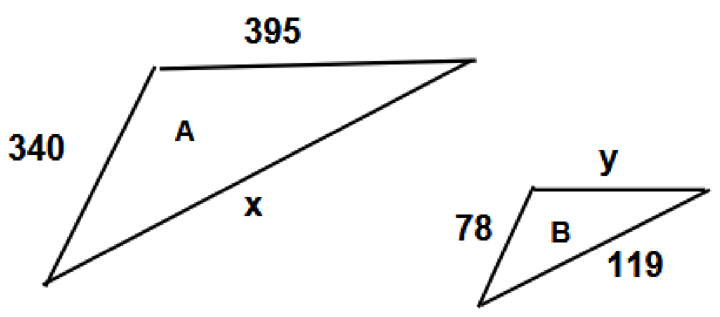
Find the unknowns, x and y.



Assume that the two triangles to the left are similar. Using this knowledge, find the unknown lengths.

G6ESA

SIMILAR TRIANGLES



Assume that the two triangles to the left are similar. Using this knowledge, find the unknown lengths.

x = 518.72, y = 90.62

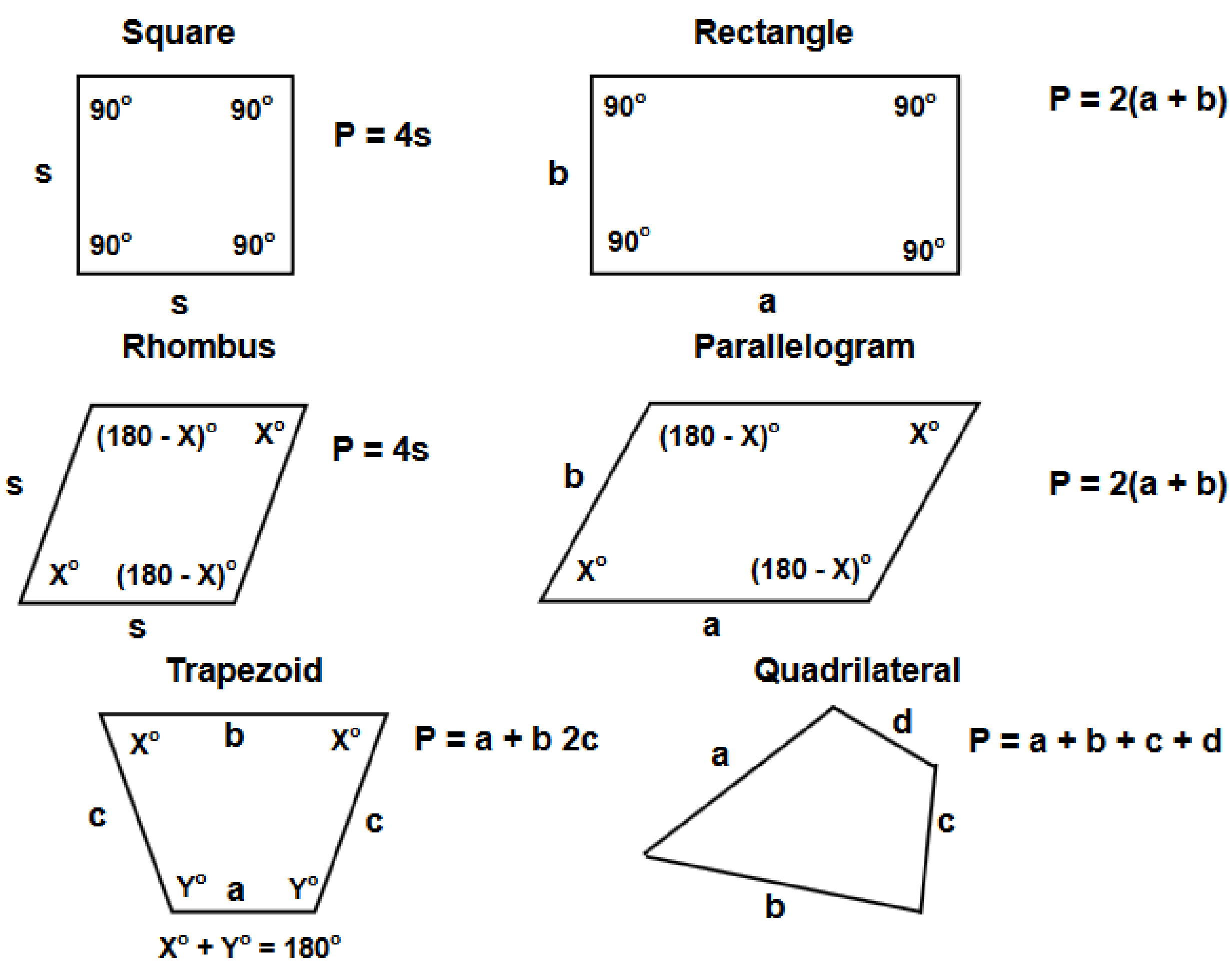
G7 LESSON: QUADRILATERALS, POLYGONS, PERIMETERS (P)

A **Polygon** is a closed geometric figure whose boundary is straight line segments. The **Perimeter** (**P**) is the distance around the polygon.

A **Quadrilateral** is a **polygon** with four sides.

Common **Quadrilaterals** are **Square**, **Rectangle**, **Rhombus**, **Parallelogram**, and **Trapezoid**.

There are three things one is usually interested in for any **quadrilateral**: **Dimensions**, **Perimeter** and **Area**.



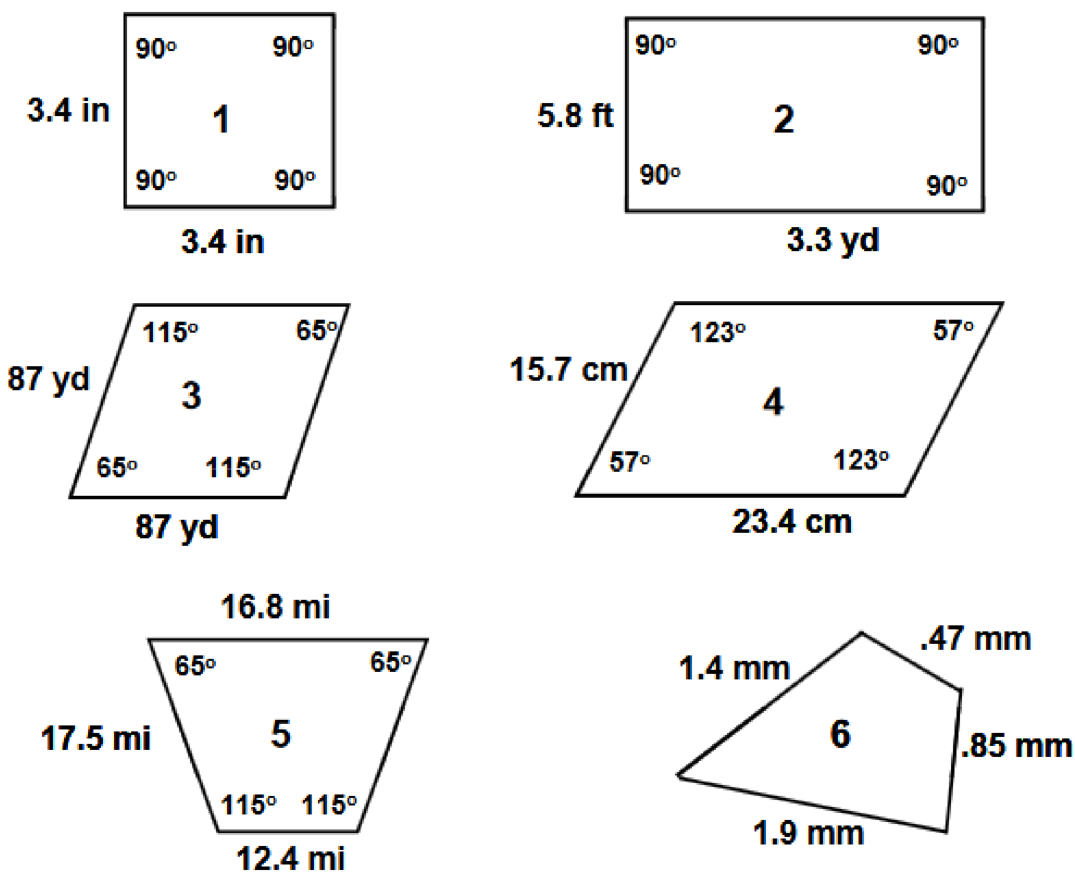
G7 Quadrilaterals, Polygons, Perimeters (P) Problems

Identify the figures below and compute their **Perimeters.**

Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.

Suppose a **rectangle** has one side 1 1/2 feet, and the other side 8 inches. Then, convert feet to inches or inches to feet.

Answers are at bottom of page - Number, name, Perimeter.



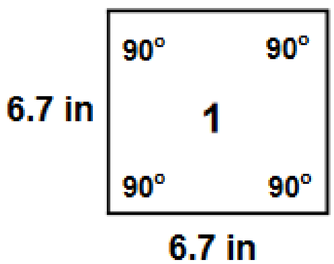
Answer: 1. Square 13.6 in 2. Rectangle 31.4 ft or 10.5 yd 3. Rhombus 348 yd
4. Parallelogram 78.2 cm 5. Trapezoid 64.2 mi 6. Quadrilateral 4.62 mm

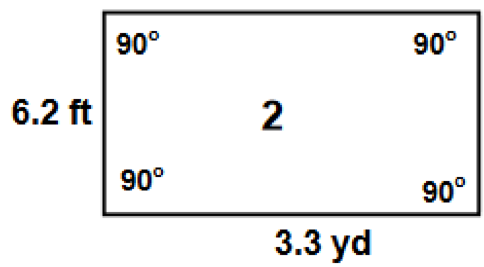
G7E

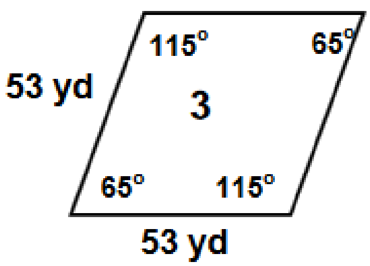
QUADRILATERALS, POLYGONS, PERIMETERS (P)

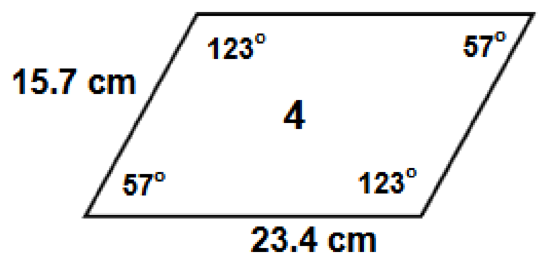
Identify the figures below and compute their Perimeters

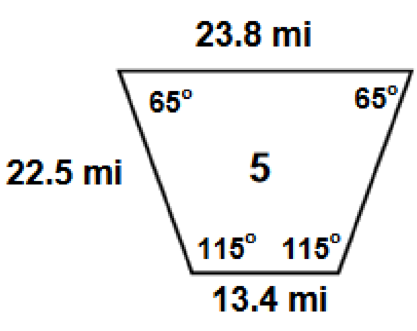
Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.

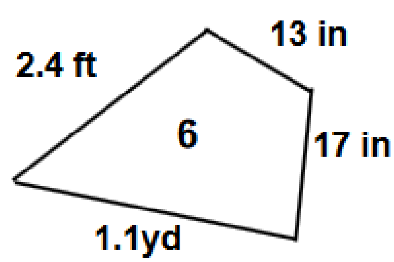


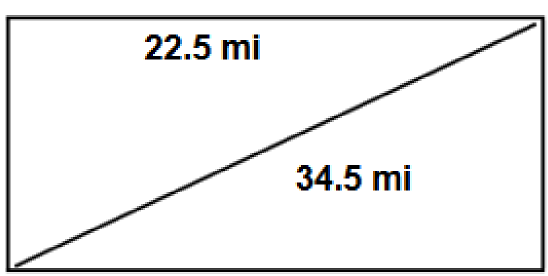












Hint: Use Pythogorean Theorem first

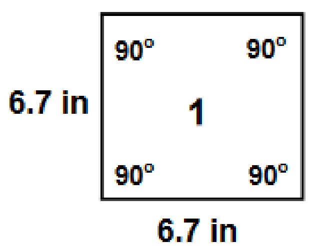
G7EA

QUADRILATERALS, POLYGONS, PERIMETERS (P)

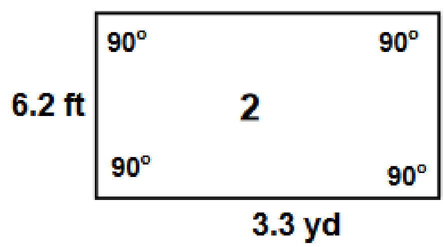
Identify the figures below and compute their Perimeters

Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.

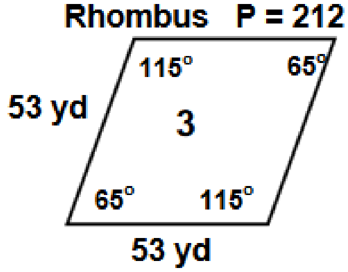
Square P = 26.8



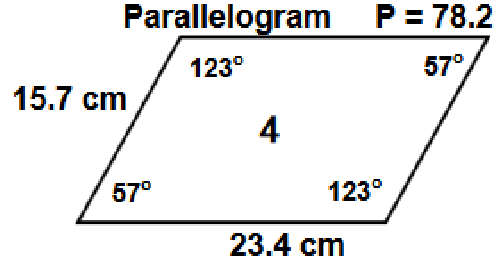
Rectangle P = 32.2 ft = 10.7 yd



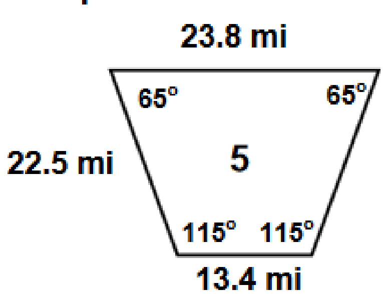
Rhombus P = 212 yd



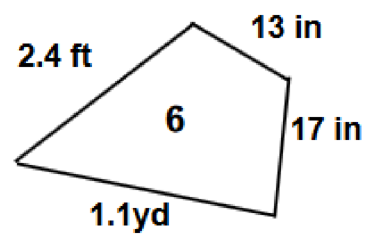
Parallelogram P = 78.2

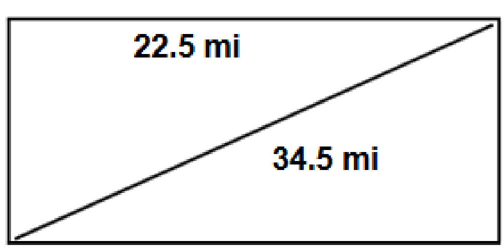


Trapezoid P = 82.2 mi



Polygon P = 98.4 in = 8.2 ft = 2.7 yd



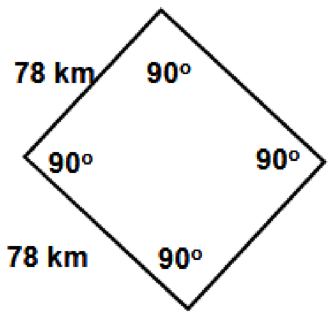


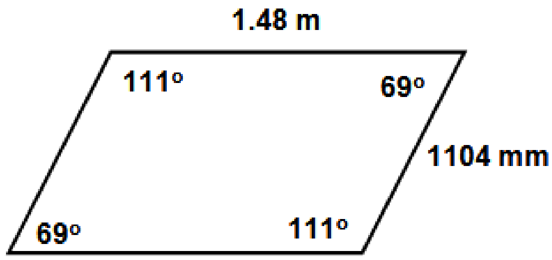
Rectangle P = 97.3 mi

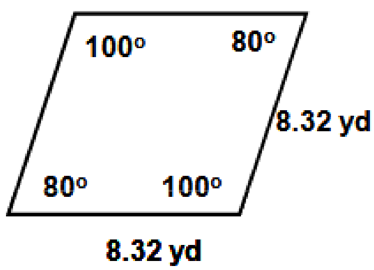
G7ES

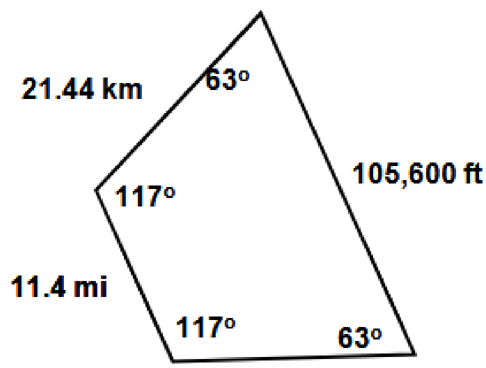
QUADRILATERALS, POLYGONS, PERIMETERS (P)

Identify the figures and calculate their perimeters. Be sure to check units and convert all numbers to the same unit where necessary.





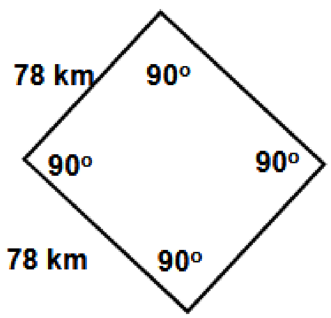




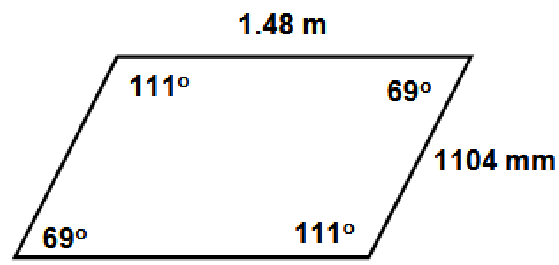
G7ESA

QUADRILATERALS, POLYGONS, PERIMETERS (P)

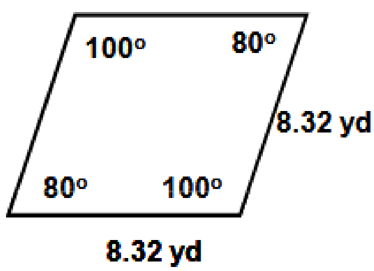
Identify the figures and calculate their perimeters. Be sure to check units and convert all numbers to the same unit where necessary.



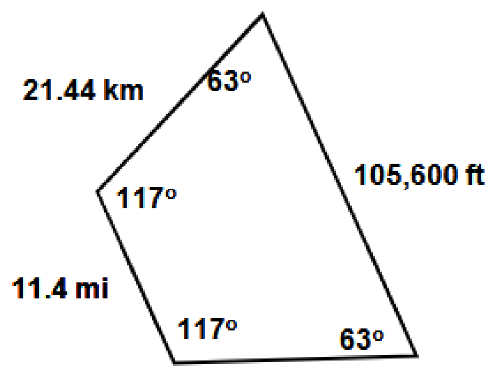
Square, P = 312 km



Parallelogram, P = 5.168 m = 5168mm



Rhombus, P = 33.28 yd



Trapezoid, P = 93.12 km = 58.2 mi = 307,296 ft

G8 LESSON: AREA OF TRIANGLES AND RECTANGLES

The **Area** of any **polygon** is a measure of its size.

The **Rectangle** is the simplest **polygon** and its **Area** is defined to be:

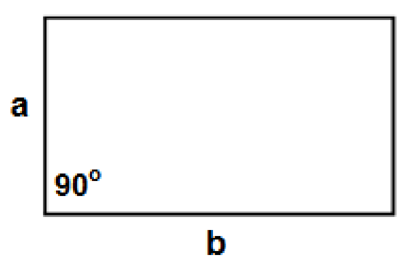
Area = ab where a and b are the lengths of its two sides.

A **Parallelogram** is a "**lopsided**" rectangle whose two adjacent sides have an angle **X°** instead of 90**°**.

Its **Area** can be calculated with a "**Correction Factor**" which is **SIN**(**X°**)

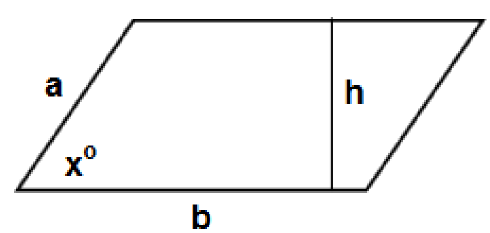
A **Triangle** is one-half of a **parallelogram**. So, its **Area** can be expressed with this same correction factor. **See Below.**

Of course, if one does know the "**height**" then one can use an alternative formula for the **Area**, which is usually given.

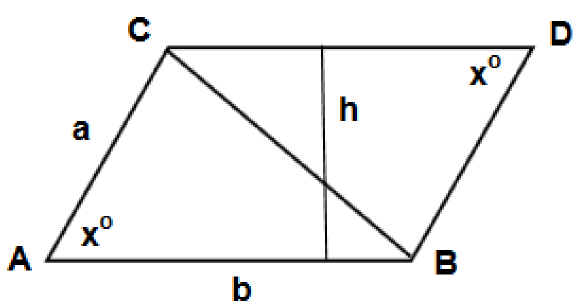


Area = ab

h = height



Area = abSIN(x°)
Area = hb



Triangle ABC = Triangle BDC

Area = .5abSIN(x°)

Area = (1/2)hb

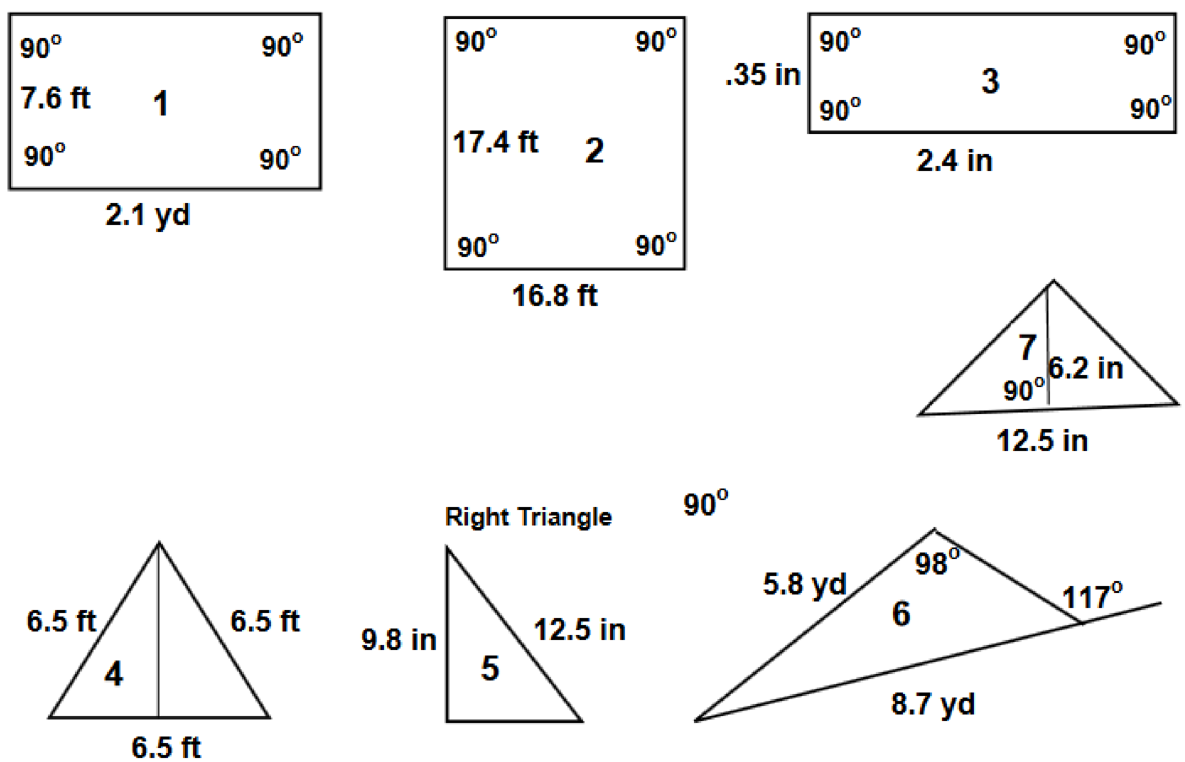
G8 Area of Triangles and Rectangles Problems

Calculate the areas of the triangles and rectangles.

Note: The lateral units of measurement must be the same.

DO NOT multiply **ft** times **yd** for example.

Answers: # Area.



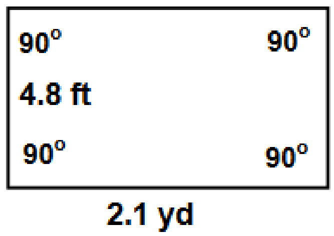
Answers: 1. 47.9 ft^2 or 5.3 yd^2 2. 292 ft^2 3. .84 in^2
4. 18.3 ft^2 5. 76 in^2 6. 8.2 yd^2 7. 38.8 in^2

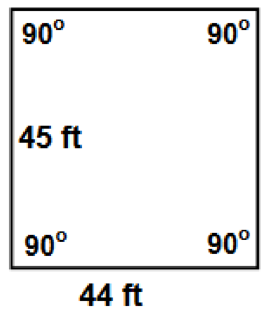
G8E

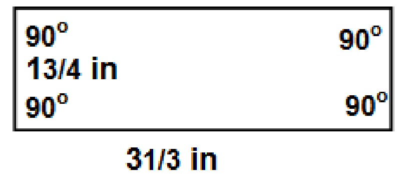
AREA OF TRIANGLES AND RECTANGLES

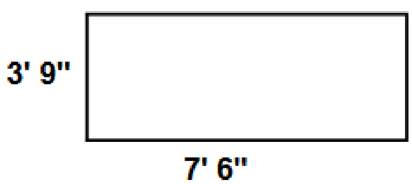
Calculate the areas of the triangles and rectangles.

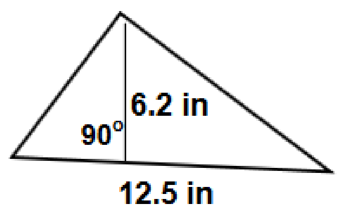
Note: The lateral units of measurement must be the same.

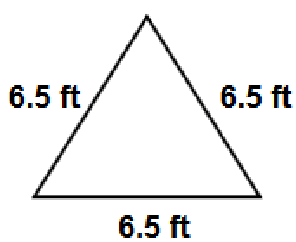


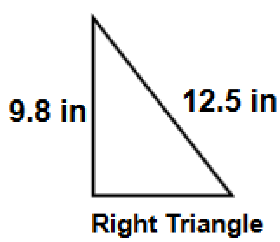


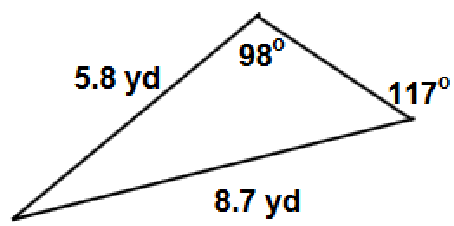












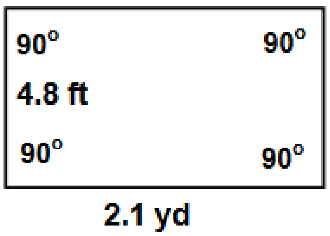
G8EA

AREA OF TRIANGLES AND RECTANGLES

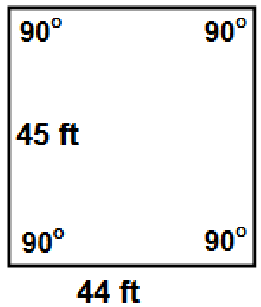
Calculate the areas of the triangles and rectangles.

Note: The lateral units of measurement must be the same.

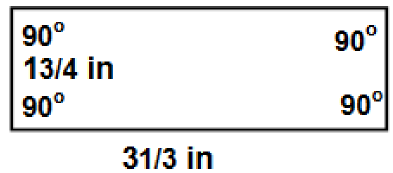
$A = 30.2\ \text{ft}^2 = 3.4\ \text{yd}^2$



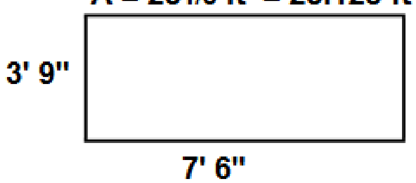
$A = 1980\ \text{ft}^2$



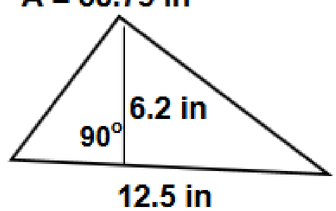
$A = 5\tfrac{5}{6}\ \text{in}^2$



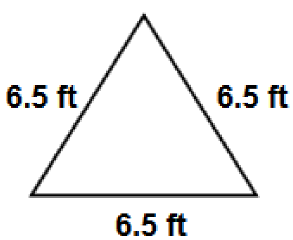
$A = 28\tfrac{1}{8}\ \text{ft}^2 = 28.125\ \text{ft}^2$



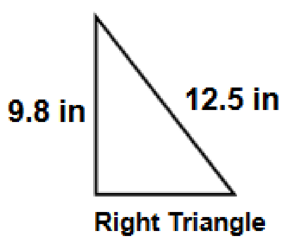
$A = 38.75\ \text{in}^2$



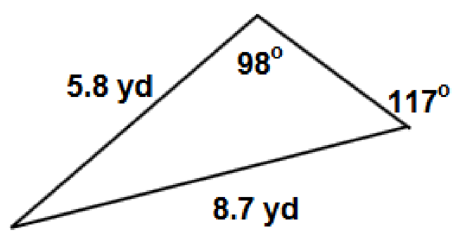
$A = 18.3\ \text{ft}^2$



$A = 38.0\ \text{in}^2$



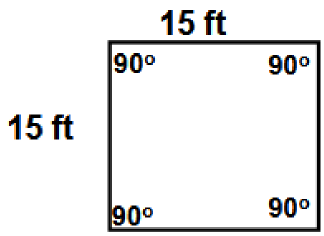
$A = 8.2\ \text{yd}^2$

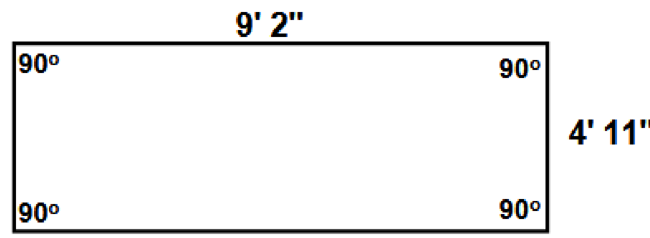


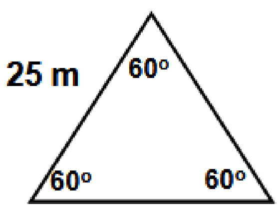
G8ES

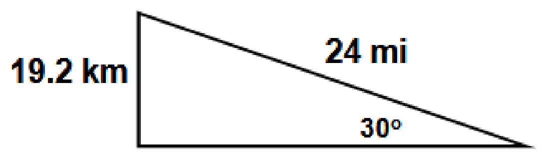
AREA OF TRIANGLES AND RECTANGLES

Identify the figures and calculate their areas. Be sure to check units and convert all numbers to the same unit where necessary.





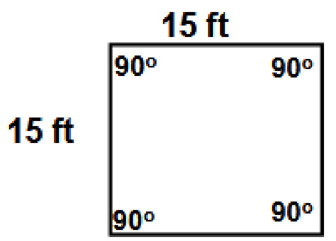




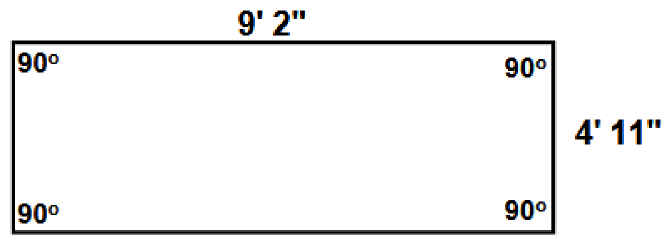
G8ESA

AREA OF TRIANGLES AND RECTANGLES

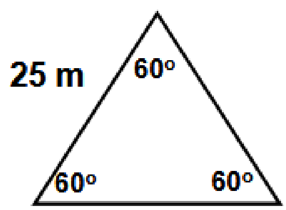
Identify the figures and calculate their areas. Be sure to check units and convert all numbers to the same unit where necessary.



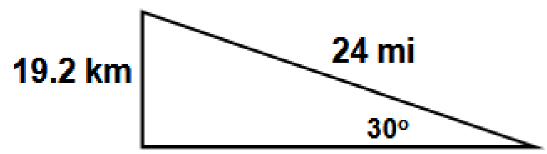
Square, A = 225 ft²



Rectangle, A = 45.07 ft²



Triangle, A = 270.6 m²



Right triangle, A = 124.7 mi² = 199.5 km²

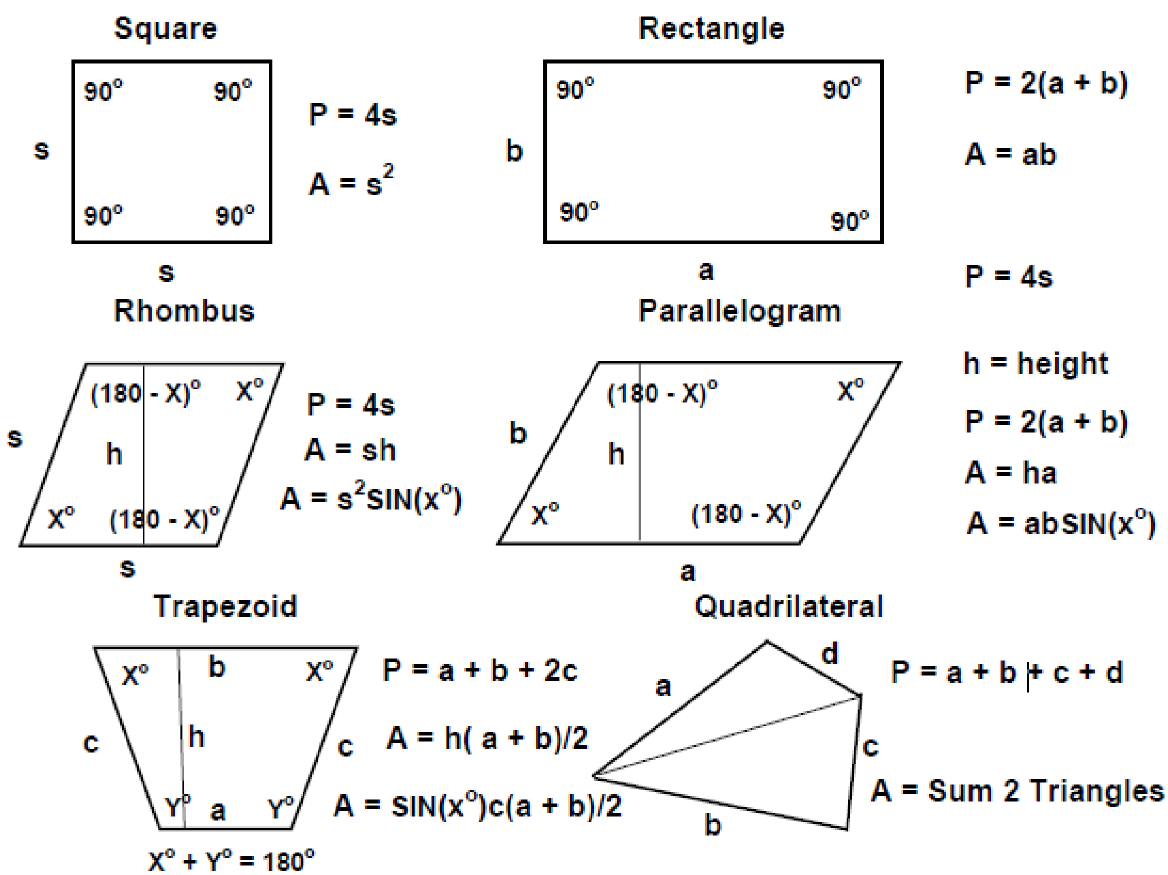
G9 LESSON: FORMULAS FOR POLYGONS

The **Area** of any geometric object is a measure of its size.

The basic unit of **Area** measure is a **square** which measures **one linear unit (U) per side**. Then, by definition, the **Area** of such a **square is 1 U^2** of **1 Square Unit**.

The **Area** of any other closed geometric figure is defined to be the sum of **areas** of inscribed, non-overlapping, squares which are so small they fully fill up the figure.

A rigorous definition is possible, but challenging. However; intuitively, the idea of **Area** is pretty easy.



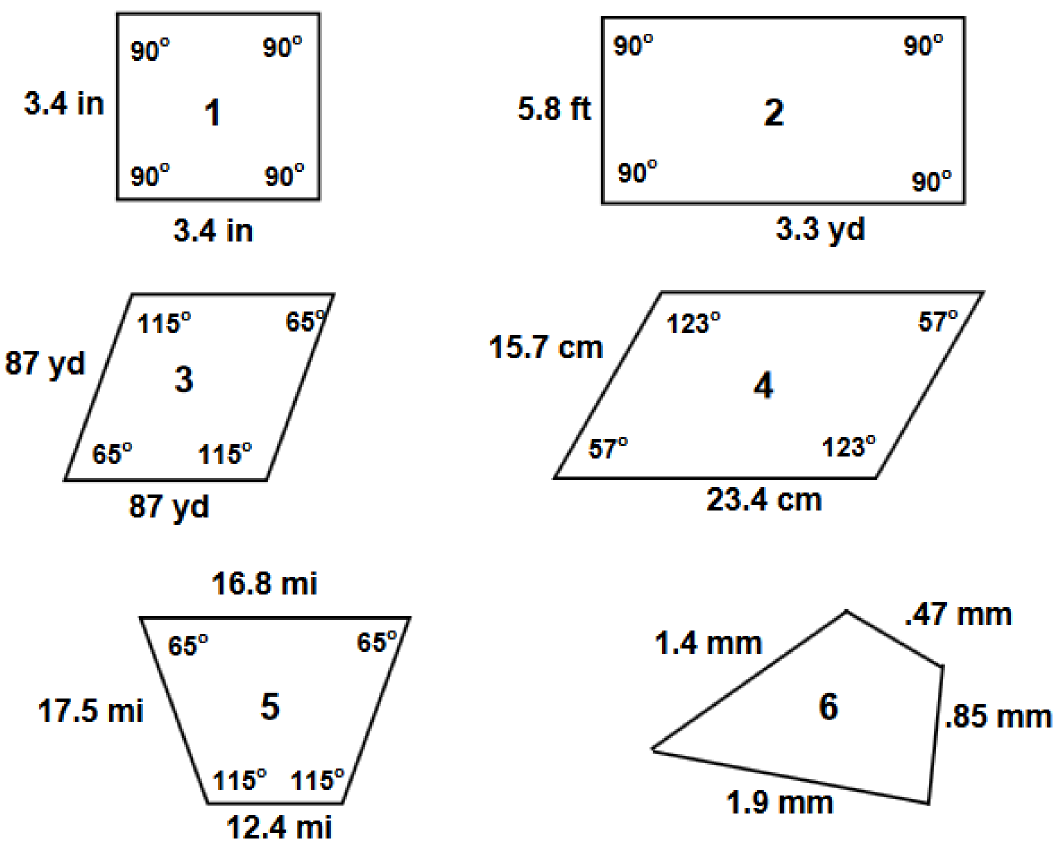
G9 Formulas for Polygons Problems

Identify the figures below and compute their Areas

Note: The Units of measure of the sides must be the same for all sides. For example, if one side is given in feet and the other side in inches, then you must convert one of the side's units accordingly. Must use same units for both sides.

Suppose a rectangle has one side 11/2 feet, and the other side 8 inches. Then, convert feet to inches.

Answers are at bottom of page # Name, Area.

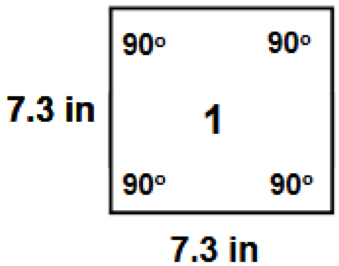


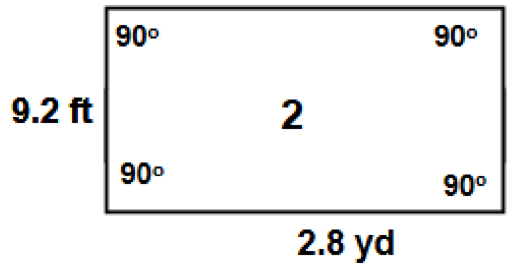
Answer: 1. Square 11.6 in^2 2. Rectangle 57.4 ft^2 or 6.4 yd^2 3. Rhombus 6860 yd^2
4. Parallelogram 308 cm^2 5. Trapezoid 231 mi^2 6. Quadrilateral Not enough Info

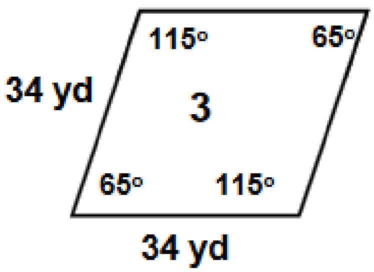
G9E

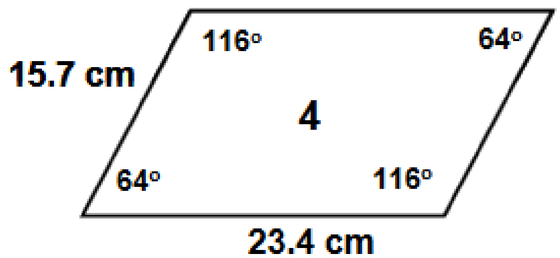
FORMULAS FOR POLYGONS

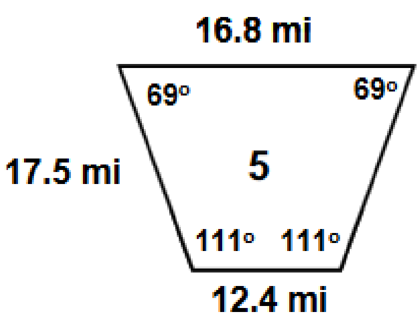
Identify the figures and calculate their areas.

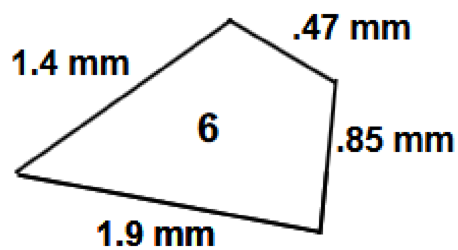


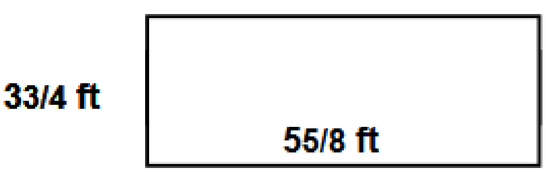


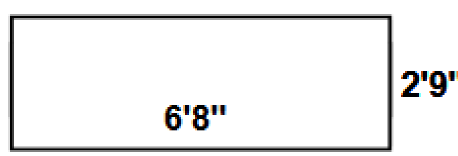








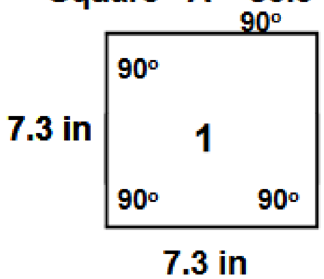




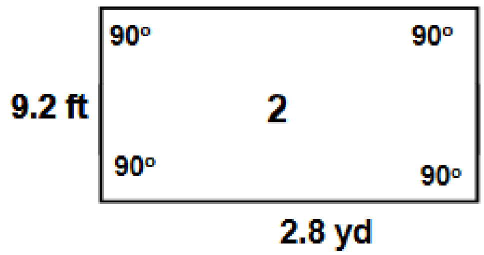
FORMULAS FOR POLYGONS

Identify the figures and calculate their areas.

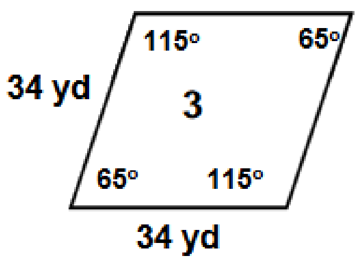
Square **A = 53.3 in^2**



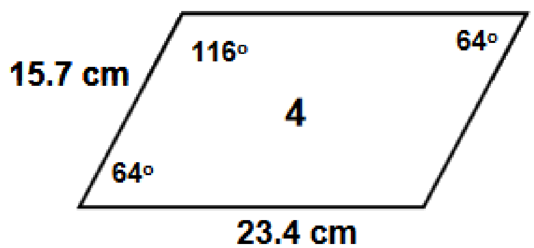
Rectangle **A = 8.6 yd^2 = 77.3 ft^2**



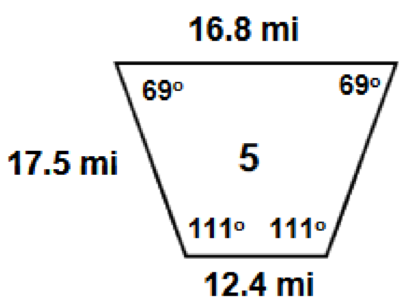
Rhombus **A = 1048 yd^2**



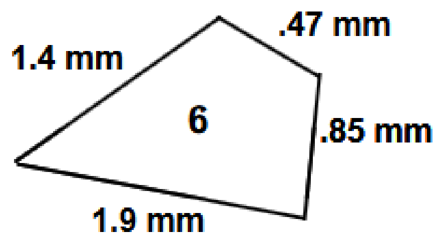
Parallelogram **A = 330.2cm^2**



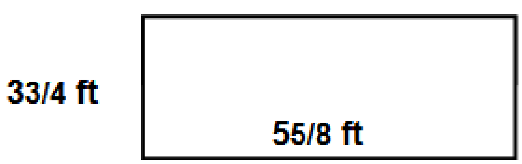
Trapezoid **A = 238.5 mi^2**



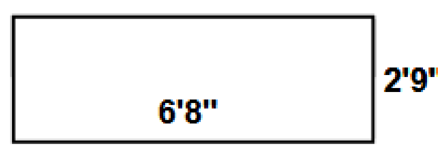
Polygon **A = Insufficient Information**



Rectangle **A = 213/32 *ft^2* = *21.1ft^2***

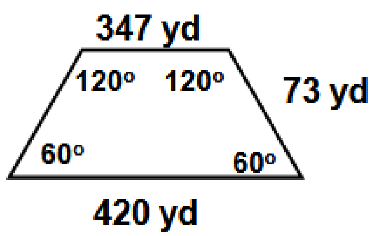


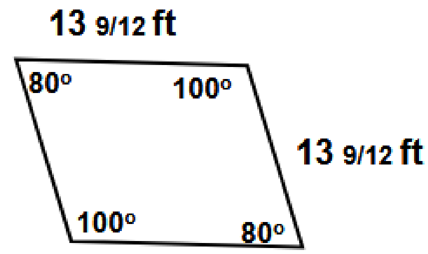
Rectangle **A = 181/3 ft^2**

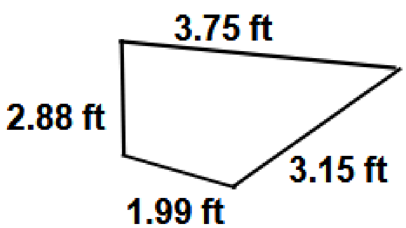


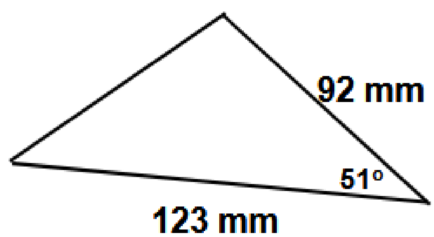
FORMULAS FOR POLYGONS

Identify the figures and calculate their areas.





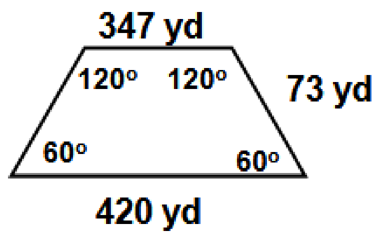




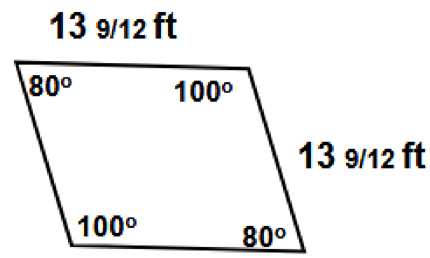
G9ESA

FORMULAS FOR POLYGONS

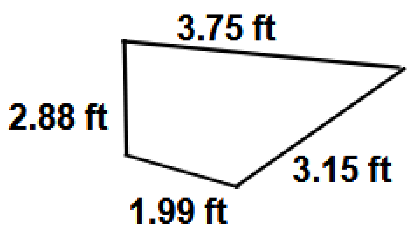
Identify the figures and calculate their areas.



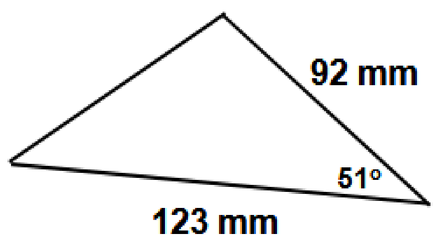
Trapezoid, A = 24,244.87 yd^2



Rhombus, A = 189.1 ft^2



Polygon, A = Insufficient Information



Triangle, A = 4,397.1 mm^2

G10 LESSON: CIRCLES π CIRCUMFERENCE

A **Circle** is a set of points equidistant from a point called the Center. This distance is called the **Radius** of the circle.

The distance across the **Circle** from one side to the other through the center is called the **Diameter** = 2x**Radius**

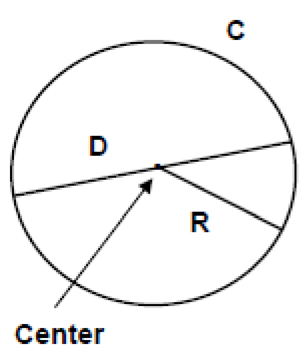
The **Circumference**, (**C**) of the **Circle** is the distance around the **Circle**, sort of its **perimeter**.

The ratio of the **Circumference** to the **Diameter** is always the same number for any circle. It is called **Pi** or π

Thus **$C = \pi D = 2\pi R$**

$\pi = 3.141592654\ldots$ 22/7 is an approximation.

I usually use 3.14 unless I need a lot of accuracy, then I use 3.1416. π is called a "**transcendental number**."



R = Radius = Distance from center to any point on the circle.

D = Diameter = Distance across circle

C = Circumference

$C = \pi D = 2\pi R$

G10 Circles π Circumference Problems

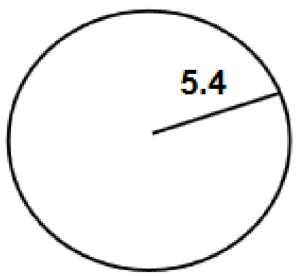
The TI 30XA has a "π **Key**" we will use for π.

The three formulas we must remember are:

$$D = 2R \quad \textbf{and} \quad C = 2\pi R \quad \textbf{and} \quad A = \pi R^2 \text{ (next lesson)}$$

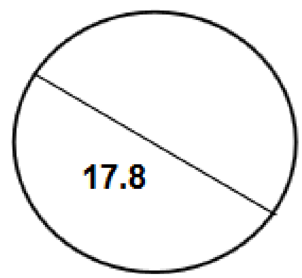
Find the unknown in the following problems.

Answers: **#, R, D, C**



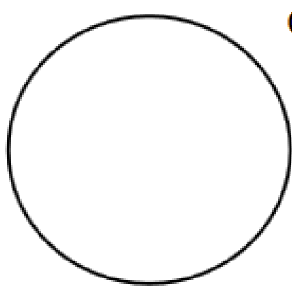
C =
D =

1



C =
R =

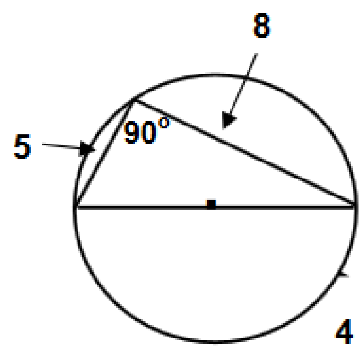
2



Circumference = 32

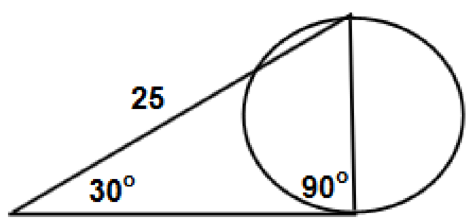
R =
D =

3



R =
D =
C =

4



R =
D =
C =

5

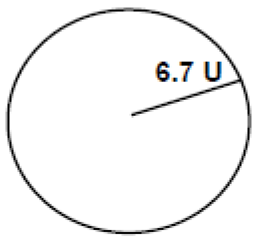
Answers 1. 5.4, 10.8, 33.9 2. 8.9, 17.8, 55.9 3. 5.1, 10.2, 32 4. 4.7, 9.4, 29.6
5. 6.25, 12.5, 39.3

G10E

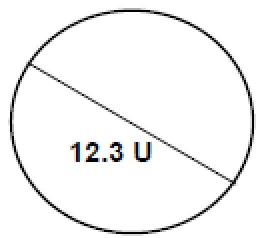
CIRCLES π CIRCUMFERENCE

R = Radius   D = Diameter   C = Circumference

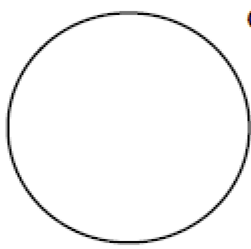
Find Unknowns



C = ? U
D = ? U

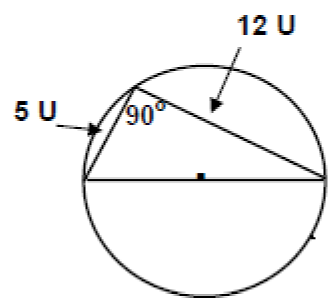


C = ? U
R = ? U

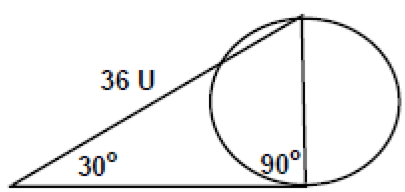


C = 53 U

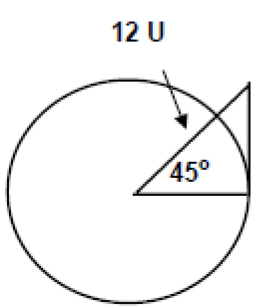
R = ? U
D = ? U



R = ? U
D = ? U
C = ? U

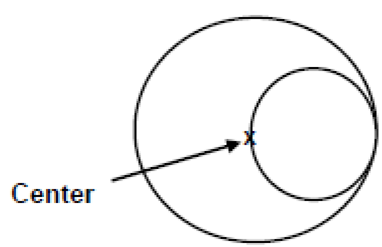


R = ? U
D = ? U
C = ? U



R = ? U
D = ? U
C = ? U

Inner Circle C = 15



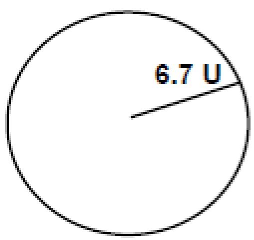
R = ? U
D = ? U
C = ? U

G10EA

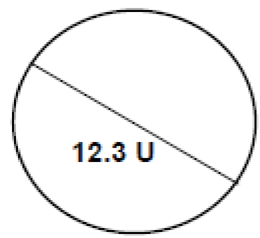
CIRCLES π CIRCUMFERENCE

R = Radius **D** = Diameter **C** = Circumference

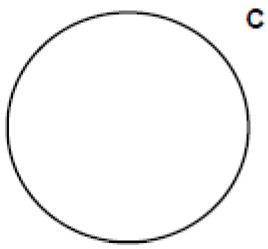
Find Unknowns



C = 42.1 U
D = 13.4 U

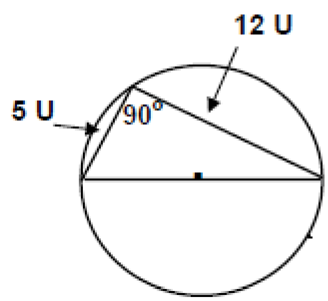


C = 38.6 U
R = 6.15 U

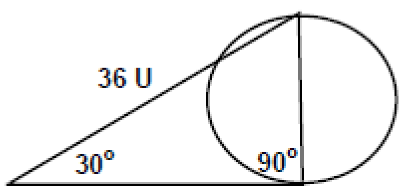


C = 53 U

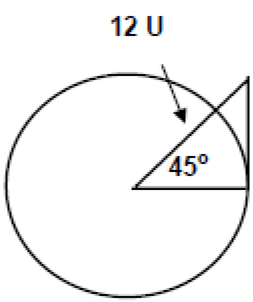
R = 8.4 U
D = 16.9 U



R = 6.5 U
D = 13 U
C = 40.8 U

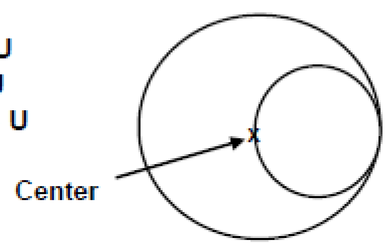


R = 9 U
D = 18 U
C = 56.6 U



R = 8.5 U
D = 17 U
C = 53.3 U

Inner Circle C = 15

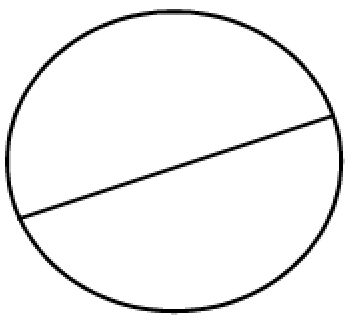


R = 4.8 U
D = 9.6 U
C = 30 U

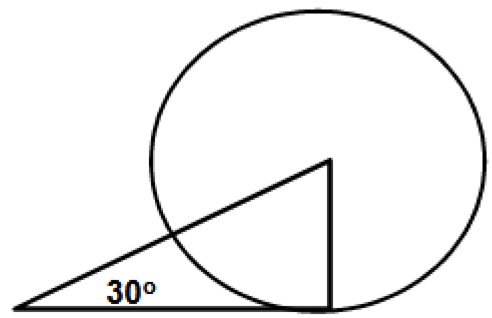
G10ES

CIRCLES π CIRCUMFERENCE

Identify the figures and calculate their perimeters. Be sure to check units and convert all numbers to the same unit where necessary.



d = 460,689 light years

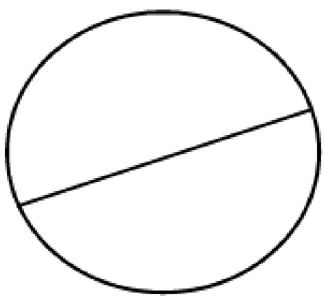


700 ft

G10ESA

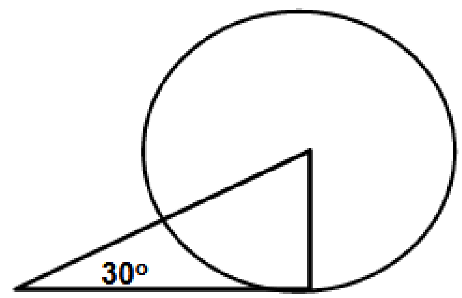
CIRCLES π CIRCUMFERENCE

Identify the figures and calculate their perimeters. Be sure to check units and convert all numbers to the same unit where necessary.



d = 460,689 light years

Circle, C = 460,689π ly = 1,447,297.2 ly



700 ft

Circle, C = 2539.32 ft

G11 LESSON: CIRCLES AREA A = π R2

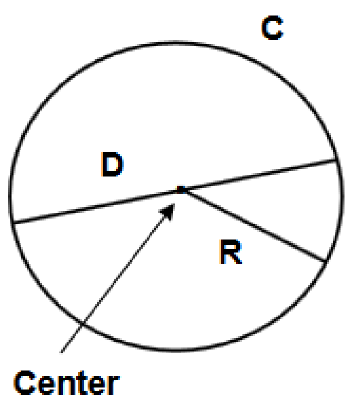
A **Circle** is a set of points equidistant from a point called the **Center**. This distance is called the **Radius** of the **circle.**

π is defined to be **C/D = Circumference/Diameter**

The **Area** (**A**) of the **Circle** turns out to be **A =** πR^2

This is a remarkable fact first discovered by the Greek genius mathematician **Archimedes.** It now is very easy to calculate the **Area** of any **Circle** using a calculator.

Remember: π is about 3.14



R = Radius = Distance from center to any point on the circle.

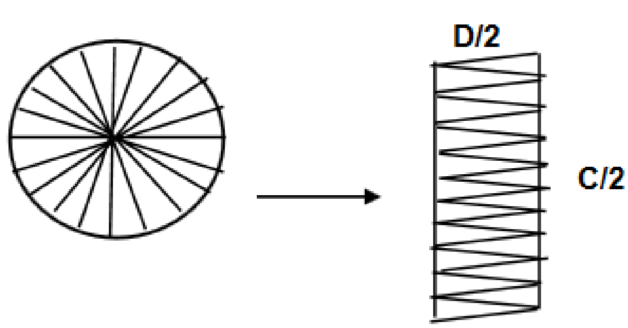
D = Diameter = Distance across circle

C = Circumference

C = πD = 2πR

A = πR^2

Archimedes "Proof" of Area. $A = (C/2)x(D/2) = (2\pi R/2)x(2R/2) = \pi R^2$



A = (D/2)X(C/2)

G11 Circles π Area Problems

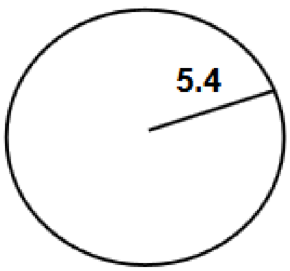
The TI 30XA has a "π Key" we will use for π.

The three formulas we must remember are:

$D = 2R$ **and** $C = 2\pi R$ **and** $A = \pi R2$ (**next lesson**)

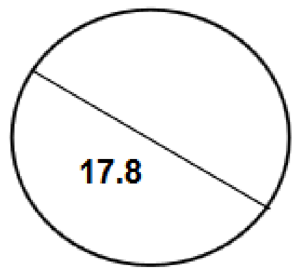
Find the Area in the following problems.

Answers: #, R, A



R =
A =

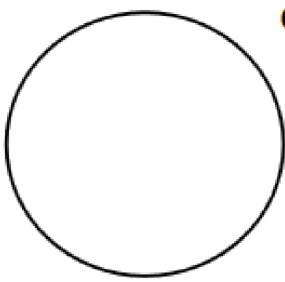
1



R =
A =

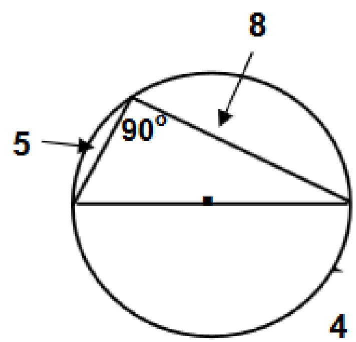
2

Circumference = 32



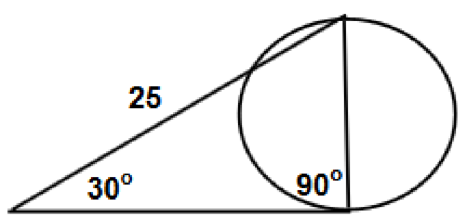
R =
A =

3



R =
A =

4



R =
A =

5

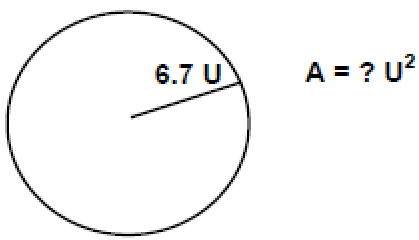
Answers 1. 5.4, 91.6 2. 8.9, 249 3. 5.1, 81.5 4. 4.7, 70
5. 6.25, 123

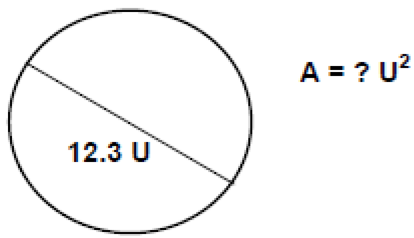
G11E

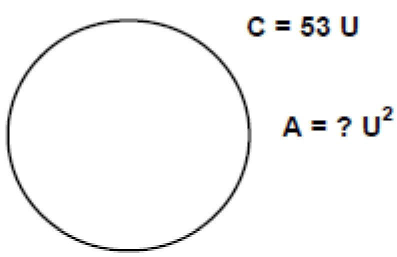
CIRCLES π AREA

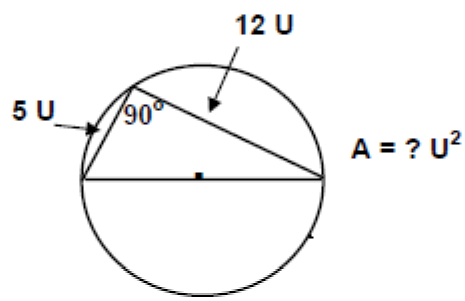
R = Radius, **D** = Diameter, **C** = Circumference

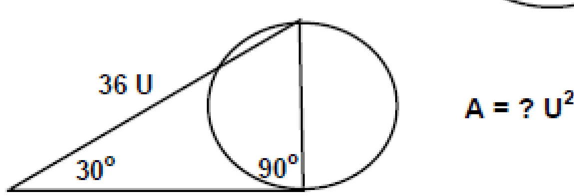
Find Area

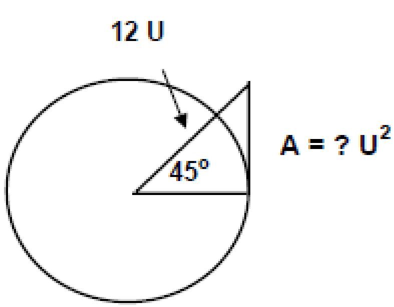


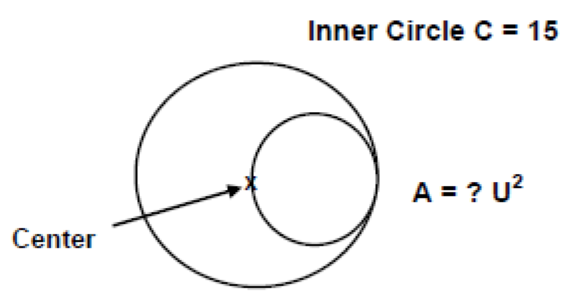










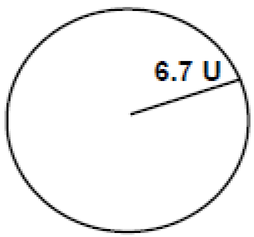


G11EA

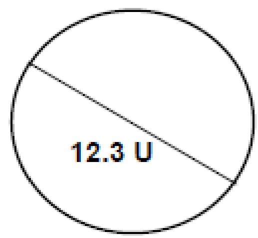
CIRCLES π AREA

R = Radius, **D** = Diameter, **C** = Circumference

Find Area

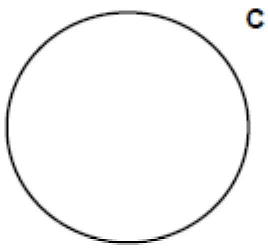


A = 141 U^2

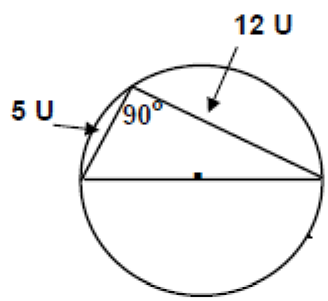


A = 118.9 U^2

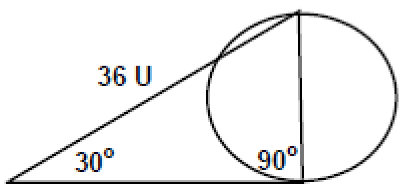
C = 53 U



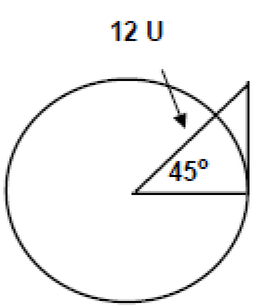
A = 223.5 U^2



A = 132.7 U^2

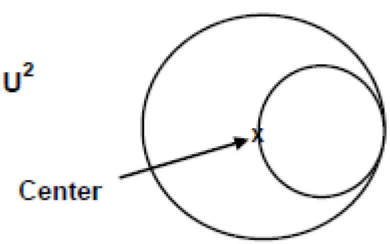


A = 254.5 U^2



A = 226 U^2

Inner Circle C = 15

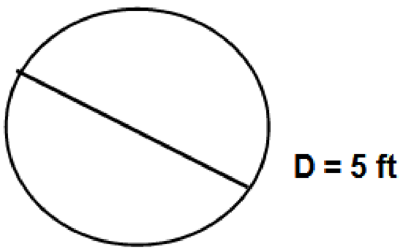


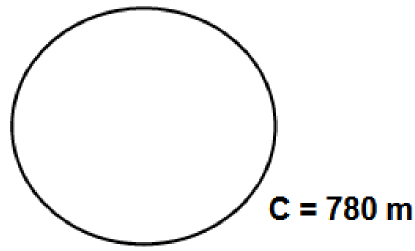
A = 71.6 U^2

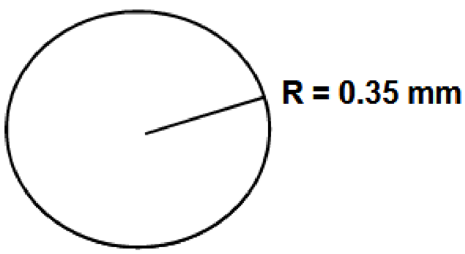
CIRCLES π AREA

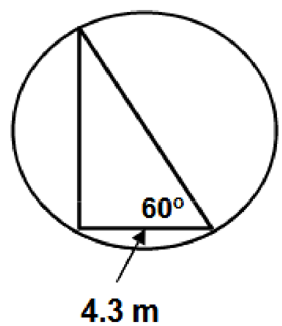
Calculate the areas of the figures below. Be sure to treat units appropriately!

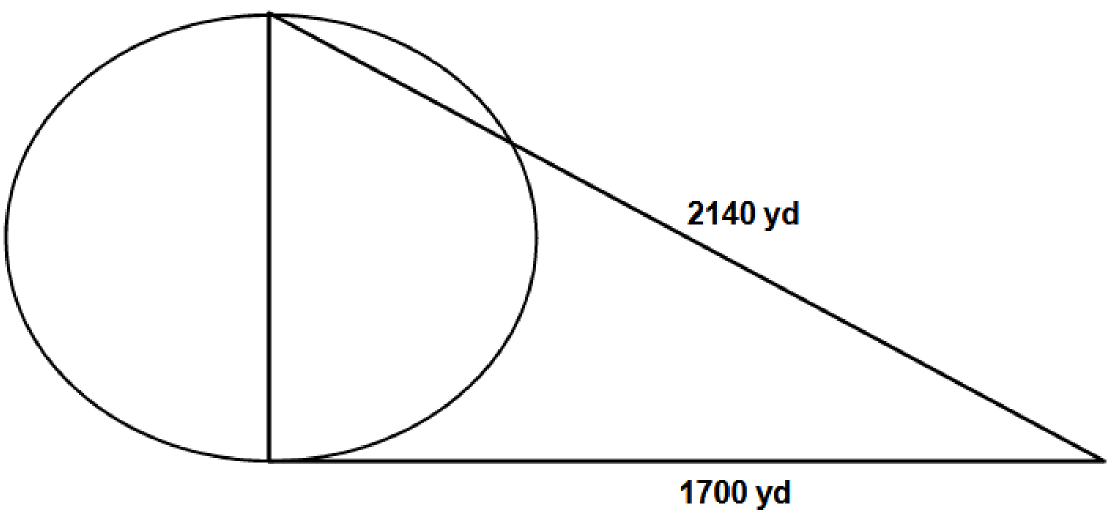
R = Radius, **D** = Diameter, **C** = Circumference









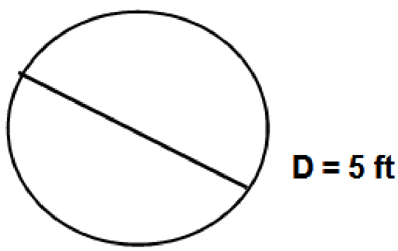


G11ESA

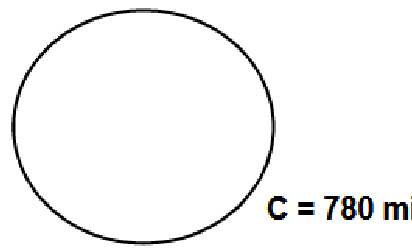
CIRCLES π AREA

Calculate the areas of the figures below. Be sure to treat units appropriately!

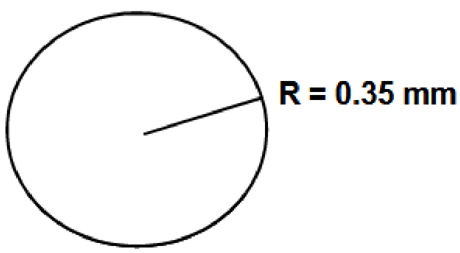
R = Radius, **D** = Diameter, **C** = Circumference



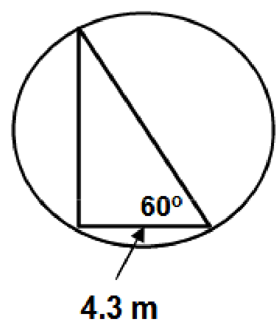
A = 19.6 ft²



A = 48,415 mi²

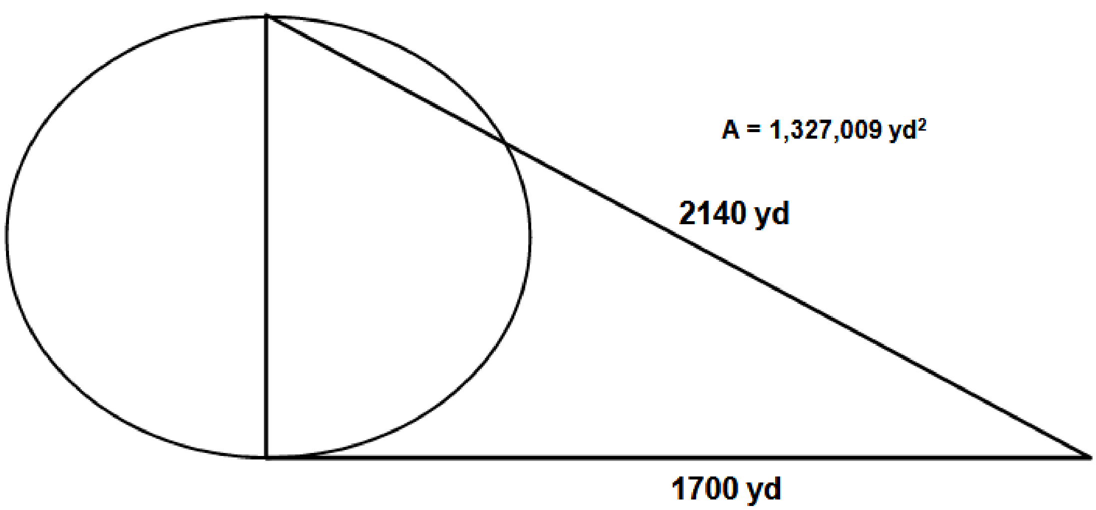


A = 0.385 mm²



A = 43.57 m²

A = 1,327,009 yd²

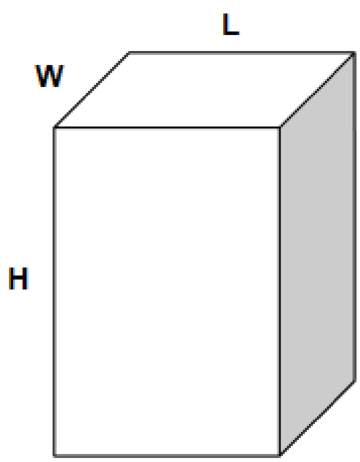


G13 LESSON: SURFACE AREAS BLOCKS AND CYLINDERS

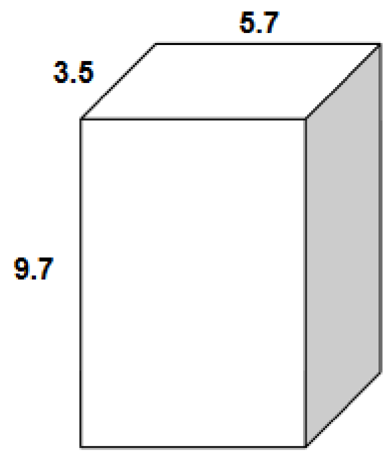
Calculate the Area of each "**face**" or "**side**" for a block.

The **Ends** and then the **Lateral Area** for the **Cylinder**

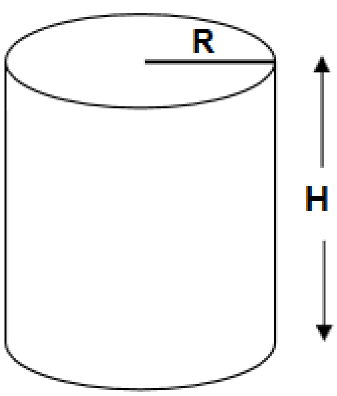
Area is measured in **Square Units, U^2**



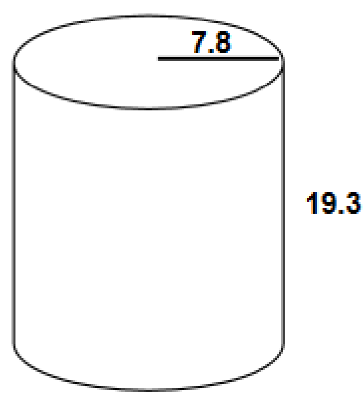
$$A = 2HL + 2HW + 2LW$$
$$= 2(HL + HW + LW)$$



$$A = 2(3.5x5.7 + 3.5x9.7 + 5.7x9.7) = 218\ U^2$$



$$A = 2\pi R^2 + 2\pi RH$$
$$= 2\pi R(R + H)$$

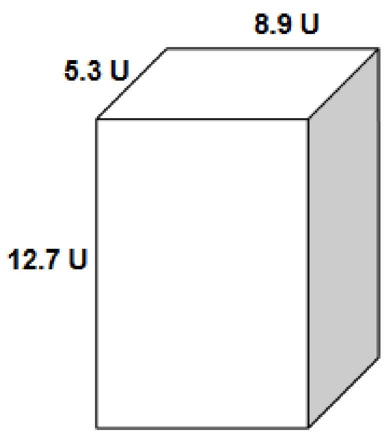


$$A = 2x\pi x7.8^2 + 2\pi x7.8x19.3 = 1328\ U^2$$

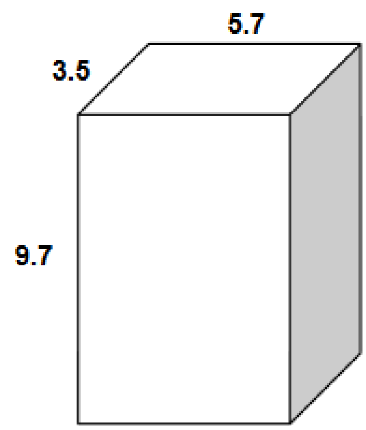
G13E

SURFACE AREAS BLOCKS AND CYLINDERS

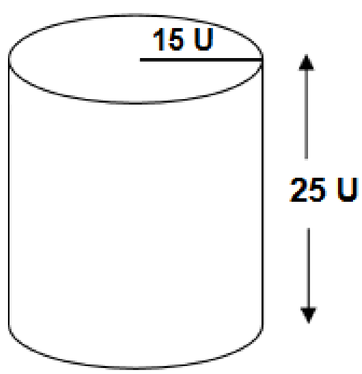
Calculate the Total Surface Area, $\mathbf{U^2}$, in each case.



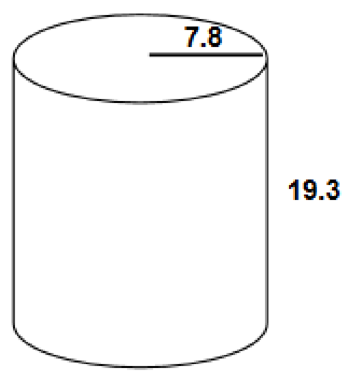
Area = ? U^2



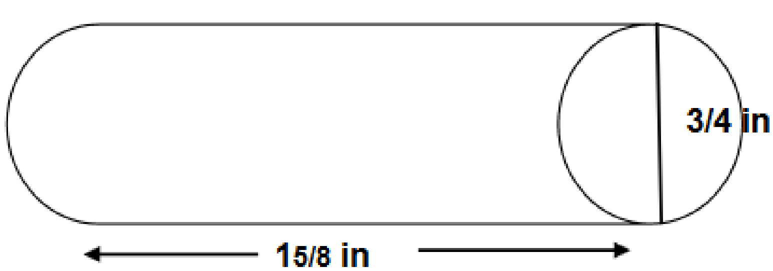
A = ? U^2



Area = ? U^2



A = ? U^2

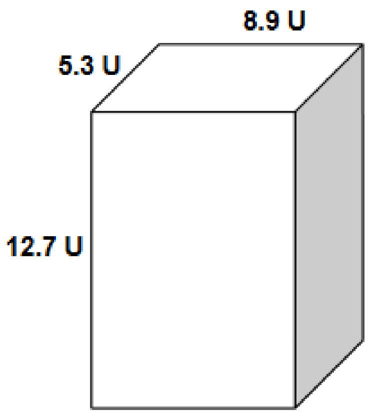


Total Surface Area = ? in^2

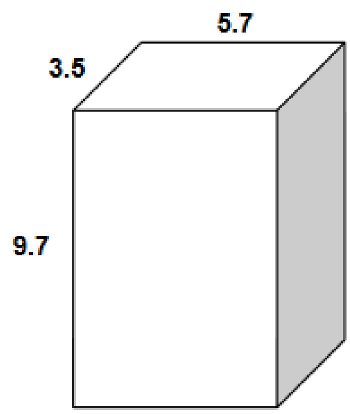
G13EA

SURFACE AREAS BLOCKS AND CYLINDERS

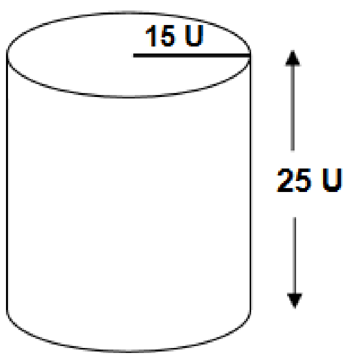
Calculate the Total Surface Area, $\mathbf{U^2}$, in each case.



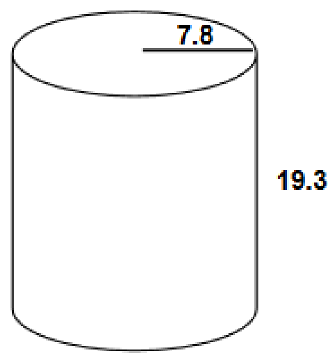
Area = 455 U^2



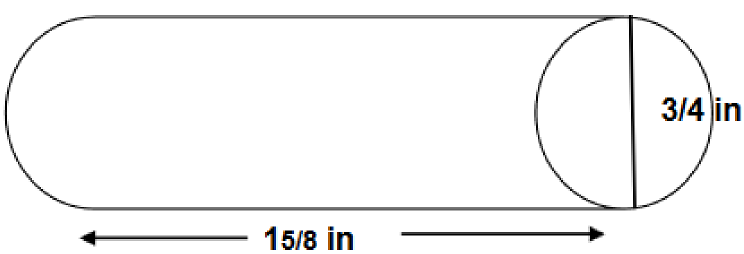
$A = 2(3.5x5.7 + 3.5x9.7 + 5.7x9.7) = 218\ U^2$



Area = 3770 U^2



$A = 2\pi x7.8^2 + 2\pi x7.8x19.3 = 1328\ U^2$

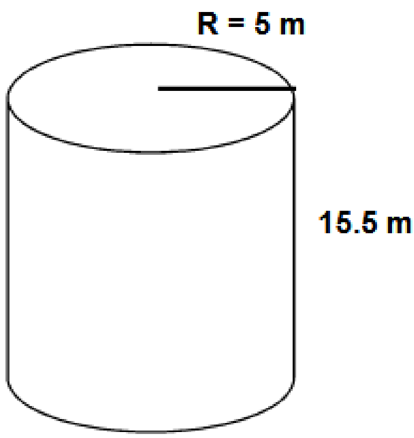


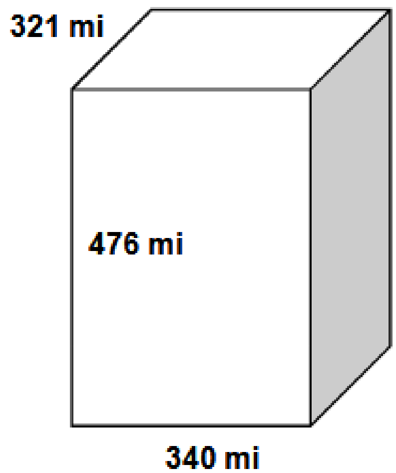
Total Surface Area = 4.71 in^2

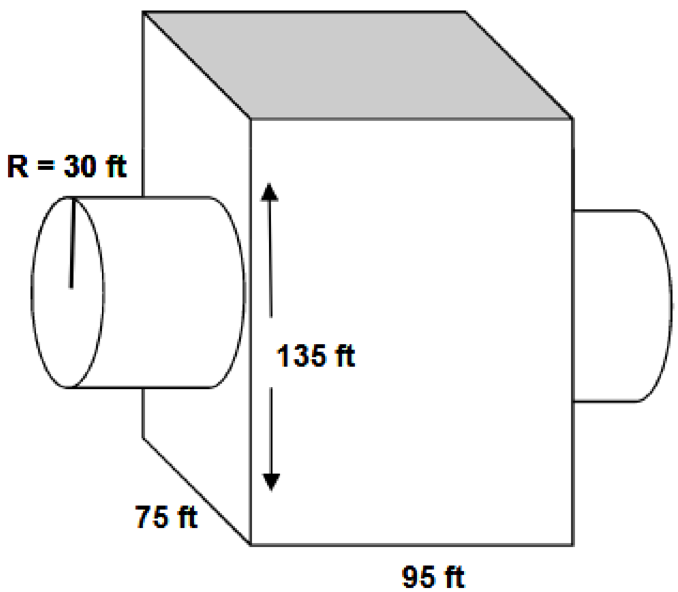
G13ES

SURFACE AREAS BLOCKS AND CYLINDERS

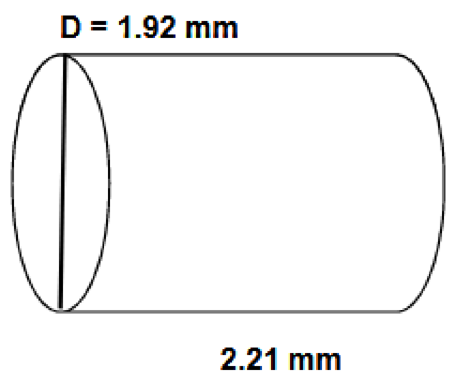
Calculate the surface area of the figures below. Be sure to treat units appropriately!







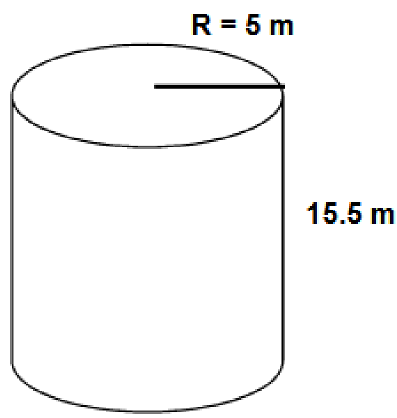
Note: the cylinder of length 140 ft is centered inside the block.



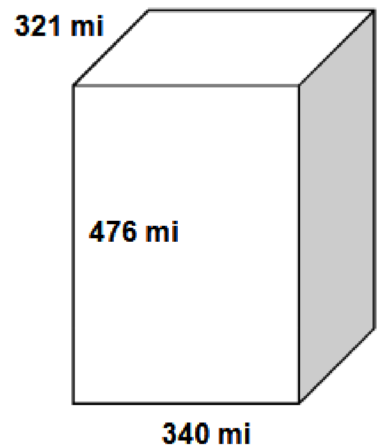
G13ESA

SURFACE AREAS BLOCKS AND CYLINDERS

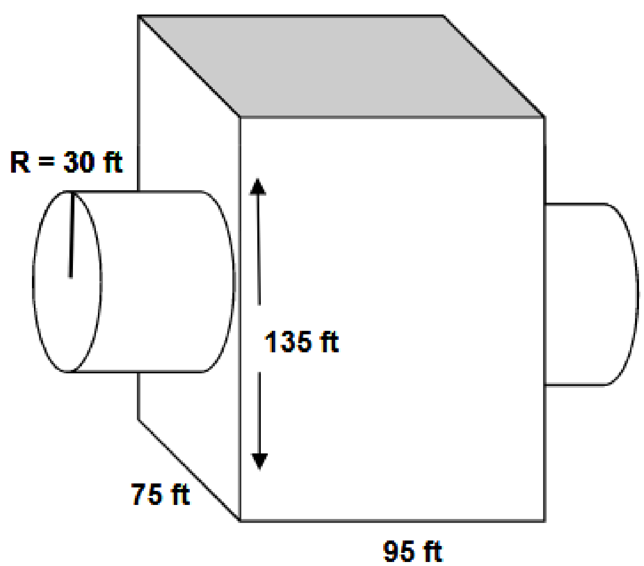
Calculate the surface area of the figures below. Be sure to treat units appropriately!



SA = 644 m^2

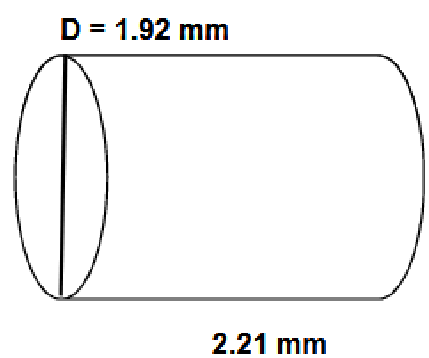


SA = 847,552 mi^2



Note: the cylinder of length 140 ft is centered inside the block.

SA = 68,632.3 ft^2

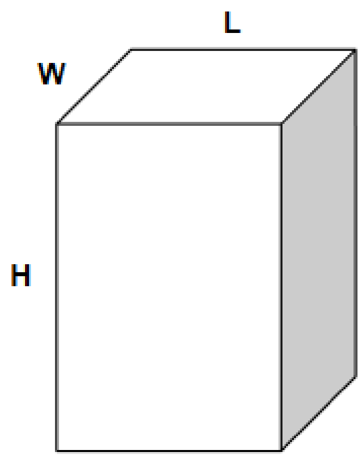


SA = 19.12 mm^2

G15 LESSON: VOLUMES BLOCKS AND CYLINDERS

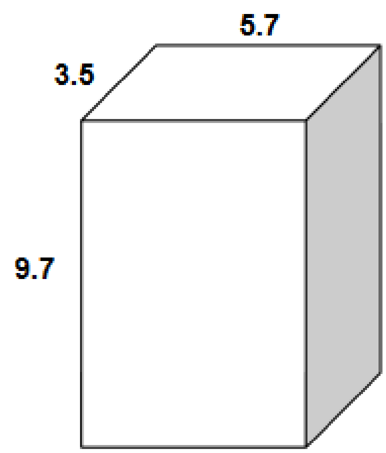
Volume = (**Area of Base**) x **Height**

Volume is measured in **Cubic Units, U^3**



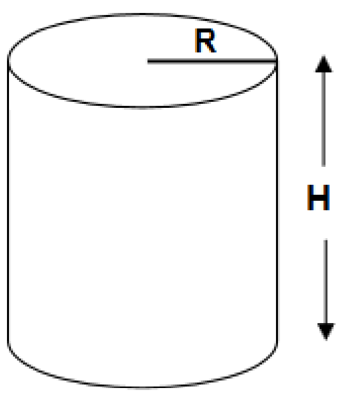
$\boxed{A = 2HL + 2HW + 2LW}$

$\boxed{V = LWH}$



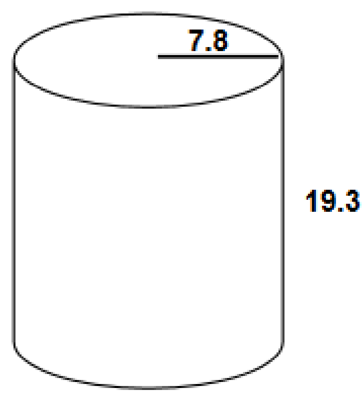
$A = 2(3.5x5.7 + 3.5x9.7 + 5.7x9.7) = 218\ U^2$

$V = 3.5x5.7x9.7 = 194\ U^3$



$$\boxed{\begin{aligned} A &= 2\pi R^2 + 2\pi RH \\ &= 2\pi R(R + H) \end{aligned}}$$

$\boxed{V = \pi R^2 H}$



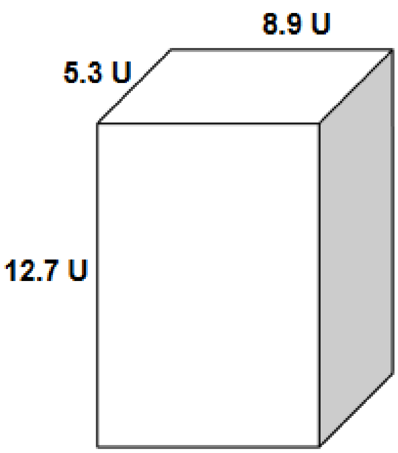
$A = 2x\pi x7.8^2 + 2\pi x7.8x19.3 = 1328\ U^2$

$V = \pi x7.8^2x19.3 = 1174\ U^3$

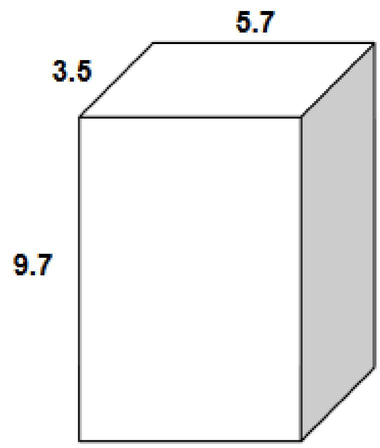
G15E

VOLUMES BLOCKS AND CYLINDERS

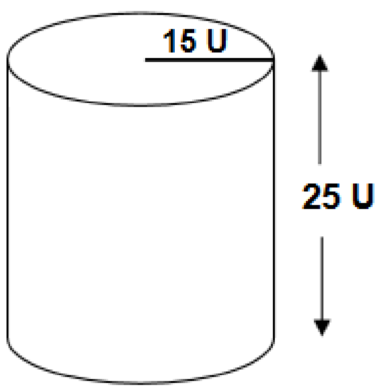
Calculate the **Volume, U^3**, in each case.



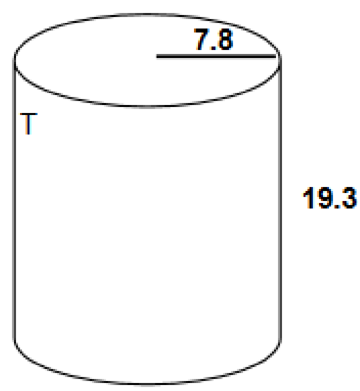
Volume = ? U^3



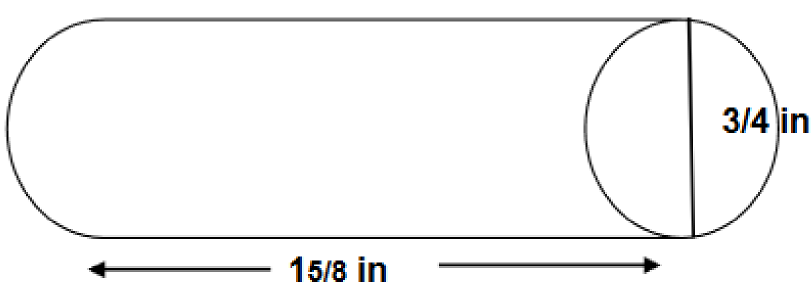
Volume = ? U^3



Volume = ? U^3



Volume = ? U^3

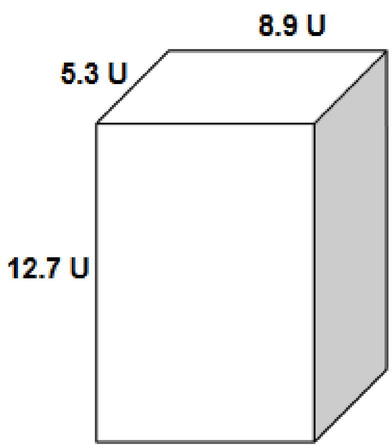


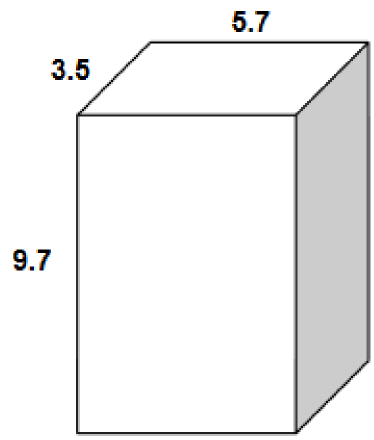
Volume = ? U^3

G15EA

VOLUMES BLOCKS AND CYLINDERS

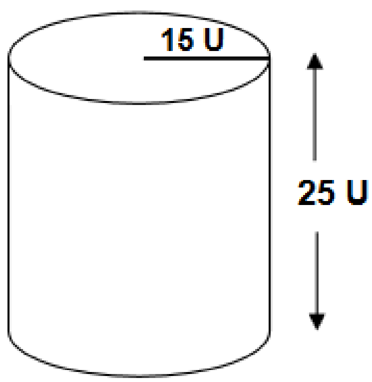
Calculate the **Volume, U^3**, in each case.

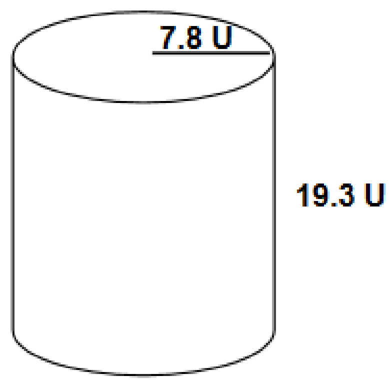




Volume = 599 U^3

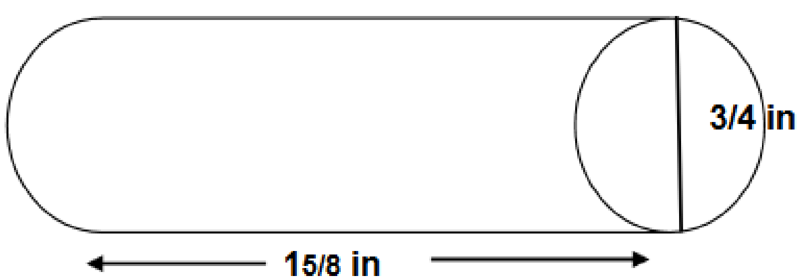
Volume = 193.5? U^3





Volume = 17,671 U^3

Volume = 3689 U^3

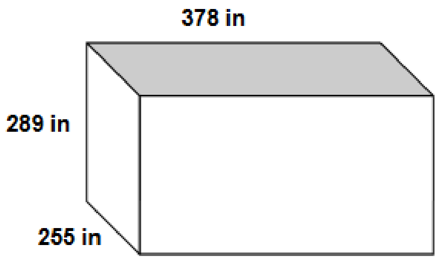


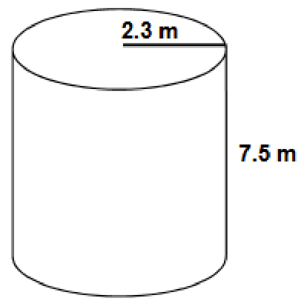
Volume = .718 in^3

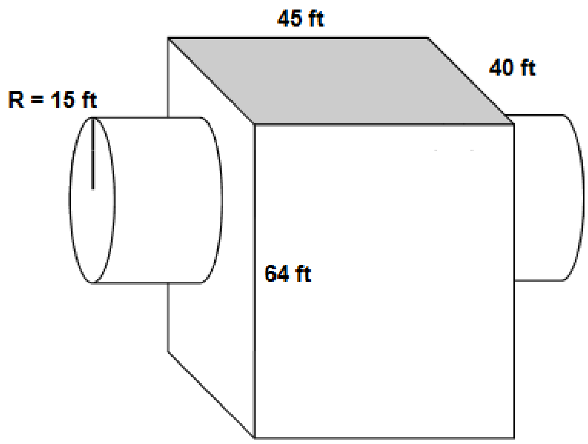
G15ES

VOLUMES BLOCKS AND CYLINDERS

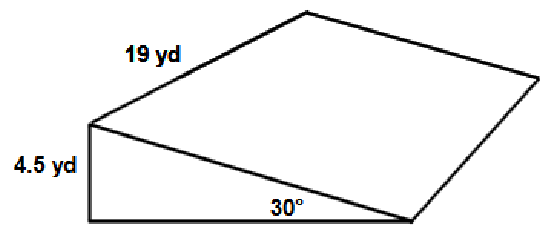
Find the volumes of the figures below. Be mindful of units!







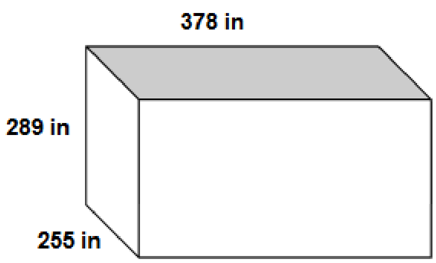
The cylinder of length 65 ft is centered inside the block.



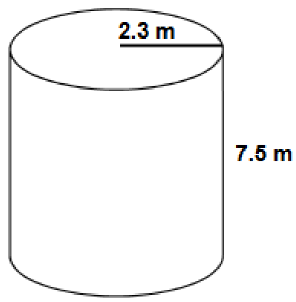
G15ESA

VOLUMES BLOCKS AND CYLINDERS

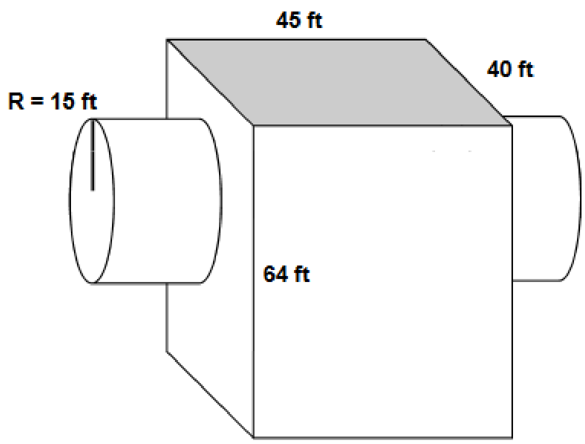
Find the volumes of the figures below. Be mindful of units!



V = 27,856,710 in^3

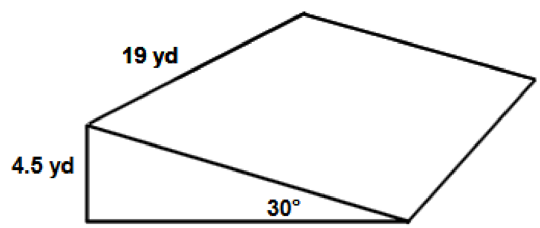


V = 124.6 m^3



The cylinder of length 65 ft is centered inside the block.

V = 129,337 ft^3



V = 333.2 yd^3

S1 LESSON: UNITS CONVERSION

Suppose you have two Units of Measurement

U_1 and U_2 and you wish to convert from one unit to the other, for example, cm and inches.

For example, you want to convert 23.4 cm to inches.

First, you must determine the conversion number.

You may look this up in some type of unit conversion table, or you can go to www.wolframalpha.com and get the answer or find the conversion number.

WA1 **Convert 1 cm to inches**

Answer: 1 cm = 0.3937 inches

Now, you have 23.4 cm = X inches and you want X.

Multiply both sides by 23.4 and get:

23.4 cm = 23.4x0.3937 inches = 9.2 inches

Of course, we could have gotten this directly from www.wolframalpha.com

WA2 **Convert 23.4 cm to inches**

Answer: 9.213

Suppose you wanted to convert 15.7 inches to cm?

1 cm = 0.3937 inches same as 1/0.3937 cm = 1 inch

Or, 1 inch = 2.54 cm since 1/0.3937 = 2.54

Then, 15.7 inches = 15.7x2.54 cm = 39.88 cm

Of course,

WA3 **convert 1 inch to cm**

Answer: 2.54

WA4 **convert 15.7 inches to cm**

Answer: 39.88

This type of process applies to any type of conversion of units. Of course, the units must be measuring the same thing like length or weight.

Example 1: convert 18.3 grams to ounces

First you must find a conversion factor for grams to ounces:

1 gm = .0353 oz you find somewhere.

Then, 18.3 gm = .0353x18.3 oz = .646 oz

WA5 **1 gram to ounce**

Answer: .03527

WA6 **18.3 gram to ounce**

Answer: .6455

The same process applies to any type of unit conversion.

For example, square feet to square meters:

1 sq meter = 10.76 square feet

Thus, 1 square foot = 1/10.76 sq m = .093m^2

Example 2: 4.7 sq m are how many sq ft?

Answer: 4.7x10.76 ft^2 = 50.57 ft^2

WA7 **4.7 sq m to sq ft**

Answer: 50.6

To get more accuracy:

WA8 **4.70 sq m to sq ft**

Answer: 50.59

WA9 **1 square meter to square feet**

Answer: 10.76

Example 3: 12.3 Kilograms is how many pounds?

WA10 **12.3 kilograms to pounds**

Answer: 27.12 lb = 27 lb 1.9 oz

Example 4: 3.4 cubic meters is how many cubic yards

1 m = 1.094 yd

1 m^3 = 1.0943^3 yd^3 = 1.309 yd

So 3.4 cu m = 3.4x1.309 cu yd = 4.45 cu yd

WA11 **3.4 cubic meter to cubic yard**

Answer: 3.45 cu yd

In general, if you have two units which measure the same quantity, U_1 and U_2, and you wish to convert from one unit to the other, then:

If you have access to www.WolframAlpha.com, you simply enter the command:

convert N U_1 to U_2

where N is the amount of the quantity you have expressed in U_1 and you will get the amount expressed in U_2.

If you don't have access to Wolfram Alpha, then you must find the conversion factor, C, where:

$1\ U_1 = CU_2$

Multiply both sides by N to obtain the answer:

$N\ U_1 = CxN\ U_2$

Example: you know 1 mile = 1.609 kilometers

60 miles = 1.609x60 km = 96.54 km

So, you can see for example that:

100 km/hr is about 60 m/hr.

S1E

Units Conversion

1. Given the conversion factor 1 ft = 12 in, how many inches are in 1.5 ft?

2. Given the conversion factor 1 ft = 12 in, how many feet are in 14 in?

3. Given the conversion factor 1 m = 39.37 in, how many inches are in 2.8 m?

4. Given the conversion factor 1 m = 39.37 in, how many meters are in 76 in?

5. Given the conversion factor 1 in^2 = 6.452 cm^2, how many cm^2 are on an 8 ½ in x 11 in sheet of paper?

6. Given the conversion factor 1 in^2 = 6.452 cm^2, how many in^2 are in 100 cm^2?

7. Given the conversion factor 1 gal = 3.785 L, how many liters are in 19 gal?

8. Given the conversion factor 1 km^2 = 0.3861 mi^2, how many mi2 are in 15 km2?

9. Given the conversion factor 1 gal = 3.785 L, how many gallons are in 2 L?

10. If I want to pour a concrete house slab that is 52 feet long by 28 feet wide by 4 inches deep, how would I determine how many cubic yards of concrete would be needed?

UNITS CONVERSION

1. Given the conversion factor 1 ft = 12 in, how many inches are in 1.5 ft?

   1 ft = 12 in (You will also see this written as 12 in/ft.)

   1.5 ft = X in

   (12 in/ft)*(1.5 ft) = 18 in

   or

   WA **convert 1.5 ft to in**

   18 in

2. Given the conversion factor 1 ft = 12 in, how many feet are in 14 in?

   1 ft = 12 in

   1/12 ft = 12/12 in

   0.0833 ft = 1 in (You will also see this written as 0.0833 ft/in.)

   14 in = X feet

   (0.0833 ft/in)*(14 in) = 1.167 ft

   or

   WA **convert 14 in to ft**

   1.167 ft

3. Given the conversion factor 1 m = 39.37 in, how many inches are in 2.8 m?

   2.8 m = X in

   (39.37 in/m)*(2.8 m) = 110.24 in

4. Given the conversion factor 1 m = 39.37 in, how many meters are in 76 in?

   1 m = 39.37 in

   1/39.37 m = 39.37/39.37 in

   0.0254 m = 1 in (You will also see this written as 0.0254 m/in.)

   76 in = X m

   (0.0254 m/in)*(76 in) = 1.930 m

   or

   WA **convert 76 in to m**

   1.93 m

5. Given the conversion factor 1 in^2 = 6.452 cm^2, how many cm^2 are on an 8 ½ in x 11 in sheet of paper?

   (8 ½ in)*(11 in) = 93.5 in^2

   (6.452 cm^2/in^2)*(93.5 in^2) = 603.262 cm^2

   or

   WA **convert 93.5 inches^2 to cm^2**

   603.2 cm^2

   or

   WA **convert (8.5 inches)*(11 in) to cm^2**

   603 cm^2

   Note: The answers are actually the same. The slight differences occur during rounding.

6. Given the conversion factor 1 in^2 = 6.452 cm^2, how many in^2 are in 100 cm^2?

   1 in^2 = 6.452 cm

   1/6.452 in^2 = 6.452/6.452 cm^2

   0.155 in^2 = 1 cm^2 (You will also see this written as 0.155 in^2/ cm^2.)

   100 cm^2 = X in^2

   (0.155 in^2/ cm2)*(100 cm^2) = 15.5 in^2

   or

   WA **convert 100 cm^2 to in^2**

   15.5 in^2

7. Given the conversion factor 1 gal = 3.785 L, how many liters are in 19 gal?

   19 gal = X L

   (3.785 L/gal)*(19 gal) = 71.915 L

   or

   WA **convert 19 gal to L**

   71.92 L

8. Given the conversion factor 1 km^2 = 0.3861 mi^2, how many mi^2 are in 15 km^2?

   15 km^2 = X mi^2

   (0.3861 mi^2/km^2)(15 km^2) = 5.7915 mi^2

   or

   WA **convert 15 km^2 to mi^2**

   5.792 mi^2

9. Given the conversion factor 1 gal = 3.785 L, how many gallons are in 2 l?

1 gal = 3.785 L

1/3.785 gal = 3.785/3.785 L

0.2642 gal = 1 L (You will also see this written as 0.2642 gal/L.)

2 L = X gal

(0.2642 gal/L)*(2 L) = 0.5284 L

or

WA **convert 2 L to gal**

0.5283 L

10. If I want to pour a concrete house slab that is 52 feet long by 28 feet wide by 4 inches deep, how would I determine how many cubic yards of concrete would be needed?

27 ft^3 = 1 yd^3

27/27 ft^3 = 1/27 yd^3

1 ft^3 = 0.0370 yd^3

1 ft = 12 in

1 in = 0.0833 ft. (See A1 for math conversion.)

First, convert in to ft.

4 in = X ft

(0.0833 ft/in)(4 in) = 0.3332 ft

Next, calculate number of ft^3.

(52 ft)(28 ft)(0.3332 ft) = 485.1392 ft^3

Finally, convert ft^3 to yd^3

485.1392 ft^3 = X yd^3

(0.0370 yd^3/ft^3)(485.1392 ft^3) = 17.968 yd^3

S2 LESSON: DMS Degrees – Minutes - Seconds

There are 360°, or Degrees, in one revolution or circle.

In the DD (decimal degrees) system we express degrees with decimal notation. 37.45 degrees means 37 and 45/100 degrees.

In the DMS system, 1 degree = 60 minutes, or 1° = 60′

And 1 minute = 60 seconds, or 1′ = 60″

So, 1′ = (1/60)° and 1″ = (1/60)′ = (1/3600)°

We can express degrees in either DD or DMS format and convert degrees from DD to DMS and DMS to DD using the TI30Xa calculator.

DMS → DD is 2^{nd} +

DD → DMS is 2^{nd} =

Example:

6.5° = 6°30′ 00″00

6.55° = 6°33′ 00″00

6.57° = 6°34′ 12″00

6.573° = 6°34′ 22″80 (this means 22.80″)

127.875° = 127°52′ 30″

57.382° = 57°22′ 55″2 (this means 55.2″)

To apply the DMS → DD conversion you must enter the angle in the following format:

6°34’ 22”80 is entered: 6.342280 2nd +

Answer: 6.573°

26°4’ 2”50 is entered: 26.040250 2nd +

Answer: 26.06736

Now enter 26.06736° and get 26°04’ 02”5

It is possible to do these conversions manually with formulas, but it is best to do it with a calculator.

S2E

DMS Degrees – Minutes - Seconds

Convert the following decimal degree (DD) numbers to degrees-minutes-seconds (DMS).

1. 87.625
2. 137.6489
3. 65.475698
4. 19.01325
5. 45.4557

Convert the following degrees-minutes-seconds (DMS) to decimal degree (DD) numbers.

6. 66°18’12”0
7. 78°45’06”4
8. 180°04’07”
9. 97°09’45”7
10. 54°57’27”4

S2EA

THE NUMBER LINE, NEGATIVE NUMBERS

Answers: [ ]'s

Convert the following decimal degree (DD) numbers to degrees-minutes-seconds (DMS).

1. 87.625

    [**87°37'30"00**]

2. 137.6489

    [**137°38'56"**]

3. 65.475698

    [**65°28'32"5**]

4. 19.01325

    [**19°00'47"7**]

5. 45.4557

    [**45°27'20"5**]

Convert the following degrees-minutes-seconds (DMS) to decimal degree (DD) numbers.

6. 66°18'12"0

    [**66.30333333**]

7. 78°45'06"4

    [**78.75177778**]

    Note: If you get an answer of 78.75167778, what you did is enter into your calculator "78.450604" instead of "78.45064" before you hit the DMS → DD key. Anything after the " symbol, in this case 06"4, should be treated as 6.4 seconds, therefore, entering a 0 before the 4 would be incorrect.

8. 180°04′07″

[**180.0686111**]

9. 97°09′45″7

[**97.162269444**]

Note: If you get an answer of 97.16251944, what you did is enter into your calculator "97.094507" instead of "97.09457" before you hit the DMS → DD key.

10. 54°57′27″4

[**54.95761111**]

Note: If you get an answer of 54.95751111, what you did is enter into your calculator "54.572704" instead of "54.57274" before you hit the DMS → DD key.

S3 LESSON: y^x EXPONENTS

y^x means y times itself x times

y is called the base,

x is called the exponent

Examples:

$2^3 = 8$; $3^2 = 9$; $5^4 = 625$; $10^5 = 100{,}000$

The y^x key is the east way to calculate this.

Clear the calculator

Enter 2 and press the y^x

Enter 3 and press the = key

Answer: 8

Do all of the above.

y can be any positive number

x can be any number

$\sqrt[x]{y}$ means the x^{th} root of y

same as $y^{(1/x)}$ $[\sqrt[x]{y}]^x = y = \sqrt[x]{(y^x)}$

$\sqrt[3]{8} = 2 = 8^{1/3}$

$1.7^{2.7} = 4.19$

$2^{10} = 1024$ Kilo $\sqrt[10]{1024} = 2 = 1024^{1/10}$

Metric		Digital
$10^3 = 1000$	Kilo	$2^{10} = 1024$
$10^6 = 1{,}000{,}000$	Mega	$2^{20} = 1{,}048{,}576$
$10^9 = 1{,}000{,}000{,}000$	Giga	$2^{30} = 1{,}073{,}741{,}824$
$10^{12} = 1{,}000{,}000{,}000{,}000$	Tera	$2^{40} = 1{,}099{,}511{,}627{,}776$

Compound interest at 5% for 40 years:

$1.05^{40} = 7.04$

$1.06^{40} = 10.3$

$1.25^{25} = 265$ Kmart growth rate 25%/yr

$1.56^{25} = 67{,}315$ Walmart growth rate 56%/yr

$(1 + 1/1{,}000{,}000)^{1{,}000{,}000} = 2.718 = e$

Negative exponents

$y^{-x} = 1/y^x$

$9^{-2} = 1/9^2 = 1/81 = .012345679$

$9^{-1/2} = 1/3 = 1/9^{1/2}$

$5.7^{-1.3} = .104$

$.58^{-3.2} = 5.715$

$-3^{.5} = \text{Error}$

Exponents are very common in many situations. The calculator makes it very easy to deal with them. Just follow the rules.

Of course, Wolfram Alpha also will deal with them very easily.

S3E

y^x EXPONENTS

Use your calculator to solve the following exercises.

1. $4^7 =$

2. $10^9 =$

3. $4.2^{3.6} =$

4. $8 \sqrt{256} =$

5. $6 \sqrt{1{,}000{,}000} =$

6. $^{3.2}\sqrt{8.3} =$

7. $7^{-2} =$

8. $56^{-2.4} =$

9. $0.47^{-3.1} =$

10. If production increases at a rate of 6.5%/year, what is your production after 15 years?

11. If production increases at a rate of 7.5%/year, what is your production after 15 years?

12. For the following exponents, match them with their name:

1. $10^3 = 1{,}000$	a. Giga (Digital)
2. $10^6 = 1{,}000{,}000$	b. Tera (Digital)
3. $10^9 = 1{,}000{,}000{,}000$	c. Giga (Metric)
4. $10^{12} = 1{,}000{,}000{,}000{,}000$	d. Tera (Metric)
5. $2^{10} = 1{,}024$	e. Mega (Metric)
6. $2^{20} = 1{,}048{,}576$	f. Kilo (Metric)
7. $2^{30} = 1{,}073{,}741{,}824$	g. Mega (Digital)
8. $2^{40} = 1{,}099{,}511{,}627{,}776$	h. Kilo (Digital)

S3EA

y^x EXPONENTS Answers: [ ]'s

Use your calculator to solve the following exercises.

1. 4^7 = [**16,384**]

2. 10^9 = [**1,000,000,000**]

3. $4.2^{3.6}$ = [**175.266**]

4. 8 √256 = [**2**]

5. 6 √1,000,000 = [**10**]

6. $^{3.2}\sqrt{8.3}$ = [**1.937**]

7. 7^{-2} = [**0.020**]

8. $56^{-2.4}$ = [**0.0000637**]

9. $0.47^{-3.1}$ = [**10.387**]

10. If production increases at a rate of 6.5%/year, what is your production after 15 years?
    [**$1.065^{15} = 2.572$**]

11. If production increases at a rate of 7.5%/year, what is your production after 15 years?
    [**$1.075^{15} = 2.959$**]

12. For the following exponents, match them with their name:
    [**1f, 2e, 3c, 4d, 5h, 6g, 7a, 8b**]

S4 LESSON: Density = Weight/Volume

How much does 55 gallons of water weigh (in lbs)?

How much does 55 gallons of gasoline weigh?

How much does 55 gallons of cement weigh?

How much does 55 gallons of mulch weigh?

Weight is measured in units such as:
Grams (gm), pound (lb), ounce (oz),
kilograms (kg), stone (st), etc

Volume is measured in such units as:
gallons(gal), quarts (qt), fluid ounces (fl oz),
liters (ltr), cubic inches (cu in or in^3),
cubic feet (cu ft or ft^3), or in general cubic U (cu U or U^3) where
U is a linear length, etc.

Suppose 1 gallon of water weighs 8.345 lbs

Then, 55 gallons would weigh 55x8.345 = 459 lbs

How do you find out what 1 gallon of water weighs?

Well, you could weigh a quart of water and multiply by 4, since 4 quarts equals one gallon.

Or, you could weigh 1 oz of water and multiply by 128 since one gallon is 128 oz.

Or, you could weigh a container full of water whose volume is 12 oz and then multiply by 128/12

Of course, you must subtract the weight of the empty container!

The Density of water is what you are computing.

Density = Mass/Volume = Weight/Volume

D = W/V or W = DV or V = W/D

So, if you know any two of these, then you always can calculate the third.

The units must always match up.

If W is lb and V is ft^3, the D must be lb/ft^3

D could be lb/gal, or oz/quart, or gm/liter, etc.

Above we determined a W and V in an experiment and calculated D, and then used this D to calculate the W when we were given the V.

What you always want to do first is learn the D for a substance.

For example, D for gasoline is 6.06 lb/gal

So, 55 gallons of gasoline would weigh:

55x6.06 = 333 lbs VxD = W galx(lb/gal) = lb

BUT, how do we know D for gasoline?

1. We could look it up in some table of densities.
2. We could find out on the Internet. My favorite is www.wolframalpha.com
3. We could do the experiment by weighing a known volume, usually pretty small.

WA1 **density of gasoline in lb/gal**

Answer: 6.06 lb/gal

But, suppose you did the experiment and found that 24.7 cu in of gasoline weighed 10.4 oz?

10.4/24.7 = .42 oz/in^3

WA2 **convert .42 oz/in^3 to lb/gal**

convert this to lb/gal

Answer: 6.06 lb/gal as it should be.

Note: Do you think I actually did this experiment?

Of course not, I just used WA backwards

WA3 **convert 6.06 lb/gal to oz/in^3**

Answer: .42 oz/in

But, in many cases, you won't be able to find the Density of a substance in any handbook, or even on Wolfram Alpha. So then, you simply must do the experiment with a convenient container.

1. Compute its volume.
2. Fill it up with the substance.
3. Calculate the Density of this substance.

Then you can find either V or W if you know the other one.

For example, how many cubic yards will one ton of insulation material fill up?

Suppose we do the experiment and find that the density of some insulation material is 2.5 lbs/gal. (I have no idea what it really would be.)

Then, WA tells us the density would be:

WA4 **convert 2.5 lbs/gal to lbs/yd^3**

Answer: 505 lbs/cu yd

So, V = W/D yields 2000/505 = 4 yd^3 as the answer.

How much does 55 gallons of cement weigh?

WA5 **density of cement in lb/gal**

Answer: 16.8 lb/gal

So 55 gallons weighs 55x16.8 = 924 lbs

If in doubt, actually do the experiment and weigh a small amount and then do the calculations.

How much does 55 gallons of mulch weigh?

WA6 **density of mulch in lb/gal**

WA doesn't know. You will probably just have to do the experiment and calculate the density.

So now, you can do a bunch of problems.

Sometimes, WA will give you the density.

Sometimes you will have to find it by experiment.

Use some handy container whose volume you know or can compute. And, fill it up and weight it. Subtract the empty container weight. Then, use WA to convert it to the Units you want.

S4E

Density = Weight/Volume

Use your calculator to solve the following exercises.

1. 1 quart of seawater (salt water) weighs 2.138 lb. What is the density of seawater (lb/gal)?

2. The density of propane is 0.0156843 lb/gal. A residential tank holds 250 gal. of propane. What is the weight (lb) of the propane in that tank?

3. The density of gold is 11.2 oz/ in^3. What is the volume (in^3) of 16 oz. (or 1 lb) of gold?

4. A quart of whole milk weighs 2.3 lb. What is the density (gal) of whole milk in lb/gal?

5. An adult is recommended to limit their salt intake to no more than 2300 mg per day. If the density of salt is 10,600 mg/tsp (teaspoons), what is the volume of salt (tsp) an adult should not exceed per day?

6. A grass catcher for a mower holds 4.4 ft^3 of grass. If the density of grass is 17.4 lb/ ft^3, what is the weight (lb) of the grass in the catcher?

7. You buy a pool which is 24 ft in diameter and fills with water to 4 ft deep. The density of water is 8.345 lb/gal. How much does the water in your pool weigh (lb)? Useful information: 1 ft^3 = 7.481 gal.

8. A ream (500 sheets) of 8.5 in x 11 in standard office paper is 2 in thick, and weighs 5 lb. What is the density of the paper (oz/in^3)? Useful information: 1 lb = 16 oz.

9. If 1 lb of feathers has a density of 0.0025 g/cm^3, what is the volume of those feathers (cm^3 and ft^3)? Useful information: 1 lb = 453.6 g; 1ft^3 = 28,317 cm^3

10. A bag of concrete mix weighs 80 lb. and has a dry volume of 0.53 ft^3. If 4 liters (L) of water are added to the mix, what is the final weight (lbs.) of the concrete? Also, what is the final volume (ft^3) that the bag will fill once mixed with water? Use these numbers to calculate the density (lb/ft^3). Useful information: Density of water: 1000 g/L (grams/liter); 1 lb = 453.6 g; 1 L = 0.03531 ft^3

S4EA

Density = Weight/Volume Answers: [ ]'s

1. $D = W/V$

   $D = 2.138\ lb/1\ quart$

   $D = (2.138\ lb/quart)x(4\ gal/quart)$

   $D = 8.552\ lb/gal$

2. $W = VD$

   $W = (250\ gal)x(0.0156843\ lb/gal)$

   $W = 3.92\ lb$

3. $V = W/D$

   $V = (16\ oz)/(11.2\ oz/in^3)$

   $V = 1.43\ in^3$

4. $D = W/V$

   $D = 2.3\ lb/1\ quart$

   $D = (2.3\ lb/quart)x(4\ gal/quart)$

   $D = 9.2\ lb/gal$

5. $V = W/D$

   $V = (2300\ mg)/(10{,}600\ mg/tsp)$

   $V = 0.217\ tsp$

6. $W = VD$

   $W = (4.4\ ft^3)x(17.4\ lb/\ ft^3)$

   $W = 76.6\ lb$

7. $W = VD$

$V = \text{Height} \times \text{Area}$

$V = \text{Height} \times \pi\text{Radius}^2$ or $\text{Height} \times \pi \times (1/2\ \text{Diameter})^2$

$V = (4\ \text{ft}) \times (\pi \times (1/2 \times 24\ \text{ft})^2)$

$V = 1809.557\ \text{ft}^3$

$V = (1809.557\ \text{ft}^3) \times (7.481\ \text{gal/ft}^3)$

$V = 13{,}537.299\ \text{gal}$

$W = (13537.299\ \text{gal}) \times (8.354\ \text{lb/gal})$

$W = 113{,}091\ \text{lb}$

8. $D = W/V$

$V = (8.5\ \text{in}) \times (11\ \text{in}) \times (2\ \text{in})$

$V = 187\ \text{in}^3$

$W = (5\ \text{lb}) \times (16\ \text{oz/lb})$

$W = 80\ \text{oz}$

$D = (80\ \text{oz})/(187\ \text{in}^3)$

$D = 0.4\ \text{oz/in}^3$

9. $V = W/D$

$V = (453.6\ \text{g})/0.0025\ \text{g/cm}^3)$

$V = 181{,}440\ \text{cm}^3$

$V = (181{,}440\ \text{cm}^3)(1/28{,}317\ \text{ft}^3/\text{cm}^3)$

$V = 6.4\ \text{ft}^3)$

10. Weight:

Concrete mix: 80 lb (given)

Water:

(4 L) x (1000 g/L) x (1/453.6 lb/g) = 8.82 lb

Total:

80 lb + 8.82 lb = 88.82 lb

Volume:

Concrete mix: 0.53 ft^3 (given)

Water:

(4 L) x (0.03531 ft3/L) = 0.14 ft^3

Total:

0.53 ft^3 + 0.14 ft^3 = 0.67 ft^3

Density:

D = W/V

D = 88.82 lb/0.67 ft^3

D = 132.57 lb/ft^3

S5 LESSON: FLO SCI ENG Formats

Numbers can be expressed in three different formats:

FLO or Floating Point is the format you are familiar with.

64327.59 is an example.

Of course you know this is the same as:
$6x10^4+4x10^3+3x10^2+2x10^1+7x10^0+5x10^{-1}+9x10^{-2}$

And, $10^0 = 1$, $10^{-n} = 1/10^n$

Now we can also express this number is what is called **SCI or scientific format**

$64327.59 = 6.432759 \times 10^4$

Or in **ENG or engineering format**

$64327.59 = 64.32759 \times 10^3$

In the ENG format you will always have 10 to an exponent that is a multiple of 3. You'll see why this is when we study Prefixes in another lesson.

SCI and ENG notations are sometimes used in documentation and you can always convert from one to the other with our calculator or to FLO if the number is not too large.

However, for very large or very small numbers, SCI or ENG formats are necessary.

Frankly, if you are going to be working with very large or very small numbers you will probably be using a computer and much more powerful tools than a calculator.

It is easy to use scientific notation with a tool like Wolfram Alpha.

However, you may occasionally see them with the calculator if you multiply or divide large numbers or use the y^x key with large exponents.

$12^{21} = 4.6 \times 10^{22}$

Now multiply by 9^{13}

$1.169 \times 10^{35} = 1.169388422 \times 10^{35}$

Also, the largest exponent of 10 the calculator will accept is 99.

109^85   error

But, WA handles it just fine.

S5E

FLO SCI ENG Formats

Using your calculator, convert the following numbers to both SCI and ENG.

1. 640873.26 =

2. 2347168.002 =

3. 0.0002547 =

Using your calculator, convert the following numbers to both SCI and ENG, fixing each to the number digits past the decimal point as indicated.

4. 54178962.3 (3 digits past the decimal point) =

5. 214697.0045 (2 digits past the decimal point) =

6. 145879125 (4 digits past the decimal point) =

Using your calculator, calculate the following numbers. If you receive an error message, use Wolfram Alpha.

7. $15^{26}x2^{23}$ =

8. $26^{56}x32^{54}$ =

9. $45^{-23}x16^{-13}$ =

10. $18.45^{-56}x46.78^{-24}$ =

S5EA

FLO SCI ENG Formats Answers: [ ]’s

1. 640873.26 = [**SCI = $6.4087326x10^{5}$; ENG = $640.87326x10^{3}$**]

2. 2347168.002 = [**SCI = $2.347168002x10^{6}$; ENG = $2.347168002x10^{6}$**]

3. 0.0002547 = [**SCI = $2.547x10^{-4}$; ENG = $254.7x10^{-6}$**]

4. 54178962.3 (3 digits past the decimal point) = [**SCI = $5.418x10^{7}$; ENG = $54.179x10^{6}$**]

5. 214697.0045 (2 digits past the decimal point) = [**SCI = $2.15x10^{5}$; ENG = $214.70x10^{3}$**]

6. 145879125 (4 digits past the decimal point) = [**SCI = $1.4588x10^{8}$; ENG = $145.8791x10^{6}$**]

7. $15^{26}x2^{23}$ = [**$4.0331166x10^{34}$**]

8. $26^{56}x32^{54}$ = [**Error**
WA **26^56x32^54**
$3.28553665x10^{160}$]

9. $45^{-23}x16^{-13}$ = [**$2.101611366x10^{-54}$**]

10. $18.45^{-56}x46.78^{-24}$ =
[**Interestingly, the calculator says “0” instead of “Error”**
WA **18.45^-56x46.78^-24**
$1.053799609x10^{-111}$]

S5A LESSON: FLO SCI ENG Formats Addendum

As we learned in S5, numbers can be expressed in three different formats.

FLO or Floating Point is the format you are familiar with.
64327.59 is an example.

SCI or scientific format
$64327.59 = 6.432759 \times 10^4$

ENG or engineering format
$64327.59 = 64.32759 \times 10^3$

What we haven't learned yet is how to enter a number in a SCI or ENG format into the calculator.

It is very easy. You just use the EE Key.

To enter 6.432759×10^4:

Just enter 6.432759 and Press the EE key,

Then enter 4, and you are done.

Now you can change it into any other format, and also you can save it in memory and the recall it in this format.

Similar for ENG format:

Just enter 64.32759 and Press EE, and then enter 3

You can also enter negative numbers.

Just press the + <-> - key before you press the EE Key.

6.432759 + <-> - EE 3

Enters the negative of this number

You can also enter a negative exponent by just pressing the + <-> - key before entering the exponent.

6.432 EE + <-> - 4

Enters $6.432 \text{x} 10^{-4}$ or .00006432

Of course, you could also enter

$-6.432 \text{x} 10^{-5}$ or -.00006432

6.432 + <-> - EE 5 + <-> -